PROMETHEE METHOD AND SENSITIVITY ANALYSIS IN THE SOFTWARE APPLICATION FOR THE SUPPORT OF DECISION-MAKING

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Summary PROMETHEE is one of methods, which fall into multi-criteria analysis (MCA). The MCA, as the name itself indicates, deals with the evaluation of particular variants according to several criteria. Developed software application (MCA8) for the support of multi-criteria decision-making was upgraded about PROMETHEE method and a graphic tool, which enables the execution of the sensitivity analysis. This analysis is used to ascertain how a given model output depends upon the input parameters. The MCA8 software application with mentioned graphic upgrade was developed for purposes of solving multi-criteria decision tasks. In the MCA8 is possible to perform sensitivity analysis by a simple form – through column graphs. We can change criteria significances (weights) in these column graphs directly and watch the changes of the order of variants immediately. The graphic upgrade of MCA8 is demonstrated on the model task, which consists in determination of the order of fifteen distribution substations (DSs). These DSs are ordered according to modernization suitability from the most suitable to the least suitable. Results are different orders of these DSs depending on different criteria weights. The MCA8 software application enables easy computation of six methods of MCA. These methods are IPA, WSA, TOPSIS, CDA, AGREPREF and PROMETHEE. The model task is solved by new method - PROMETHEE, which is also theoretically described. The MCA8 runs on Windows XP with NET framework 2.0. Enter data can be imported and results of analysis, including graphs, can be exported from/to MS Excel.

1. INTRODUCTION

Multi-criteria decision-making is a modelling of complicated situations, in which set of variants and criteria is defined. Variants are evaluated according to these criteria. Multi-criteria analysis (MCA) is determined to solving complicated decision-making tasks. The term “variant” designates each of the solutions of the selection report. The “criterion” is a property that is being evaluated with the given variant. To each criterion such as weight is assigned that expresses the importance of particular criteria with regard to the others. Most of MCA methods require cardinal information concerning relative importance of criteria, which can be expressed by mean vector of criteria weights (value of each weight is from interval <0;1>). The aim of MCA may be to find a set of “good” variants, determination of the best variant, or arrangement of all variants. The MCA8 software application (described in following chapter) enables easy computation of six methods of MCA.

2. THE MCA8 SOFTWARE APPLICATION

The MCA8 software application is determined to the support of multi-criteria decision-making. This application enables easy computation of following methods of multi-criteria analysis:

- WSA (Weighted Sum Approach)
- IPA (Ideal Points Analysis)
- TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)
- AGREPREF (Aggregation Preferences)
- CDA (Concordance Discordance Analysis)
- PROMETHEE (Preference Ranking Organistion Method for Enrichment Evaluations)

This software application was developed at the Departments of Electrical Power Engineering and Computer Science at VŠB - TU Ostrava. The above mentioned methods have been implemented to the MCA8 on the basis of the type of decision-making tasks solved in the field of electrical power engineering. The MCA8 contains a graphic upgrade, which enables us to perform sensitivity analysis by a simple form – through column graphs. We can change a criteria significances (weights) in this column graphs and watch the changes of the order of variants immediately. This graphic upgrade is demonstrated on the model task, solved by PROMETHEE method, which is theoretically described in the following chapter. The MCA8 runs on Windows XP with .NET framework 2.0.

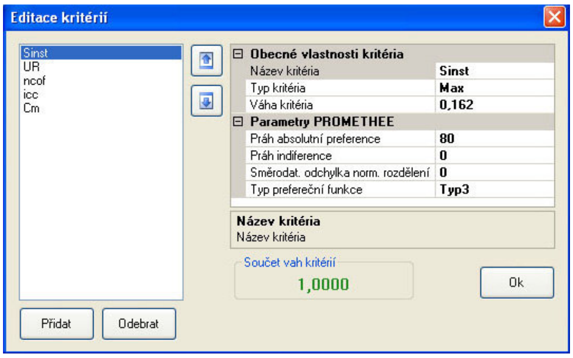

Fig. 1. MCA8 window with criteria editor

Figure 1 shows the MCA8 window for criteria editing. After each editing operation the solution of multi-criteria analysis is recomputed (button “Aktualizovat řešení”). Enter data can be imported and results of analysis, including graphs, can be exported from/to MS Excel (version 2003). [1]

3. PROMETHEE METHOD

PROMETHEE method fall into multi-criteria analysis (MCA). Basic for this method is pair comparing of variants, step by step from viewpoints of all criterions. Result is formulation intensity of preferential between pairs of variants, on valuation from viewpoint of all criterions. The coefficients $P_i(a_r,a_s)$, express intensity of preference variant a_r in relation to variant a_s from viewpoint of criterion f_i, are speculated in interval <0,1>. It’s evident, that his value will depend on difference of criterion values:

$$d_i = f_i(a_r) - f_i(a_s) \quad (1)$$

Higher difference means higher preference intensity. When we evaluate pairs of variant by some of criterion, there can come on next options:

- Variants a_r, a_s are from viewpoint of this criterion indifferent.
- Variant a_r is preferred before variant a_s.
- Variant a_s is preferred before variant a_r.

