Advanced Algorithm for Optimizing the Deployment Cost of Passive Optical Networks

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Abstract. *The deployment of passive optical networks (PONs) is slow today, especially in Europe, because completely new optical infrastructures are necessary to be installed in the last-mile segments of access networks, which is always very expensive process. One of the possibilities is to design economically effective topologies and to optimize the deployment cost. This article describes the method leading to evaluate an algorithm for designing suboptimal economic solutions and topologies for PONs by focusing on optimization of constructional length of distribution networks. While the typical PON topologies are star topologies or tree-star topologies, the first part of this article introduces new sub algorithm for estimating the minimum star topology. The next section brings the evaluation of two sub algorithms for solving minimum constructional length problems. Finally, all these parts will be merged into a complex algorithm by using clusterization technique to solve optimum topologies. However, the current version of presented algorithm is purely based on mathematical theories and was implemented in Matlab environment. Therefore, it is able to design only theoretical optimum topologies without taking external conditions and real limitations into account. These real conditions will be further implemented in the future, so the algorithm could be also used for practical applications.*

Keywords

Algorithms, costs, graphs, optical distribution networks, optimization, passive optical networks.

1. Introduction

Today, the access networks in Europe and especially in the Czech Republic still consist mostly of metallic cables running xDSL and broadband systems or wireless connections using Wi-Fi and WiMAX technologies [1]. However, the transmission capacity of these technologies will be soon not sufficient for modern multimedia services, such as IPTV, HD video broadcasting and similar video or data distribution services. Therefore, it is necessary to continually increase the performance of access networks by replacing old technologies with modern last-mile FTTx networks. One of the most promising solutions for modern high-speed access networks are passive optical networks (PONs), since they can offer significantly higher transmission capacity thanks to the usage of optical fibers [2]. The major disadvantage of deploying PONs in practice is the necessity of designing and creating completely new optical distribution networks (ODNs) and infrastructures in last-mile segment, which is usually an extremely expensive process [3], [4]. Due to that, the progression and development of PONs in practice, especially in Europe and the Czech Republic, is slow today. The cost for deploying new optical infrastructure is usually the dominant part of summary expenses [5]; therefore its optimization can significantly reduce the overall cost.

This optimization of optical distribution network can be performed in several different ways and methods by focusing on minimizing the overall network length, the length of used optical fibers (cables), the total constructional length or constructional costs. However, since the construction costs and expenses necessary for installation of optical fibers and cables are the dominant parts of the summary expenses [6], the optimization should be obviously focused on minimizing the constructional length of distribution network. For this purpose, several advanced mathematical algorithms and tools especially from the field of mathematical graphs, topologies, clusterization techniques, etc. could be useful.

This article is primarily focused on the problems concerning the optimization of optical infrastructures for PONs, nevertheless, the presented techniques might be also used for other similar types of networks. The first part is related with theoretical problems of estimating minimum star topology. This problem deals closely with minimum dilation star algorithm, Steiner tree problem etc. For the purpose of finding the center of a minimum star topology a new algorithm with sufficient accuracy and low computational complexity was designed and will be presented in the first section of this article. Nevertheless, it is evident that simple star topology suffers several serious disadvantages, especially the necessity of creating one individual path for each end-point. In practice, the cost of constructions and installations of fibers is usually significantly higher compared to the cost of optical fibers [3]. That is why this sub algorithm for minimum star is used only for initial estimations, while the rest of the complex algorithm is focused on minimizing the constructional length by merging suitable optical fibers together, as it will be presented within the second section of this article.

However, typical PON networks in practice usually have multi-star and hybrid tree-star topologies rather than simple star topologies. That is why the decomposition of the entire situation into several simple star sub clusters should be performed. Thanks to that the initial sub algorithm for minimum star can be applied individually to decide the optimum star center and topology for each sub cluster. These star clusters are further joined together during the secondary process of a complex algorithm and thanks to that the multi-stage and tree-star topology can be easily formed. Another problem comes with increasing the number of end-points (subscribers), because it is necessary to decide and divide all end-points into similar groups to achieve best economic solution. Due to that, clusterization techniques should be considered and one of the simplest method, k-means, was used in the presented algorithm with several specific parameters and conditions. The final algorithm with all previous enhancements will be presented in the last section of this paper together with an example as well.

However, the current version of presented algorithm is purely based on mathematical theories and evaluations and it does not take into account real external conditions and limitations. The process of planning and designing of new networks and infrastructures is usually based on the set of given possible paths, where the optical fibers and cables could be placed (e. g. existing networks, road network, etc.), and the set of given possible points, in which the optical splitter could be located. These real conditions and limitations will be further evaluated and implemented into the future version of presented algorithm, so it will be useful for real applications as well.

