SLEW RATE AND A STEP RESPONSE OF THE NONINVERTING STRUCTURE WITH AN OPERATIONAL AMPLIFIER

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Summary When a large-step input voltage is applied to an operational amplifier (OP AMP) input, the output waveform reises with a finite slope called the slew rate which is due to an amplifier input stage current limiting (I_M) and because of a compensating capacitor C_K. We will solve this step response for large-step input voltages which cannot be done by means of the linear analysis only.

1. MODEL OF OP AMP

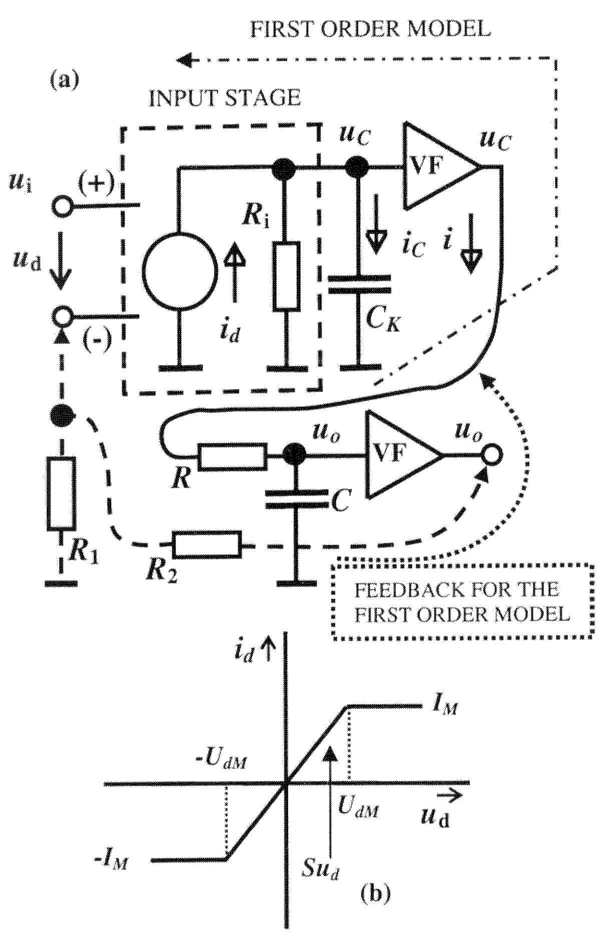

Fig.1. a) The second order model of OP AMP; b) transfer characteristics of the input stage; u_i - input voltage; u_d - differential input voltage; i_d - voltage controlled current source- INPUT STAGE; u_C - output voltage on C_K; VF - voltage follower; u_o - output voltage of the OP AMP; R_i, C_K - model a dominant (first) pole and dc gain; R, C - model the second pole of the OP AMP; i_C (i) - current in C_K (R).

Static behavior of an OP AMP can be described by the dc transfer characteristic which is for a differential OP AMP shown in Fig.1b (piecewise linear approximation). This dc characteristic can model the global behavior of any differential operational amplifiers. The compensating capacitor C_K is used to capture the dynamic behavior (one pole model). Circuit elements R and C describes a second op amp pole - see Fig.1a.

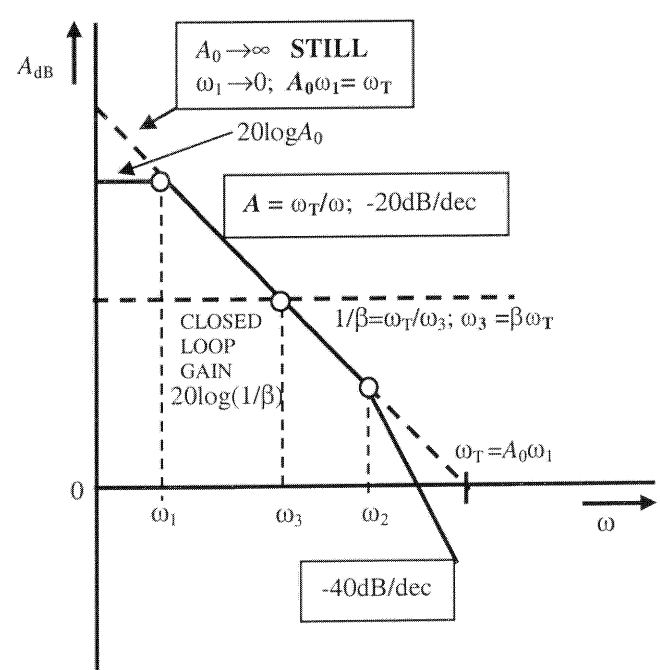

Fig.2. The frequency response of op amps (open loop gain; two pole model).

It was determined [1, 2, 3, 4] that

$$R_i = 1/(\omega_1 C_K), \tag{1}$$

$$S = A_0\omega_1 C_K = |A_0\omega_1 = \omega_T| = \omega_T C_K, \tag{2}$$

$$\omega_2 = 1/\tau_2 = 1/(RC) \tag{3}$$

where A_0 (an open loop gain), ω_1 (a dominant pole), C_K and ω_2 (sometimes) we are usually able to determine from data sheets - see Fig.2, too. S is the **transconductance** of the model.

From the equality (see Fig.1b)

$$SU_{dM} = I_M$$

we can easy determine a "nonlinear differential voltage"

$$U_{dM} = I_M / S = I_M / (\omega_T C_K) = = (I_M / C_K) / \omega_T = \rho / \omega_T \quad (4)$$

where

$$\rho = I_M / C_K \quad (5)$$

is the OP AMP slew rate. It describes the **nonlinearity** of the OP AMP and can be seen in data sheets, too.