If we want evaluate pairs of criterions and at the same time evaluating intensity preference of variant a_r before a_s, we start function $Q(d_i)$ - generalized criterion:

$$Q(d_i) = P_i(a_r,a_s), \ for\, d_i >= 0 \quad (2)$$

$$Q(d_i) = P_i(a_s,a_r), \ for\, d_i =< 0 \quad (3)$$

PROMETHEE methods offer six types of preferential functions. For every of evaluated criterions user will choose any type of preferential function and parameters of this function. Basic parameters are:

- Threshold of indifference q, it is upper limit for difference evaluate of two variants and these variants are still indifferent.
- Threshold of absolute preferential p, it is lower limit for difference evaluate of two variants and one variant is preferred before second variant.
- Mean-root-square-error from normal sorting σ

Definition of generalized criterions:

1. type:

$$Q(d_i) = 0, \ for\, d_i = 0 \quad (4)$$

$$Q(d_i) = 1, \ for\, |d_i| > 0 \quad (5)$$

This function doesn’t need enter of any parameter.

2. type:

$$Q(d_i) = 0, \ for\, |d_i| =< q \quad (6)$$

$$Q(d_i) = 1, \ for\, |d_i| > q \quad (7)$$

This function need enter threshold of indifference.

3. type:

$$Q(d_i) = \frac{|d_i|}{p}, \ for\, |d_i| =< p \quad (8)$$

$$Q(d_i) = 1, \ for\, |d_i| > p \quad (9)$$

This function need enter threshold of preference.

4. type:

$$Q(d_i) = 0, \ for\, |d_i| =< q \quad (10)$$

$$Q(d_i) = \frac{1}{2}, \ for\, q < |d_i| =< p \quad (11)$$

$$Q(d_i) = 1, \ for\, |d_i| > p \quad (12)$$

This function need enter threshold of indifference and preference.

5. type:

$$Q(d_i) = 0, \ for\, |d_i| =< q \quad (13)$$

$$Q(d_i) = \frac{|d_i| - q}{p - q}, \ for\, q < |d_i| =< p \quad (14)$$

$$Q(d_i) = 1, \ for\, |d_i| > p \quad (15)$$

This function need enter threshold of indifference and preference.

6. type:

$$Q(d_i) = 1 - \exp\left(\frac{-d_i^2}{2 \times \sigma^2}\right) \quad (16)$$

This is Gauss’s function, whose value is getting near to value 1, when the difference is growing. Parameter σ is a mean-root-square-error from normal sorting.

When we get a presumption that for every couple variants was quantifying, on basic of chosen preferential function, preferential intensity, than we can calculate global preferential indexes:

$$P(a_r,a_s) = \sum_{i=1}^{k} v_i \times P_i(a_r,a_s); \quad r,s = 1,2,...,p \quad (17)$$

where v_i are weights of criterions $(v_1+v_2+...+v_K = 1)$. Values $P(a_r,a_s)$ are from interval <0,1> and we can interpret it like global degree variant preferential a_r under a_s. When we can get final relation, there are for every variant calculated positive stream and negative stream:

$$F^+(a_r) = \sum_{s=1}^{p} \frac{P(a_r,a_s)}{(p-1)}; \quad r = 1,2,...,p \quad (18)$$

$$F^-(a_r) = \sum_{s=1}^{p} \frac{P(a_s,a_r)}{(p-1)}; \quad r = 1,2,...,p \quad (19)$$

Positive stream formulated preference intensity variant a_r according to all other variants, negative stream formulated preference intensity all variants in face of variant a_r.

