

## ON THE DAMPER CAGE BARS' CURRENTS CALCULATION FOR SALIENT POLE LARGE SYNCHRONOUS MACHINES

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**Summary** The damper cage bars' currents occur only when the synchronous machine operates in a dynamic regime and its rotor speed differs from the stator field speed. Basically there are two ways of calculating the damper cage currents, by using the machine equivalent circuit or by employing a 2D or 3D finite element method (FEM) analysis. In this paper are discussed two methods to calculate the damper cage currents, one based on a coupled field-circuit approach when all the machine dimensions and winding should be known and another based on DC-decay tests conducted with the rotor on  $d$ , respectively  $q$  axis when all the transient parameters and time constants are obtained. Both methods are quite simple and offer an acceptable accuracy.

### 1. INTRODUCTION

Obtaining parameters of a synchronous machine is and will be an important step of the designing procedure. The study of the machine transient is also requiring a good estimation of the parameters. An appropriate determination allows knowing precisely the machine behaviour in different operating regimes.

There are many proposed methods to obtain the parameters from tests [1, 2], identification and estimating methods [3, 4] while the analytical approaches are lately neglected; it is one of the reasons why this paper tries to prove that some analytical approaches can give satisfactory results depending on the considered assumptions.

The most important aspect in computing the damper winding is to obtain the current distribution in each damper bar.

In this paper two methods for calculating the parameters and the damper bar currents are presented. The first method, a field-circuit approach, is based on grouping two symmetrically located bars into loops. A sinusoidal distribution of currents in the damper winding is assumed [5]. An equivalent current and voltage is defined and the damper winding will be reduced to an equivalent winding on both axes, with an equivalent resistance and inductance. The rotor parameters will be referred to the stator side.

In the second method discussed here a DC decay test will be applied with the rotor on  $d$ , respectively on  $q$ -axis, at standstill where is no coupling between the axes [6]. The time constants and transient parameters are obtained for both axes.

### 2. FIELD-CIRCUIT APPROACH

In order to obtain the damper bar current distributions, first all the parameters must be known.

The number of parameters that must be calculated depends on the number of pole bars.

In Fig.1 is presented the way the bars are grouped into loops for  $d$ - and  $q$ -axis respectively for the case with 9 bars per pole. Each bar together with another bar which lies symmetrically on the  $d$ - respective  $q$ -axis constitutes a single turn separate damper winding, named loop in this paper.

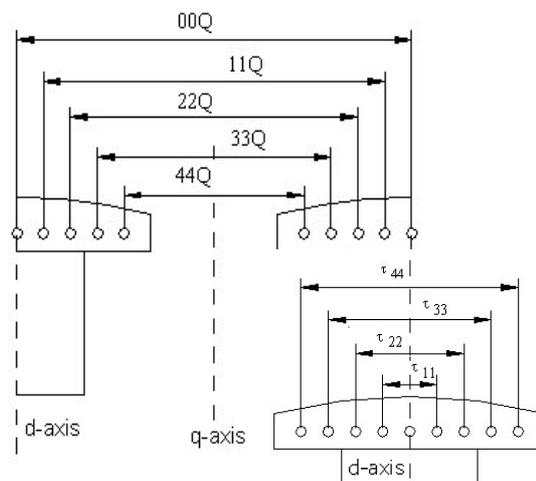


Fig.1. Defining the single turn damper windings (loops) on the  $d$ - and  $q$ -axis

The air gap field distribution in the machine can be obtained numerically by finite elements method but also is possible to be given analytically. If the analytical approach is considered to obtain the form factors that are necessary to calculate the self and mutual inductances, a permeance distribution curve along a pole pitch can be defined based on the machine geometrical dimensions (minimum and maximum air-gap length, pole pitch, stator inner

diameter, pole arc) that approximates more or less precisely the real air gap permeance or magnetic field distribution.

In Fig. 2, two permeance distribution curves are presented; the dotted one is a result of a 2D-FEM computation while the other was obtained analytically.