1.1. Existing Methods for PON Optimization

The problem of optimizing the PON topologies and their deployment process has been recently studied and several possible methods are described in e.g. [7], [8] and [9]. The solutions presented in [7] and [8] are mathematically oriented and are based on graph theories together with density analysis and clustering techniques. The density investigation represents a different approach of a PON designing problem, which uses probabilistic and heuristic methods for obtaining optimum PON design. These techniques are useful especially in periodically structured environments containing regular positions of end-users, streets (in cities) and other conditions. That is why the results achieved by [7] are illustrated for a situation inside Manhattan (New York) area with a regular set of streets and end-users. However, the application of the algorithm proposed in [7] for less populated area, where streets and buildings can be located irregularly (typically European cities), might not result into optimal solution.

The solution presented in [8] uses clusterization method to resolve groups of similar end-points within PON topology, which are further joined together. The general conclusions presented in [8] further verify the initial assumptions, which are also used in the technique presented in this paper, about decomposing the entire network into several simple similar sub elements. Thanks to that the sub algorithm for a minimum star topology together with sub algorithm for merging the optical fibers can be easily applied for each star sub cluster. Due to that the computational complexity is relatively low. Compared to the idea described in [8], the algorithm developed and presented in this article offers more complex optimization, while it combines three sub algorithms, thus ensuring complex optimization of PON topology, optical fiber merging and necessary splitter location. The article [9] also provides a complex study of optimum PON design, however is focused more on computational techniques without using mathematic derivations. These can significantly help to improve resulting complexity of proposed algorithms and to lower necessary computational capacity. Therefore, the authors use the brute-force method in [9] to decide the optimum number of splitters or PON networks in the first step of their sub algorithm, which in case of large numbers of end-users might lead into high amount of necessary iterations and computational operations, as illustrated in the example presented in the last section of article [9]. On the other hand, the solution for obtaining minimum star topology with the optimum location of a first-stage splitter together with fiber merging sub algorithm presented in this article is not very complex and requires only minor computational resources, as will be presented in the next sections.

2. Characterization of a Minimum Star Problem

Since the typical topologies of PONs are a star and hybrid tree-star topologies, the initial characterization of a problem is finding the optimized star or multi-star topology for given set of end-points [10]. Considering the usage of Cartesian coordinates and Euclidean space, the previous condition can be mathematically expressed as:

$$min \sum_{i=1}^{n} \sqrt{(X-x_i)^2+(Y-y_i)^2}, \tag{1}$$

where the center of the star S has coordinates $[X, Y]$, while the n end-points have coordinates $[x_i, y_i]$. The sum represents the sum of Euclidean distances between the center of a star S and all n end-points.

Although the definition of a problem is easy, the general solution is analytically not possible and the process of computation the coordinates of minimum star center S is classified as NP-hard [10], [11]. Several theories and methods for its estimation [12] and iterative algorithms [13] based on assumptions were already introduced and the problem is generally solvable by using several different iterative algorithms with various accuracy and computational complexity. However, for the purpose of planning and deployment of telecommunication networks, only a rough estimation of a star center S would be typically sufficient, but on the other hand, the algorithm should be fast and simple enough. The most frequent algorithms used for finding minimum value of a function and its location, such as Nelder-Mead [14] or parabolic interpolations [15], might be too complicated for use in network planning and they typically require a large amount of iterations. Therefore, the first task was to develop a simpler algorithm for using in a process of network planning, which would be able to find the coordinates of minimum star center S with sufficient accuracy and low computational complexity.

Several mathematical evaluations of minimum dilation stars were provided in [12], nevertheless, the method proposed here is based on weighted algorithm together with Delaunay triangulation. First, the proposed algorithm was designed for calculating a center of a minimum star only in a triangle (3 end-points), however, subsequently this idea was further extended by using Delaunay triangulation and the algorithm is able to estimate this center generally for any amount of end-points. The first step represents a rough estimation based on following formulas (2). Still, this first estimation is usually very accurate and thanks to that, the algorithm needs only a limited number of iterations to reach desired accuracy.

$$\begin{aligned} X_{est} &= \frac{1}{\pi(n-2)} \cdot \sum_{i=1}^{n} x_i \alpha_i [\mathrm{rad}], \\ Y_{est} &= \frac{1}{\pi(n-2)} \cdot \sum_{i=1}^{n} y_i \alpha_i [\mathrm{rad}]. \end{aligned} \tag{2}$$