We can get final sorting of variants, when variants are sorted by so-called clear stream $F(a_r)$, when is defined like difference positive stream and negative stream:

$$F(a_r) = F^+(a_r) - F^-(a_r); \quad r = 1,2,...,p \quad (20)$$

The best variant is a variant with maximum value of clear stream. We rank the variants according to this clear stream. [2]

4. MODEL DECISION-MAKING TASK

The possibilities of MCA8 software application are demonstrated on the following model decision-making task, which consists in determination of the order of fifteen distribution substations (DSs). These DSs are ordered according to modernization suitability from the most suitable to the least suitable on the basis of five selected criteria. [3] Table 1 shows the valuation of all select DSs according to these criteria. These DSs are situated in the North-Moravian region in Czech Republic.

Tab. 1. Distribution substations (DSs) evaluated according to particular criteria

Distribution substation	Criterion				
	S_{inst} [kVA]	UR [-]	n_{cof} [-]	i_{cc} [-]	C_m [1000*EUR]
DS-1	400	1	4	3	15,5
DS-2	250	2	3	2	12,5
DS-3	160	2	3	2	17
DS-4	250	3	2	3	22
DS-5	160	2	3	1	9,5
DS-6	630	3	5	4	41,5
DS-7	250	1	3	2	10
DS-8	630	2	6	4	39
DS-9	400	2	5	4	37,5
DS-10	250	1	4	2	35
DS-11	400	3	4	5	31
DS-12	250	3	3	1	19,5
DS-13	250	1	4	3	28
DS-14	400	2	6	4	31,5
DS-15	400	2	4	3	34,5

S_{inst} [kVA] - Installed capacity of DS
UR [-] - Usage rate of DS at the control of electrical power network {1;2;3}
UR = 1 ... high usage rate (city centre)
UR = 2 ... middle usage rate (city)
UR = 3 ... low usage rate (outskirts)
n_{cof} [-] - Number of controlled outgoing feeders of DS
i_{cc} [-] - Importance of connected customers to DS {1;2;3;4;5}
i_{cc} = 1 ... minimal importance (out of city)
i_{cc} = 2 ... small importance (outskirts)
i_{cc} = 3 ... middle importance (city and small business men)
i_{cc} = 4 ... high importance (city centre and wholesale customers)
i_{cc} = 5 ... very high importance (great wholesale customers)
C_m [1000*EUR] - Cost of modernization

The task is solved by PROMETHEE method. Table 2 shows all criteria information needful for computation through this method. Table contains two different weight vectors (different criteria weights), A and B, for the execution of sensitivity analysis. This analysis is a tool, which is used to ascertain how the output of a given model depends upon the input parameters (how the order of DSs depends upon the criteria weights).

Tab. 2. Criteria parameters needful for computation through PROMETHEE method

Criterion		S_{inst}	UR	n_{cof}	i_{cc}	C_m
Criterion type		max.	min.	max.	max.	min.
Pref. function type		3.	3.	4.	3.	6.
Parameter	q	–	–	1	–	–
	p	80	1	2	1	–
	σ	–	–	–	–	8
Criterion weight (A)		0,162	0,203	0,17	0,271	0,194
Criterion weight (B)		0,141	0,176	0,148	0,235	0,3

The result of PROMETHEE method for basic "A" weight vector is displayed on the right side of MCA8 window (see Figure 2). Variants (DSs) are ordered on the basis of computed values of Clear stream F_A.

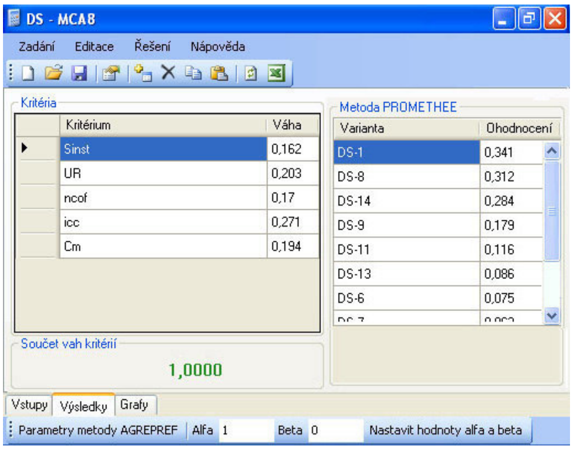

Fig. 2. MCA8 window with result of PROMETHEE method for basic "A" weight vector

The MCA8 contains a graphic upgrade (see Figure 3), which enables to perform the sensitivity analysis by a simple form – through column graphs. Figure 3 shows the MCA8 window with two column graphs.