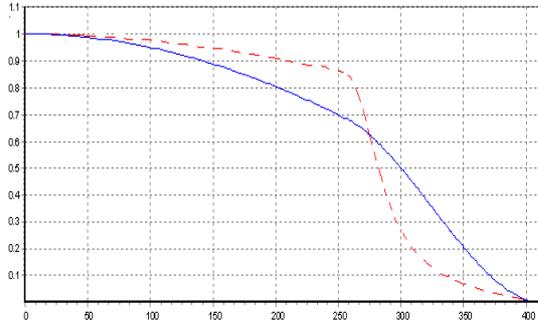


Fig.2. Permeance distribution curves

--- numerically, — analytically

The air gap flux given analytically has the following expression:

$$\phi = \iint B \cdot dl \cdot dx \quad (1)$$

$$B_x = \frac{d \phi_x}{dx} = \frac{N}{l_i} \cdot I_R \cdot \frac{d}{dx} \lambda_x \quad (2)$$

where  $N$  is the number of turns per pole of a given rotor circuit,  $l_i$  is the machine ideal length and  $I_R$  is the current from a rotor circuit (field winding or a loop). The permeance associated for an elemental length ( $dx$ ) of the pole pitch:

$$d\lambda_x = \frac{\lambda}{\lambda_0} \cdot d\lambda_0 = \frac{\lambda}{\lambda_0} \cdot \mu_0 \cdot \frac{l_i}{\delta_i} dx \quad (3)$$

The ratio  $\lambda/\lambda_0$  was introduced because the permeance distribution through the air gap is not constant. A function can be defined as the ratio  $\lambda/\lambda_0$  for the whole pole pitch, function that has two terms: a sinusoidal function for the length of the pole tip ( $f_1$ ) and a third degree function for the inter-pole region ( $f_2$ ). The permeance distribution curve is symmetrical in respect to the pole axis, than the  $\lambda/\lambda_0$  ratio:

$$\frac{\lambda}{\lambda_0} = f_1 = \cos\left(A \cdot \frac{\pi \cdot x}{\tau_p}\right), \text{ for } : x \in \left(0, \frac{b_{fa}}{2}\right) \quad (4)$$

$$\frac{\lambda}{\lambda_0} = f_2 = B\left(x - \frac{\tau_p}{2}\right)^3 + C\left(x - \frac{\tau_p}{2}\right)^2, \text{ for } : x \in \left(\frac{b_{fa}}{2}, \frac{\tau_p}{2}\right) \quad (5)$$

Constants A, B and C have to be calculated for a given machine considering the specific values of the air-gap permeance. The self and mutual inductances must be calculated separately for each rotor circuit. First, few relations will be presented in order to obtain the field winding self and mutual inductances.

The field winding self inductance is composed by two terms; the main magnetizing inductance ( $L_{hF}$ ) and the leakage inductance ( $L_{\sigma F}$ ):

$$L_F = L_{hF} + L_{\sigma F}$$

The flux produced by a pole, if only the field winding is fed, comes as:

$$\Phi_{hF} = 2 \cdot \tau_p \cdot l_i \cdot \int_0^{\frac{\tau_p}{2}} B_x dx \quad (6)$$

The total flux produced by the field winding:

$$\Psi_{hF} = N_F \cdot \phi_{hF} = L_{hF} \cdot I_F \quad (7)$$

Where the magnetizing inductance is

$$L_{hF} = \lambda \cdot N_F^2 \cdot C_{hF} \quad (8)$$

with  $\lambda$  being the permeance calculated for a constant air gap machine and  $C_{hF}$  is a form factor calculated based on relation (4),  $D_i$  being the air-gap interior stator diameter:

$$\lambda = \frac{2 \cdot \mu_0 \cdot l_i \cdot D_i}{\pi \cdot p^2 \cdot \delta_i} \quad (9)$$

$$C_{hF} = \frac{\pi^2}{8} \cdot \frac{2}{\tau_p} \cdot \int_0^{\frac{\tau_p}{2}} \frac{\lambda}{\lambda_0} \cdot dx \quad (10)$$

The coupling between the field winding and the stator phase winding exists for the fundamental of the flux on the d-axis. The mutual stator-field winding inductance is:

$$L_{SF} = N_S \cdot k_{wS} \cdot \lambda \cdot N_F \cdot C_{FS} \quad (11)$$

with  $k_{wS}$  being the stator winding factor and  $C_{FS}$  the form factor, [7].