The initial estimation of a minimum star center S $[X_{est}, Y_{est}]$ is based on the sum of all n end-points coordinates weighted by appropriate inner angles α in a polygon created using Jordan curve. In the first step, the polygon is divided into triangles by applying Delaunay triangulation technique, so all inner angles can be easily determined. The second step (3) of an algorithm uses the correction of (2).

$$\begin{aligned} X_{est2} &= X_{est} + a \cdot \sum_{i,j=1;i\neq j}^{n} \Bigg[(x_i - X_{est}) \cdot \\ &\cdot \frac{\alpha_i - \frac{\pi}{3}}{\frac{4\pi}{3}} + (x_i + x_j) \cdot \frac{\alpha_i - \alpha_j}{4\pi} \Bigg], \\ Y_{est2} &= Y_{est} + a \cdot \sum_{i,j=1;i\neq j}^{n} \Bigg[(y_i - Y_{est}) \cdot \\ &\cdot \frac{\alpha_i - \frac{\pi}{3}}{\frac{4\pi}{3}} + (y_i + y_j) \cdot \frac{\alpha_i - \alpha_j}{4\pi} \Bigg]. \end{aligned} \tag{3}$$

This second step of proposed iterative algorithm is based on correction of the first estimation (2), where the sum of differentials between all inner angles and coordinates of all n end-points are calculated. After computing the second estimation of minimum star center S $[X_{est2}, Y_{est2}]$, the sum of Euclidean distances from the current star center is calculated according to the (1) and compared with the same sum calculated for previous estimation. By comparing them, the sign of term a is decided and new estimation using (3) is performed again. This process is repeated several times, until the difference between two following estimations does not reach the tolerance, which is set at the beginning of the iterative algorithm. The proposed algorithm is converging, because the term a controls the sign of sums and their values are continually decreasing with each step. The major advantage of proposed algorithm is a relatively low amount of necessary iterations; moreover these iterations are mathematically not very complex. Therefore, the whole algorithm is fast and requires only minor computational resources. However, its main disadvantage is limited accuracy, because it is evident, that the first step based on (2) critically affects all following estimations, while next steps performed according to (3) can apply only relatively small corrections. The presented algorithm was compared with Nelder-Mead, especially its accuracy and the amount of required iterations to reach the result. For that purpose, 500 of n-points polygons were randomly generated and both algorithms were used to calculate the coordinates of their minimum star centers

S. The result of accuracy of proposed algorithm is presented in the following Fig. 1, while the comparison of the amount of iterations is shown in Fig. 2.

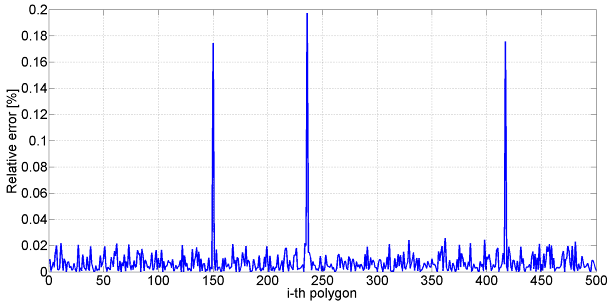

Fig. 1: The relative error of proposed angle weighted algorithm compared to Nelder-Mead.

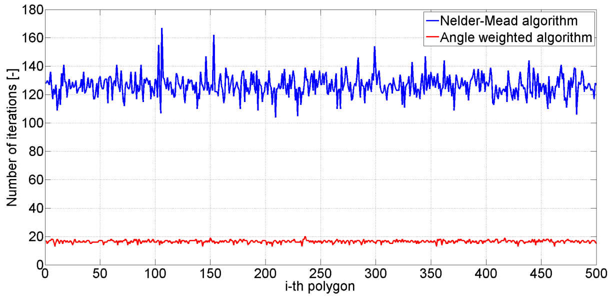

Fig. 2: The amount of necessary iterations for both algorithms.

The first graph in Fig. 1 contains the relative error (in %) of proposed angle weighted algorithm compared to Nelder-Mead algorithm. It is evident that the mean error of proposed algorithm is usually equal or better than 0,02 %; although, there are several situations, in which the error is significantly higher. Figure 2 illustrates the comparison of the amount of required iterations of both algorithms. The example shows that the Nelder-Mead algorithm required always approx. 120 iterations to estimate the coordinates of a minimum star center S for each polygon, while the maximum number of iterations required by proposed angle weighted algorithm did not exceed 20.