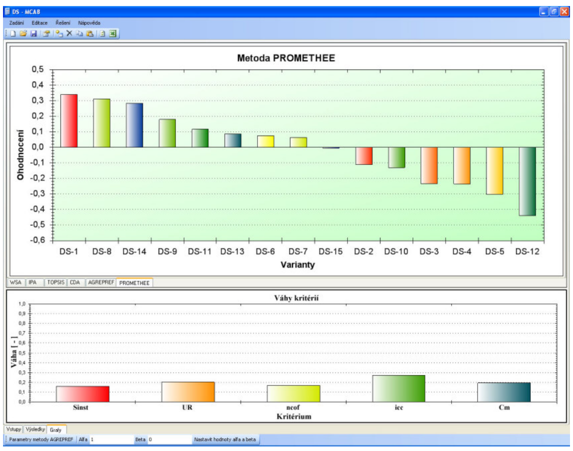

Fig. 3. The graphic output for basic "A" weight vector

The top graph displays final valuation of DSs (their Clear streams) for weights adjusted in the bottom graph. These weights correspond to the basic „A" weight vector. In the bottom graph we can change the arbitrary criterion weight (we can change the column depth by the help of mouse pointer) and watch the changes of the order of DSs in the top graph immediately. The other weights are automatically recomputed. Sum of all weights is equal „1". Figure 4 shows changed order of DSs for a new „B" weight vector.

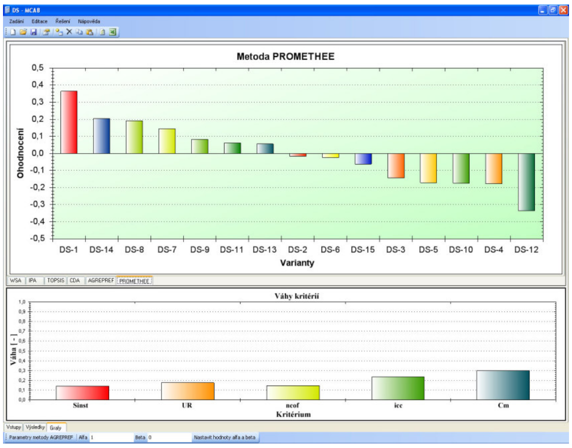

Fig. 4. The graphic output for new "B" weight vector

Tab. 3. Different orders of DSs in dependance on various weight vectors

order of DSs	weight vector (criteria weights) (A) Distribu-tion substation	(A) Clear stream $F_{(A)}$ [-]	(B) Distribu-tion substation	(B) Clear stream $F_{(B)}$ [-]
1.	DS-1	0,341	DS-1	0,364
2.	DS-8	0,312	DS-14	0,204
3.	DS-14	0,284	DS-8	0,192
4.	DS-9	0,179	DS-7	0,144
5.	DS-11	0,116	DS-9	0,083
6.	DS-13	0,086	DS-11	0,061
7.	DS-6	0,075	DS-13	0,055
8.	DS-7	0,063	DS-2	-0,016
9.	DS-15	-0,004	DS-6	-0,024
10.	DS-2	-0,111	DS-15	-0,062
11.	DS-10	-0,13	DS-3	-0,144
12.	DS-3	-0,234	DS-5	-0,171
13.	DS-4	-0,236	DS-10	-0,174
14.	DS-5	-0,303	DS-4	-0,177
15.	DS-12	-0,439	DS-12	-0,336

Table 3 shows the results of analysis - two different orders of DSs ordered according to modernization suitability. The most suitable is distribution substation DS-1 (for both weight vector).

As it is clear from the analysis that distribution substations DS-1, DS-8 and DS-14 are always on the first three places of these orders for increasing the weight of criterion „Cost of modernization" from value 0,194 to value 0,3. Whereas the order of further DSs is changing in dependence on this (except DS-12 that is the least suitable for modernization in both cases). The changes of other weights are less significant.

5. CONCLUSION

The MCA8 software application was described in this paper. This application is determined to the support of multi-criteria decision-making in the field of electrical power engineering. The MCA8 enables easy computation of six methods of multi-criteria analysis and includes a graphic upgrade to perform the sensitivity analysis by a simple form. The model multi-criteria decision-making task was solved by new method (PROMETHEE) and results were submited to the sensitivity analysis. This task consisted in determination of the orders of fifteen distribution substations (DSs) in dependance on two various weight vectors (criteria weights). These DSs were ordered according to modernization suitability from the most suitable to the least suitable on the basis of five selected criteria. From the results of sensitivity analysis we can obtain an idea of possible behaviour of solution (the orders of DSs) in consequence of the changes of inputs (criteria weights).

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