$$C_{FS} = \frac{\pi}{\tau_p} \cdot \int_0^{\frac{\tau_p}{2}} \frac{\lambda}{\lambda_0} \cdot \cos\left(\frac{\pi \cdot x}{\tau_p}\right) dx \quad (12)$$

One of the methods which is proposed in this paper calculates the damper bar currents by considering a sinusoidal distribution in loops. This analytical calculation reduces the damper winding to two equivalent windings, one with its magnetic axis in line with the direct- and one with the quadrature axis. The method is applicable when the damper

windings have the bars with the same chemical composition and cross section. The flux density has a pulsating sinusoidal distribution in space with its magnetic axis in the pole axis. The currents induce in each pair of bars located at the same distance from the pole axis will have the same magnitude and an opposite flowing direction.

The definition of a loop (Fig.1) remains valid in this situation but will be assumed that the currents with the maximum magnitude are induced in the loop located at a half pole pitch from the flux wave axis.

The currents from each loop positioned at the electric angle  $\theta/2$  from the flux wave axis will have a magnitude equal to  $\sin(\theta/2)$  from the maximum magnitude. If  $I_{Dd}$  is the direct axis current with the maximum magnitude, then a current from the  $n^{\text{th}}$  loop will be:

$$I_{nD} = I_{Dd} \cdot \sin\left(\frac{\theta_n}{2}\right) \quad (13)$$

An important simplifying assumption introduced in this approach is that the **mmf** produced by any loop has a rectangular distribution. This assumption is not entirely true at a machine with salient poles and a variable air gap along the pole tip, but only in this way the damper winding can be reduced to an equivalent one circuit winding. Thus, the magnitude of the fundamental direct axis MMF component corresponding to the  $n^{\text{th}}$  loop will be:

$$F_{nd\max} = \frac{4}{\pi} I_{nD} \cdot \sin\left(\frac{\theta_n}{2}\right) \quad (14)$$

and the resultant space fundamental **mmf** of all the loops is:

$$F_{Dd} = \frac{4}{\pi} I_{Dd} \cdot \sum_{k=1}^{N_{Dd}} \sin^2\left(\frac{\theta_k}{2}\right) \quad (15)$$

where  $N_{Dd}$  is the number of loops. Based on the number of bars per pole, the following discussion must be made:

$$\text{if } n_b = 2 \cdot N_{Dd}, \text{ then } \theta_n = (2n-1) \cdot \alpha_b \quad (16)$$

$$\text{if } n_b = 2 \cdot N_{Dd} + 1, \text{ then } \theta_n = 2n \cdot \alpha_b \quad (17)$$

$\alpha_b$  is the angle between two bars and

$$\theta_n = \frac{\tau_{nn}}{\tau_p} \cdot \pi \quad (18)$$

Considering this, for both odd and even number of bars per pole, the resultant space fundamental **mmf** on d-axis can be expressed as:

$$F_{Dd} = \frac{1}{\pi} \cdot n_b \cdot I_{Dd} \cdot (1 - k_b) \quad (19)$$

$$k_b = 1 - \frac{\sin(n_b \cdot \alpha_b)}{n_b \cdot \sin(\alpha_b)} \quad (20)$$

For the q-axis, the resultant space fundamental **mmf** is obtained by a similar reasoning.

$$F_{Dq} = \frac{1}{\pi} \cdot n_b \cdot I_{Dq} \cdot (1 + k_b) \quad (21)$$

In order to refer the rotor parameters to the stator side, the fundamental armature **mmf** and the equivalent damper winding **mmf** yield equal air gap fundamental flux density distributions when their magnetic axis are aligned.

$$B_{ad} = B_{Dd} \quad (22)$$

$B_{ad}$  is the flux density produced by an equivalent damper winding current circulating in the stator phase ( $I_{Dr}$ ).