3. Optimum Constructional Length

Previous section described the problem and algorithm for estimating minimum star topology for n end-points. However, the simple star topology with minimized fiber length is usually not the optimal one in practice, especially in case of PONs [16]. That is why the advanced methods of clusterization and optimization of constructional length are needed. First, it is necessary to decide between simple star topology, hybrid tree-star topology and the solution based on clusterization. This problem is related with passive optical splitters and their attenuation. It can be deduced that cascading more splitters together will result into increasing their overall attenuation, because the residual loss (uniformity) and the loss of connectors or splices will be cascaded as well [2]. Another problem of star topology is the necessity of creating one individual path for each end-point, which in case of a star topology means the summary fiber length is always equal to summary constructional length. However, the expenses necessary for the installation of optical fibers and cables are usually significantly higher compared to the cost of the fibers and cables themselves. One of the key parameters for optimizing the economical effectiveness of PONs is the ratio between constructional and fiber cost for 1 meter of a proposed distribution network. Reference [5] for example calculates with the price of $26 USD for installation of 1 meter of optical fiber in highly populated city area compared to the price of $1,3 USD for 1 meter of optical fiber itself, therefore the ratio in this case is approx. 20. However, this value should be usually higher in practice, because the price for construction and installation of 1 meter of optical network can easily achieve $100 USD, making the resulting value of the ratio around 80. It is evident that minimizing constructional length of distribution network can significantly reduce the overall expenses, even when the length of optical fibers may increase. One of the possible solutions is using optical cables and thanks to that optimizing the summary constructional network length by merging several individual fibers together. The problem is schematically illustrated in Fig. 3.

Although, the summary fiber length in the second case in Fig. 3 is greater compared to the first solution, the total constructional length is shorter and therefore the second solution is economically more profitable [17].

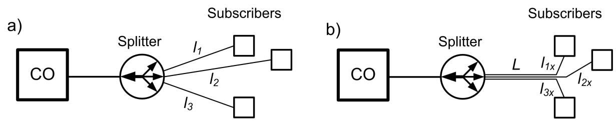

Fig. 3: The illustration of difference between minimum fiber length vs. minimum network constructional length.

3.1. Sub Algorithm for Merging Optical Fibers

For the purpose of optimizing the constructional length of optical distribution network according to the Fig. 3, following sub algorithm for merging the suitable individual fibers together was proposed.

Step 1: Obtain the coordinates of all n end-points, as well as the coordinates of central office (if the location of CO is not preassigned, skip this) and place them into a graph.

Step 2: Apply Delaunay triangulation to divide the graph into triangles to obtain all combinations of inner angles inside the polygon. This polygon is created by using Jordan curve.

Step 3: Estimate the optimum line in a graph, for which the sum of Euclidean distances from all n end-points to this projected line is minimum possible.

Step 4: Calculate the projections of all n end-points towards the line using minimum distance between the original point and the projected line. Connect these projections with their originals and connect all projections to form a line.

Step 5: Decide the center of a projected line as a mid-point (center of a minimum star) and place a splitter in it.

Step 6: Calculate the Euclidean distances between all projections and compare them with a given value. If the distance of two projections is shorter than this value, merge these points together.

Step 7: Route newly merged points with their projections and eventually repeat step 6 until all end-points are connected with the central splitter.

The algorithm was implemented in Matlab environment and was tested for various scenarios, while the results were further compared. The following example was performed for 24 end-points (users), which were randomly distributed in an area of 300×300 meters (this could simulate the situation, when 24 subscribers (households) should be connected into one PON network). The first step consists of applying Delaunay triangulation (step 2), followed by iterations and estimations of minimum line inside the whole graph containing all end-points. This line corresponds with the segment marked L in the Fig. 3b). When this minimum line is estimated, the projections of all end-points are calculated towards this line, as presented in Fig. 4 (step 5).

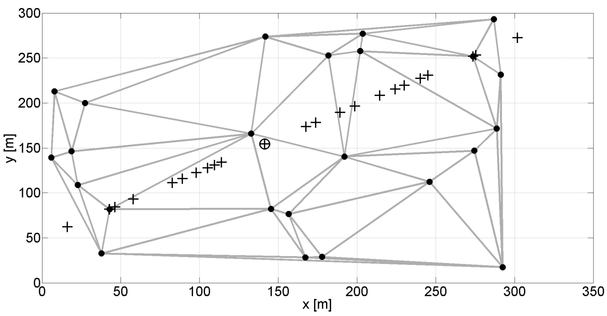

Fig. 4: The original end-points and their projections on a minimum line with the position of a central splitter marked by a circle.

