ESTIMATES OF QUANTITIES IN A HALL EFFECT GEODYNAMO THEORY

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Summary Currents, resistances, dynamo constant, Hall voltage coefficient and inductances are estimated for the author's geodynamo theory incorporating the Hall Effect. It is concluded that the Hall Coefficient in the bulk liquid core of the Earth is approximately 1.512×10^{-1}, orders of magnitude greater than in normal liquid metals. The ordering effect of enormous pressure is a possible cause.

1. INTRODUCTION

de Paor [1] published a radical theory of the Earth's magnetic field and Sunspots, in which the idea of the self-excited dynamo [2] is integrated with the Hall Effect [3]. Conventional models of the geodynamo are very complicated and are simulated on supercomputers [4]. The author's treatment may be understood from two first order nonlinear ordinary differential equations, describing currents in RL circuits, resulting from mass circulation of the liquid outer core, with coupling provided by Hall voltage generators. Other theories are silent on the most prominent secular variation of the Earth's magnetic field, a cycle of declination taking about 624 years, and an associated cycle of dip. These follow directly from the author's theory [1], although the calculations need to be corrected, because of the accidental use of dip poles rather than geomagnetic poles. Relationships between these poles were presented in [5].

The author's theory was criticised [6] on two grounds. The first was a misconception of the Hall Effect as a perfect orthogonal axis power transfer mechanism and clarification was given [7], leading to correction of the classical expression for the two-species Hall Coefficient [8]. The second criticism was that the Hall Coefficient is orders of magnitude too small to support the field. This was based on invocation of the resistivity of liquid steel at 1550^0C and atmospheric pressure, and on the electron density in iron at room temperature and pressure. The pressure in the Earth's liquid core however has a mean value of 232.3×10^9 Pa, and the mean temperature is 4650K. The pressure "ordering energy" per atom is about 17eV, compared to the thermal "disordering energy" of 0.4eV per atom. So dominant is pressure that the core solidifies at the centre, and so a dramatic effect on electromagnetic properties cannot be ruled out. Nobody has yet measured the Hall Coefficient under the pressures and temperatures involved, and the matter must remain open until that can be done. Allen [6] suggested a value of -4.7×10^{-7}, whereas the author's current estimate is $+1.512 \times 10^{-1}$.

The author's theory is filled out here by estimation of values for the currents involved, resistances, dynamo and Hall voltage constants, inductances, power relationships, and the Hall Coefficient. All units used here are S.I.

2. ESTIMATION OF QUANTITIES

The basic equations of the theory [1] are

$$\begin{aligned} \frac{L_a di_a}{dt} &= -R_a i_a + k_h i_a i_d \\ \frac{L_d di_d}{dt} &= -R_d i_d + k_d \omega i_a - k_h i_a^2 \end{aligned} \qquad (1)$$

The first current, i_a, sustained by the power $k_h i_a^2 i_d$ injected from the i_d circuit, encircles the solid inner core, flowing around an annular path of depth $2r_1$, whose inner radius is that of the solid inner core, $r_1 = 1.217 \times 10^6$m [9]. The outer radius of the annulus is not quite that of the liquid core, $r_2 = 3.485 \times 10^6$m, but of a postulated transition zone at the core-mantle boundary, assigned the value

$$r_3 = r_1 + 0.9[r_2 - r_1] = 3.258 \times 10^6 \qquad (2)$$

Variation of current density with radius in the annulus is [1]

$$j(r) = \frac{1}{r^2}\left[\frac{r_3 i_a}{2(r_3 - r_1)}\right] \qquad (3)$$

The contribution of j(r) to the Earth's magnetic dipole moment (area by current) is

$$dm = \pi r^2 j(r) 2r_1 dr \qquad (4)$$

Integrating this from r_1 to r_3 gives the total dipole moment as

$$m = \pi r_1 r_3 i_a \tag{5}$$

Currently [11]

$$m = 7.835 \times 10^{22} \tag{6}$$

and so

$$i_a = 6.29 \times 10^9 \tag{7}$$

The increment of power dissipated between r and dr is

$$dP = \frac{[j(r)2r_1 dr]^2 \rho 2\pi r}{2r_1 dr} \tag{8}$$

Integrating this from r_1 to r_3 and equating to $P = R_a i_a^2$ gives

$$R_a = \frac{[\rho\pi(r_3 + r_1)]}{[2r_1(r_3 - r_1)]} \tag{9}$$

The resistivity estimate [9], $\rho = [1/6]\mathrm{x}10^{-5}$, gives

$$R_a = 4.717 \times 10^{-12} \tag{10}$$

The resistance of a frustum with base area A_b, apex area A_a and height h is

$$R = \frac{[\rho h.\ln(A_b / A_a)]}{[A_b - A_a]} \tag{11}$$

Eqn. (11) is applied to two components of R_d, firstly the resistance R_{d1} presented to current flow across the annulus, perpendicular to i_a. Here

$$\begin{aligned} A_b &= 2\pi r_3 2r_1 \\ A_a &= 2\pi r_1 2r_1 \\ h &= r_3 - r_1 \end{aligned} \tag{12}$$

$$R_{d1} = 1.073 \times 10^{-13} \tag{13}$$

i_d enters the annulus over two parallel paths, from north and south. In the transition zone hugging the core-mantle boundary, each of these is modelled by a resistive frustum with the areas

$$\begin{aligned} A_b &= \pi[r_2^{\,2} - r_3^{\,2}] \\ A_a &= 2\pi r_1\left[\sqrt{r_2^{\,2} - r_1^{\,2}} - \sqrt{r_3^{\,2} - r_1^{\,2}}\right] \end{aligned} \tag{14}$$