$C_{ad}$  and  $C_{aq}$  are the form factors [7].

$$B_{ad} = \frac{4}{\pi} \cdot \frac{N_s \cdot k_{ws}}{p} \cdot I_{Dr} \cdot \frac{\mu_0}{\delta_i} \cdot C_{ad} \quad (23)$$

while  $B_{Dd}$  is the flux density produced by the equivalent damper winding current  $I_{Dd}$ .

$$B_{Dd} = \frac{n_b}{\pi} \cdot I_{Dd} \cdot (1 - k_b) \cdot \frac{\mu_0}{\delta_i} \cdot C_{ad} \quad (24)$$

From rel. (35) the ratio of actual and referred to the stator side currents will be obtained:

$$\frac{I_{Dd}}{I_{Dr}} = \frac{4 \cdot N_s \cdot k_{ws}}{p \cdot n_b \cdot (1 - k_b)} = k_{iD} \quad (25)$$

$$\frac{I_{Dq}}{I_{Qr}} = \frac{4 \cdot N_s \cdot k_{ws}}{p \cdot n_b \cdot (1 + k_b)} = k_{iQ} \quad (26)$$

The expressions of the referred parameters are [7]:

- the equivalent damper winding leakage inductance referred to the stator, with  $L_{d\sigma}$  being the equivalent single bar direct axis leakage:

$$L_{D\sigma r} = \frac{8 \cdot (N_s \cdot k_{ws})^2}{p \cdot n_b \cdot (1 - k_b)} \cdot L_{d\sigma} \quad (27)$$

- the mutual inductance between the stator phase and the equivalent damper winding referred to stator:

$$L_{SDr} = \frac{(N_s \cdot k_{ws})^2}{p} \cdot \lambda \cdot C_{ad} \quad (28)$$

$$L_{SQr} = \frac{(N_s \cdot k_{ws})^2}{p} \cdot \lambda \cdot C_{aq} \quad (29)$$

- the referred stator-field, field-equivalent damper winding and stator-direct axis equivalent damper winding mutual inductances have the same value (28).

$$L_{SDr} = L_{SFr} = L_{FDr} \quad (30)$$

- the equivalent damper winding resistance referred to stator:

$$R_{Ddr} = 8 \cdot \frac{(N_s \cdot k_{ws})^2}{p \cdot n_b \cdot (1 - k_b)} \cdot r_{bed} \quad (31)$$

- the referred field winding leakage inductance and resistance:

$$L_{F\sigma} = L_{\sigma F} \cdot \left( \frac{4}{\pi} \cdot \frac{C_{ad}}{C_{hF}} \cdot \frac{N_s \cdot k_{ws}}{p \cdot N_F} \right)^2 \quad (32)$$

$$R_{Fr} = R_F \cdot \left( \frac{4}{\pi} \cdot \frac{C_{ad}}{C_{hF}} \cdot \frac{N_s \cdot k_{ws}}{p \cdot N_F} \right)^2 \quad (33)$$

Also the ration between the actual and referred field currents is:

$$\frac{I_F}{I_{Fr}} = \frac{4}{\pi} \cdot \frac{C_{ad}}{C_{hF}} \cdot \frac{N_s \cdot k_{ws}}{p \cdot N_F} \quad (34)$$

With the above presented relations, all the rotor parameters were referred to the stator side. Thus, if a *p.u.* system is adopted, than only the stator reference system will be used. Independently from the number of damper bars per pole, a voltage equation system with 5 equation and 5 unknown quantities is obtained at a given slip (26); the unknown quantities being the stator d- and q-axis currents, the field current and the equivalent damper winding d- and q-axis currents.

The sample machine considered as example is a three phase one with rated power of 7.5 MVA, two pole pairs and 9 bars per poles. After obtaining all the machine parameters, the obtained voltage systems, with 12 respectively with 5 voltage equations, were solved for a given slip ( $s=1$ ) and voltage ( $u=0.186$  p.u. corresponding to  $U=12.656 \cdot 10^3 V$  rated phase voltage). The considered operating regime was an asynchronous starting with short-circuited field winding. In Fig.2., the actual values of damper bar currents are presented, obtained with the above presented method and respectively obtained via a numerical approach [8].

As can be seen, with only one exception the proposed method gives closed values to that calculated via the numerical approach.