The mid-point of a line is an optimum location for optical splitter and is marked by a circle. The next step performs calculations of Euclidean distances between all projections and compares them with a value, which was decided at the beginning of the optimization process. For the purpose of this example, the value was set to 30 meters. If the distance of two projections is shorter than 30 meters, these projections are merged together and the end-points are routed together. The final solution (after step 7) is presented in the Fig. 5.

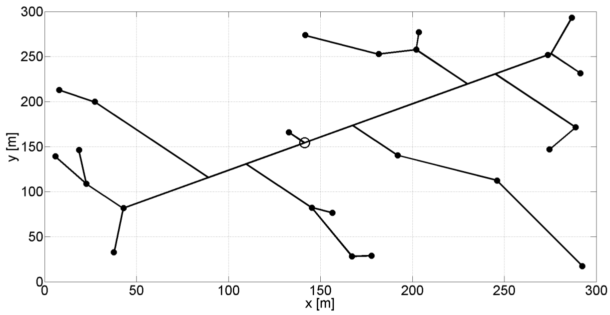

Fig. 5: The result of optimum constructional length sub algorithm.

To evaluate the previous solution of proposed algorithm, a simple comparison between topology provided in Fig. 5 and a simple one-stage star topology calculated by angle weighted algorithm was performed. This simple star topology was created using angle weighted algorithm and is presented in the following Fig. 6.

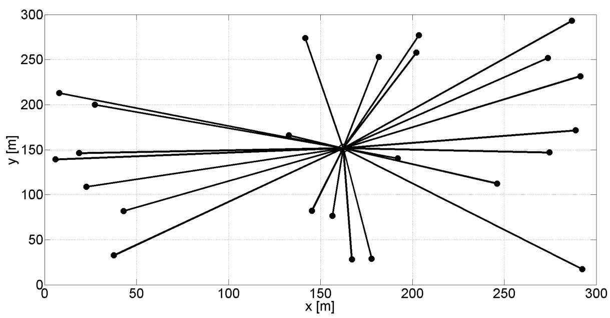

Fig. 6: A simple one-stage star topology for the previous example.

It is also possible to assume following prices of optical fibers and their installation for performing economical comparison: e.g. \$1,3 USD for 1 meter of optical fiber and \$50 USD for installation of 1 meter of optical fiber (cable). The comparison between previous scenarios in Fig. 5 and Fig. 6 is presented in Tab. 1, which contains the calculated fiber lengths, constructional lengths and economical calculations.

It is evident that the total fiber length was increased in case of the presented algorithm for merging the optical fibers; on the other hand, the constructional length was reduced and thanks to that this scenario is sig-

Tab. 1: Comparison of both previously presented scenarios.

	Algorithm for merging optical fibers (Fig. 5)	Simple one-stage star topology (Fig. 6)
Fiber length	3935, 617 meters	30071, 025 meters
Constructional length	1239, 518 meters	30071, 025 meters
Summary cost	$67092, 201 USD	$154264, 357 USD

nificantly better economical solution for the previous example. However, the algorithm can be further enhanced, as it will be presented in the next section.

3.2. Sub Algorithm for Excluding the Far-Points

The previous algorithm is useful in situations of merging only limited number of end-points in a given area, moreover these points are usually not far from each other. However, there are also situations in practice, when several end-points can be located diametrically far from the rest and the selected area contains a lot of end-points (subscribers). In this case, the previous algorithm needs to be enhanced by following steps.

Steps 1-2: Are the same as in the previous version of the algorithm.

Step 3: Estimate the center of a minimum star for all end-points and calculate the Euclidean distances between this center and each of the end-points.

Step 4: Select the most distant end-point from the center and compare it with a limit (value), which was set at the beginning of the algorithm. If the distance of this point is greater, link it with another closest end-point and exclude it from further calculations. Recalculate the center of a new minimum star for remaining points.

Step 5: Repeat the previous step 4 until all distant far-points are excluded. If no far end-points left (the distances of all remaining end-points from the center of a minimum star are lower than a given value, which was set at the beginning of an algorithm), continue with step 3 of previous merging algorithm (see previous section 3.1.).

Steps 6-9: Continue with steps 4-7 from previous merging algorithm (see previous section 3.1.).

The enhancement of the previous algorithm is useful mainly in situations with more end-points, that is why a following situation with 32 randomly generated end-points in the area of 200 × 200 meters was prepared to compare both solutions. The next Fig. 7 contains the locations of all 32 randomly generated end-points together with Delaunay triangulation (step 2).