The effective height of this bent frustum is the mean radius $(r_2 + r_3)/2$ subtending an angle δ:

$$\begin{aligned} h &= \frac{\delta[r_2 + r_3]}{2} \\ \delta &= \arccos\left(\frac{2r_1}{[r_2 + r_3]}\right) \end{aligned} \tag{15}$$

The resistivity here is taken as twice that for the bulk liquid core, giving

$$R_{d2} = 2.174 \times 10^{-12} \tag{16}$$

The next components are cylinders in the transition region, of area πr_1^2 and physical length $r_2 - r_3$. However, not all current passes through this length: we scale by 0.75. Using the higher resistivity noted above gives

$$R_{d3} = 6.098 \times 10^{-14} \tag{17}$$

Finally, there are two cylindrical paths coming up and down onto the solid inner core, which is assigned the same resistivity as the bulk liquid core. Each of these cylinders has height r_3 and area πr_1^2:

$$R_{d4} = 5.835 \times 10^{-13} \tag{18}$$

Adding these four,

$$R_d = 2.926 \times 10^{-12} \tag{19}$$

The dynamo emf is $e_d = k_d \omega i_a$. If the angular velocity of fluid in the annulus at radius r is $\Omega(r)$, and flux density $B(r)$, the increment of dynamo emf over dr is given by the famed "Blv" law:

$$de_d = B(r).dr.r\Omega(r) \tag{20}$$

where

$$B(r) = \frac{\mu_0 r_3 i_a}{r[2(r_3 - r_1)]} \tag{21}$$

This gives

$$e_d = k_d \omega i_a = [\mu_0 r_3 / 2] i_a \,.\, mean\ of\ \Omega(r) \tag{22}$$

Taking the mean value of $\Omega(r)$ as just less than $\omega/2$ gives the estimate

$$k_d = 1 \tag{23}$$

Latest estimates of ω [10] have a mean of 0.4 degrees per year which, in S.I. units is

$$\omega = 2.212\times10^{-10} \tag{24}$$

In [1], there are the relations

$$x_1 = x_3 + \sqrt{x_3^{\,2} - 1} \tag{25}$$

$$x_3 = \frac{k_d \omega}{2\sqrt{R_a R_d}} \tag{26}$$

$$x_1 = \frac{k_h i_a}{\sqrt{R_a R_d}} \tag{27}$$

The condition for self-excitation is $x_3>1$: the actual value of x_3 is 29.78. The unknown here is k_h, and so

$$k_h = 3.516\times10^{-20} \tag{28}$$

From Eqn. (1), the equilibrium value of dynamo current i_d is

$$i_d = \frac{R_a}{k_h} = 1.341\times10^{8} \tag{29}$$

The power sustaining the field is thus

$$P_f = k_h i_a^{\,2} i_d = 1.866\times10^{8} \tag{30}$$

The power injected into the system from whatever processes drive rotation of the solid inner core, $k_d\omega i_a i_d$, exceeds this by only $5.264\text{x}10^4$ (dissipated in R_d). What these processes are the author does not know, but they are probably thermal in origin, and may come from nuclear reactions in the inner core.

In [1], k_h and Hall Coefficient R_h are related by

$$R_h = \frac{8r_1^{\,2}[r_3 - r_1]k_h}{\mu_0(r_3 + r_1)} \tag{31}$$

giving

$$R_h = 1.512\times10^{-1} \tag{32}$$

This applies in the bulk liquid core, but the value in the transition region at the core-mantle boundary is orders of magnitude smaller, so that there is no significant dipolar current circulating there.

Finally, we estimate the two inductances L_a and L_d. L_a is calculated from an expression which the author originally found on Wikipedia, but he has confirmed independently:

$$L_a = \mu_0 r\left(\ln(\frac{8r}{a}) - 2 + 0.125\right) \tag{33}$$

This is the inductance of a single turn of mean radius r with conductor radius a. The 0.125 indicates that the current density distribution is intermediate between uniform and concentrated in the skin. From eqn. (5) we have

$$r = \sqrt{r_1 r_3} = 1.991\times10^{6} \tag{34}$$

The cross-sectional area of the annular current path is $2r_1[r_3 - r_1]$ and if we equate this to an equivalent circular area πa^2 we get

$$a = \sqrt{\frac{2r_1[r_3 - r_1]}{\pi}} = 1.257\times10^{6} \tag{35}$$

Applying these figures, there results

$$L_a = 1.662 \tag{36}$$

Finally, L_d is estimated by tuning it to set the period of the very lightly damped oscillation on the (i_a, i_d) spiral equal to 624 years. The result is

$$L_d = 0.289 \tag{37}$$

3. DISCUSSION

The theory presented in [1], analysed there in dimensionless units, is filled out by estimating values for quantities in S.I. units. The most startling finding is the very large, positive Hall Coefficient. The theory can neither be firmly established nor rejected until experimental measurements of this are available—and that is likely to take many years.

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