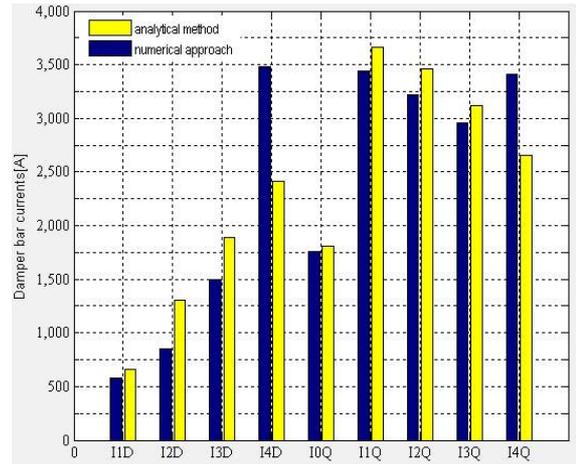


Fig.2. Actual damper bar current values from analytical and numerical methods

### 3. STANDSTILL DC DECAY TEST

The standstill DC decay test consists in cutting off the constant DC supply, allowing the current from the coils to reach zero and keeping the rotor in a fix position. Initially, the switch  $K_2$  is closed,  $K_1$  is open. The DC source is supplying two phases of the stator with a known current  $i_0$ , whereas the excitation is short-circuited. Once the switch  $K_2$  is opened and  $K_1$  is closed simultaneously, the decreasing of the current is acquired and using a dedicated program, the reactances and time constants of the synchronous machine can be obtained. A low power salient pole synchronous machine was tested in the Electric Machines Laboratory at EPFL, Switzerland.

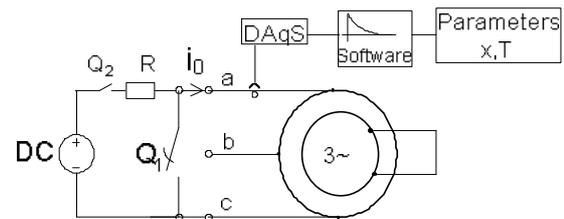


Fig. 3. Standstill DC decay test setup

The classical DC decay test requires also that the rotor at standstill to be either in transversal axis, either in longitudinal one. This work treats this case in particular, as well as, a general case, when the rotor is in a random position. The validation is done for a small laboratory machine. After this, the procedure is developed and applied for a simulated large machine with the rotor in various positions.

1. Identification procedure for d- and q- axis

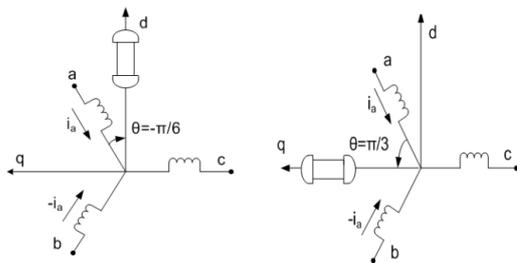


Fig. 4. d- and q- axis diagram

The identification procedure starts from a mathematical model, which consists of the machine voltage equations for the d- and q-axis. In the transient period of time, the voltage input becomes zero,  $r_s$  is the resistance of two series connected phases (in Fig. 2 – a and b phases),  $i$  is the resultant current of the two windings,  $\psi$  is the resultant flux coupled with the two phases,  $L$  stands for the inductance and  $p$  expresses the operator  $d/dt$ .

$$\begin{cases} 0 = r_s i + p\psi \\ \psi = L(p)i \end{cases} \quad (35)$$

For writing the expression of the current variation  $i'$  in the transient interval of time, Laplace transformations and operational inductances are used:

$$i'(p) = -\frac{r_s i_0}{p} \frac{1}{r_s + pL(p)} \quad (36)$$

The d- and q- axis current time-variation is considered as being given by a sum of exponential functions, which means, in a per-unit variant,

$$\begin{aligned} i_d(t) &= A_d e^{\alpha_d t} + B_d e^{\alpha_{2d} t} + C_d e^{\alpha_{5d} t} \\ i_q(t) &= A_q e^{\alpha_q t} + B_q e^{\alpha_{2q} t} \end{aligned} \quad (37)$$

where the coefficients A, B, C and  $\alpha$  are functions of time-constants and reactances for each of the axis [2].