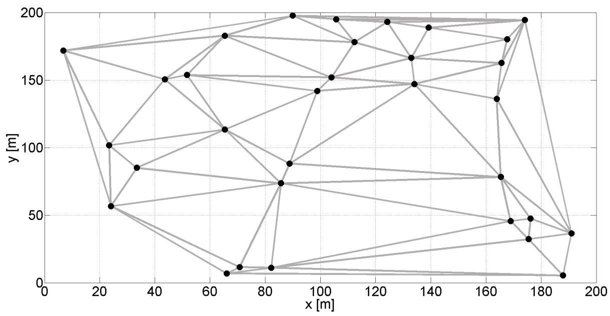

Fig. 7: An example with 32 randomly generated end-points.

First, the previous algorithm from section 3.1. was performed with the following result in Fig. 8. Then, newly enhanced algorithm version with excluding far-points from the calculation process was used and its result is presented in Fig. 9. The comparison of summary fiber length, constructional length and resulting cost of both algorithms is calculated in Tab. 2.

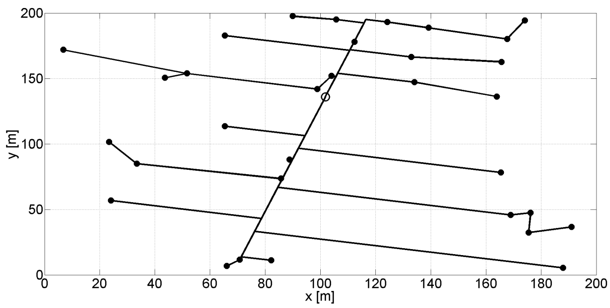

Fig. 8: The resulting network topology after using the algorithm for merging optical fibers (section 3.1.).

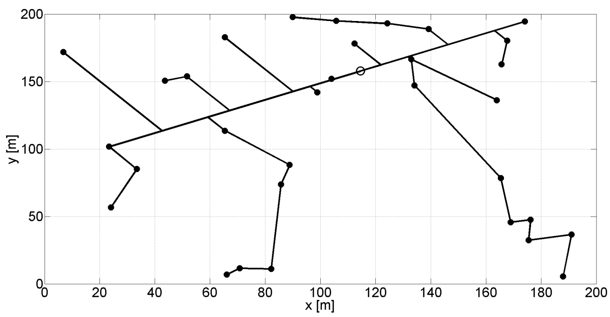

Fig. 9: The result of using enhanced algorithm with excluding far-points and merging optical fibers (section 3.2.).

Obviously, the enhanced version of proposed algorithm with a process of excluding far end-points was able to reach better solution with less summary cost. It is also evident that the value of the parameter, which is used to decide, if a specific point is far or not from the center of a minimum star (see step 4), influences the resulting solution. Therefore, the algorithm should be performed several times with various values of this pa-

Tab. 2: Comparison of both versions of the proposed algorithm (Fig. 8 and Fig. 9).

	Previous version of the algorithm for merging optical fibers (Fig. 8)	Enhanced version of the algorithm with excluding far points and merging optical fibers (Fig. 9)
Fiber length	6497, 437 meters	5008, 696 meters
Constructional length	1096, 986 meters	893, 116 meters
Summary cost	\$63295, 944 USD	\$51167, 120 USD

rameter to be able to select the best solution. However, the typical situations for real PON networks with more end-users would require the implementation of a clusterization technique to divide the end-points in several separate clusters and to create multi-stage distribution network [21], as it will be described in the following section.

4. Solving Real PONs

The PON networks in practice usually contain more end-points compared to the previous examples. The previous generation, such as EPON (Ethernet PON) or GPON (Gigabit PON), was able to cover usually 32 or 64 users (enhanced GPON version up to 128 users) [2]. Moreover, the modern versions 10GEPON [22] and XG-PON [23] can connect up to 128 or 256 users into one passive infrastructure for distances up to 20 or 40 km. That is why it is necessary to test previously described algorithm and to implement specific features for solving situations with more subscribers. Since the multi-stage topologies with more optical splitters usually offer economically better solutions [21], the usage of proper clusterization techniques is optimal. One of the most versatile and simple method is k-means technique [18]. Nevertheless, it is necessary to decide the optimum number of clusters (centroids) for using k-means method to obtain the best solution. The final algorithm with k-means clustering principle and previous algorithms can be described in following steps.

Steps 1-2: Are the same as in the previous version of sub algorithm (section 3.1.).

Step 3: Use proper clusterization algorithm, such as k-means, hierarchical clustering, distribution-based clustering (EM-clustering) or density-based clustering methods to resolve sub-clusters of end-points. Thanks to that, the topology can be divided into several multi-stage topologies by finding similar clusters and their centroids. The clusterization method should be repeated several times to ensure the best result was achieved [18].