The developed identification program is based on the MATLAB curve-fit procedure. Starting from the expressions of the currents, the identification is applied on parts of the curves. Estimated data are used as initial values.

Firstly, the procedure was tested by using known values for simulating a small machine. The results of the simulation are fed to the identification program, Fig.5. The identified parameters were afterward compared to the input values used in the simulation program. There was a very good accuracy.

The d-axis current is more difficult to calculate accurately than that on the q-axis, because of the three time constants. But, with this method of identification (in different intervals) the problem can be solved with good accuracy and consistency of the results.

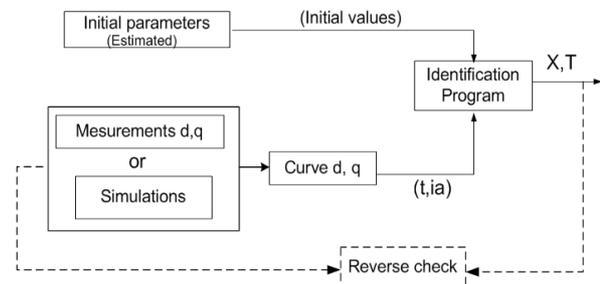


Fig. 5. Procedure lay out

2. Procedure's validation

The identified constants A, B, C and  $\alpha$ , obtained by curve-fitting, are directly used to calculate the time constants T and reactances X.

Tab. 1. d - axis parameters for simulated files

Parameters	Simulation values [p.u.]	Identified Parameters [p.u.]
$x_d$	1.1390	1.1390
$x_d'$	0.2543	0.2543
$x_d''$	0.1480	0.1480
$T_d$	2.0031	2.0031
$T_d'$	1.7789	1.7789
$T_d''$	0.0440	0.0440
$T_{d0}'$	7.9661	7.9661
$T_{d0}''$	0.0756	0.0756

Tab. 2. q - axis parameters for simulated files

Parameters	$x_q$	$x_q''$	$T_q$	$T_q''$	$T_Q$
Simulation values [p.u.]	0.714	0.1809	1.2556	0.0339	0.1341
Identified parameters [p.u.]	0.7139	0.1808	1.2556	0.0338	0.1338

A difference between known values and identified ones of less than 0.01% was obtained. The identification was made for both axes, fig.6.

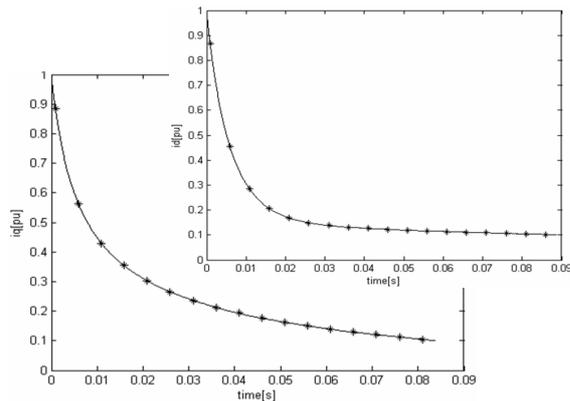


Fig. 6. Currents identification (1 p.u.)

#### 4. CONCLUSION

The goal of this paper was the determination of the parameters and the damper bar currents distribution using analytical computation method for a salient pole synchronous machine and a DC decay test.

Assuming that the damper currents have a sinusoidal distribution made possible the reduction of the damper winding to equivalent circuits in both d- and q-axis defined by only one resistance and one inductance. Thus, a synchronous machine with variable air gap and unsymmetrical damper winding was reduced by the equivalent winding to a constant air gap and continuous damper winding similar with the model of a squirrel cage induction machine.

The field-circuit approach is simple to use but is based on simplifying assumptions that influences the results.

The DC decay test allows the identification of all the transient parameters of the machine. The calculated values of the reactance and time constants by this identification method have been found to be in a quite good agreement with the simulation ones.

This identification method is quick and easy to perform once the equations are written, and with some carefully chosen initial conditions can give

good values for transient and sub-transient parameters. It is an attractive alternative to other tests because of the equipment simplicity and because of the simulation and identification CPU required time.

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