Step 4: Apply angle weighted algorithm to find coordinates of minimum star center for each cluster from step 3. Check the total number of end-points in each cluster and calculate the Euclidean distances between the center of each cluster and its end-points. If the number of points in each cluster exceeds a given limit or the maximum Euclidean distance is greater than a specified value, increase the number of clusters and run step 3 again until both conditions are fulfilled.

Step 5: Estimate the center of a minimum star for each cluster and apply previously presented sub algorithm for excluding the far-points (chapter 3.2.). Recalculate all centers of new minimum stars for remaining points in all clusters. Repeat this step until all distant far-points in all clusters are excluded.

Step 6: Apply steps 4, 5 and 6 of previously presented sub algorithm for merging optical fibers (section 3.1.) for each cluster from the previous step 5. Place second-stage splitters in centers of projected lines in all clusters.

Step 7: Route all merged points with their projections in all clusters and eventually repeat step 6 until all end-points in all clusters are connected with their second-stage splitters.

Step 8: Route the centers (second-stage splitters) of all clusters with algorithm for merging the optical fibers. Decide the location of a central optical splitter (first-stage splitter) in a center of a projected line and connect all second-stage splitters with a central optical splitter.

This algorithm was tested during numerous randomly generated situations. The following example contains 96 randomly generated end-points in an area of 400×400 meters. First, it is necessary to set several initial conditions, mainly the maximum number of points in a single cluster and a maximum distance between the center of a cluster and all its points. In this example, the splitting ratios of the second-stage splitters (maximum number of points in clusters) were set to 16 and the maximum length of a single line in a cluster to 100 meters, respectively. First, the situation with randomly generated end-points is illustrated in the next Fig. 10.

Next, the Delaunay triangulation, k-means method and initial conditions were applied according to the steps 2-4. The resulting clusters are presented in the Fig. 11 and are distinguished by different colors.

The optimum number of clusters determined by the step 4 is 10 in this example. Now, the angle weighted algorithm is applied to determine the centers of minimum stars in all clusters and Euclidean distances are calculated to exclude all far end-points in each cluster according to the step 5. This process is followed by steps 6 and 7, in which the optimum lines and projec-

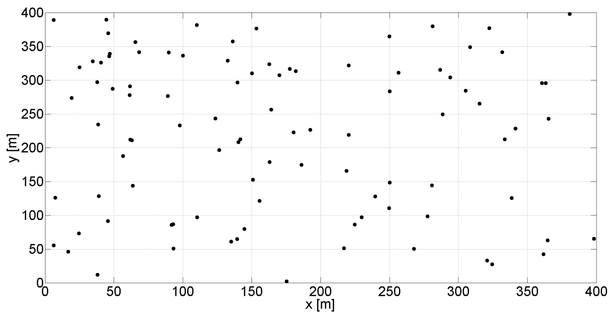

Fig. 10: The example with 96 randomly generated points.

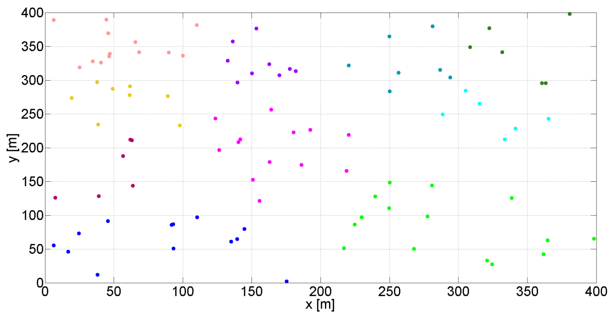

Fig. 11: All end-points were separated into clusters according to the steps 2-4 of presented algorithm.

tions are estimated, as well as, the location of second-stage splitters. The result of this part is presented in the following Fig. 12.

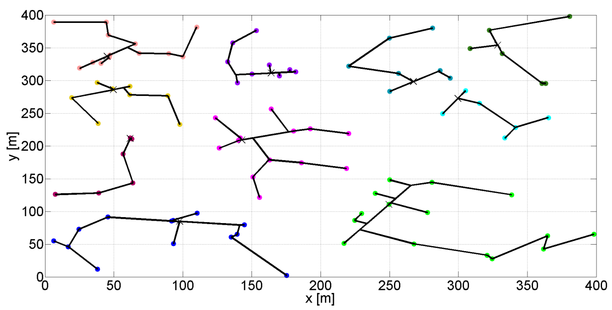

Fig. 12: Locations of second-stage splitters and optimum topologies in all clusters performed by the algorithm.

Finally, the process is completed by performing the last step of the algorithm by routing all second-stage splitters again by calculating their optimum line and projections. The algorithm also determines the location of a first-stage splitter, which is marked in the next Fig. 13 by a circle. The first-stage topology connecting the second-stage splitters is represented by a red line in Fig. 13.

To compare the effectiveness of k-means clusterization method and the whole algorithm, the previous example was also solved by using previously presented sub algorithm from section 3.2. without using any clustering method. This result is presented in Fig. 14.

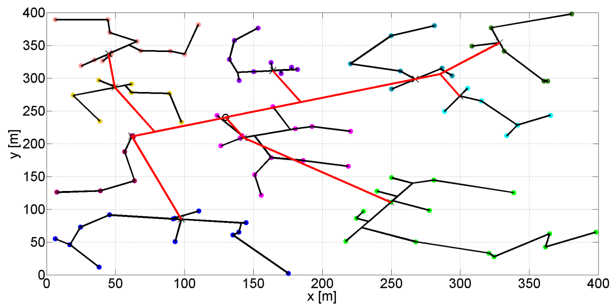

Fig. 13: The final result of presented algorithm for a given example.

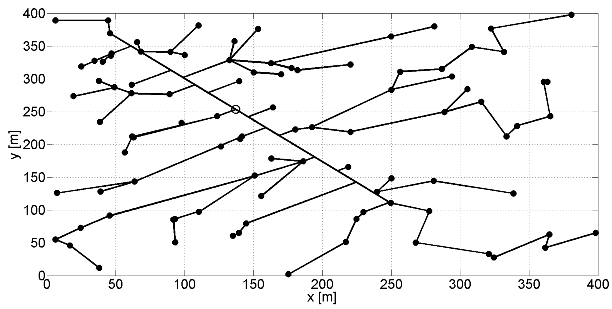

Fig. 14: Resulting topology created by the algorithm without k-means clusterization.

Tab. 3: Comparison of both previously presented scenarios.

	Complex algorithm with k-means (Fig. 13)	Previous sub algorithm without k-means (Fig. 14)
Fiber length	10662, 064 meters	28115, 770 meters
Constructional length	3692, 275 meters	3686, 892 meters
Summary cost	$198474, 442 USD	$220895, 136 USD

The difference in summary cost is significant in this example; therefore it clearly illustrates the necessity of using clusterization and process of separating the end-points into appropriate clusters, which helps mainly with the optimization of resulting topology.

5. Conclusion

The process of optimizing the optical distribution networks can significantly reduce the overall costs, as it was presented within this article. The critical parameter seems to be the ratio between constructional and fiber cost for 1 meter of a proposed network. The main goal of this paper was focused on describing useful mathematical algorithms and tools, which can be potentially used to determine optimum network topologies. First, a new algorithm for estimating the center of minimum star topology was proposed and compared with another similar method. The newly proposed angle weighted algorithm offers sufficient accuracy together with low computational complexity and thanks to that this algorithm was subsequently used in following methods for estimating optimum topologies. The next part of the article discussed the development of two possible sub algorithms for finding optimum constructional length infrastructure. Both presented algorithms were tested and compared for numerous randomly generated sets of end-points and one example was also presented within this paper. However, the algorithm should also contain some clusterization technique to be able to solve real PON topologies with the typical amount of end-points (subscribers). This enhancement was presented in the last section of this article by using k-means method. The resulting complex algorithm was again tested during randomly generated situations and one of the solutions was presented in this paper as well.

However, the current version of presented algorithm is purely based on mathematical apparatus and can design the networks only theoretically. That is why the application of presented algorithm in practice would require further implementation of real external conditions and limitations, such as following existing networks, road networks, specific locations for splitters etc. Another possible improvement of presented algorithm should result into using existing optical fiber conduits to further minimize the deployment costs. The current version of the algorithm was implemented in Matlab environment and will be further improved to take into account real conditions, so the algorithm would be useful for practical applications as wellHowever, the current version of presented algorithm is purely based on mathematical apparatus and can design the networks only theoretically. That is why the application of presented algorithm in practice would require further implementation of real external conditions and limitations, such as following existing networks, road networks, specific locations for splitters etc. Another possible improvement of presented algorithm should result into using existing optical fiber conduits to further minimize the deployment costs. The current version of the algorithm was implemented in Matlab environment and will be further improved to take into account real conditions, so the algorithm would be useful for practical applications as well.

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