In the linear domain is $|u_d| \leq U_{dM}$ and $i_d = S \cdot u_d$. In the nonlinear domain is $|u_d| \geq U_{dM}$ and $i_d = \pm I_M$.

Because always is valid

$$u_d(t) = u_i(t) - \beta \cdot u_o(t), \quad (6)$$

$$\beta = R_1 / (R_1 + R_2), \quad (7)$$

we can determine for the step input voltage $u_i(t) = U_i \cdot \sigma(t)$ that at the time $t = t_N$ (see Fig.3)

$$\pm U_{dM} = U_i - \beta \cdot u_o(t_N)$$

thus

$$u_o(t_N) = (U_i \pm U_{dM}) / \beta = U_i / \beta \pm \rho / \omega_3, \quad (8)$$

where

$$\omega_3 = \beta \omega_T = 1 / \tau_3 \quad (9)$$

is very well known "3 dB frequency" of the linear circuit action.

It was determined that the step response in the nonlinear domain ($|u_d| \geq U_{dM}$; $t \leq t_N$) of the analyzed two pole model is [3, 4]

$$u_o(t \leq t_N) = \rho \cdot [t - \tau_2 (1 - \exp(-t / \tau_2))] \quad (10)$$

and if $U_i \rangle\rangle U_{dM}$

$$t_N \cong (U_i / \beta) / \rho - \tau_3 + \tau_2 \quad (11)$$

see Fig.3 (for illustration depicted $\rho.t$, too)

2. LINEAR DOMAIN

In the linear domain (Fig.3, $t \geq t_N$) are valid equations (Fig.1a; $R_i \to \infty$ - then $A_0 \to \infty$ and $\omega_1 \to 0$ but $A_0\omega_1$ is ω_T, still, see Fig.2; VF - an ideal voltage follower)

$$i_C = C_K du_C / d\tau = i_d = S \cdot u_d,$$

$$u_C = R \cdot i + u_o,$$

$$i = C du_o / d\tau$$

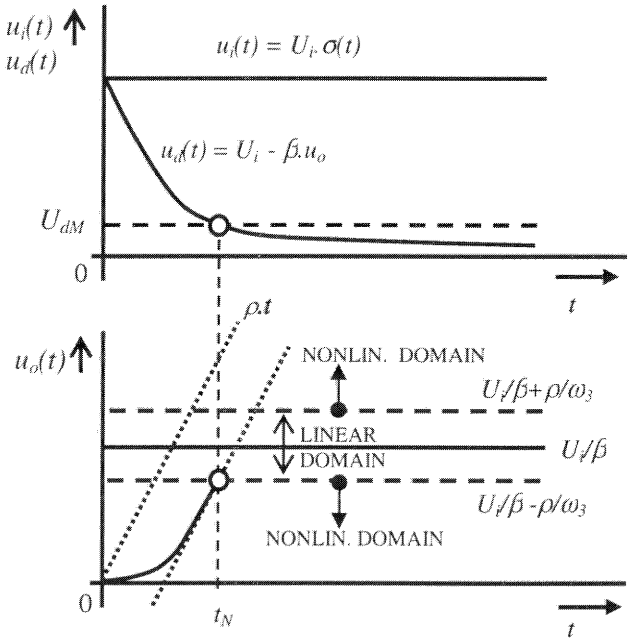

Fig.3. Depicting of the output voltage $u_o(t)$and the differential voltage $u_d(t)$for the input step $u_i(t) = U_i.\sigma(t)$.

where $\tau = t - t_N$ is a "delayed linear region time".

From this and eqs. (2) and (6) we obtain ($t \to \tau$) an obvious linear second order mathematical model

$$u_o'' + 2\xi\omega_k u_o' + \omega_k^2 u_o = \omega_2 \omega_T U_i, \quad (12)$$

where

$$\omega_k^2 = \omega_2 \omega_3; \; 2\xi = \omega_2 / \omega_k = \sqrt{\omega_2 / \omega_3}. \quad (13)$$

A complete solution consists of a complementary solution

$$u_{oh}(\tau) = K_1 \exp(\lambda_1 \tau) + K_2 \exp(\lambda_2 \tau),$$

where (roots of a characteristic equation)

$$\lambda_{1,2} = -\omega_k\left(\xi \pm \sqrt{\xi^2 - 1}\right)$$

and of a particular solution

$$u_{op}(\tau) = U_i / \beta$$

- it is very simple for $u_i(t) = U_i\sigma(t)$. Thus

$$u_o(\tau) = K_1 \exp(\lambda_1\tau) + K_2 \exp(\lambda_2\tau) + U_i / \beta . \qquad (14)$$

At the "linear time" $\tau = 0$ (it means the "all time" $t = t_N$) is

$$u_o(\tau = 0) = u_o(t_N) = U_i / \beta - \rho / \omega_3$$

and

$$u_o'(\tau = 0) = u_o'(t = t_N) \cong \rho$$

(see Fig.3). Using this initial conditions we find K_1 and K_2 and inserting the values in the complete solution, we obtain (after rearranging)

$$u_o(\tau) = \frac{\rho}{\omega_k} \cdot \exp(-\omega_k\xi\tau) \times ...$$

$$... \times \left[\frac{1 - 2\xi^2}{\sqrt{\xi^2 - 1}} \cdot \sinh(\omega_v\tau) - 2\xi\cosh(\omega_v\tau)\right] + \frac{U_i}{\beta} \qquad (15)$$

where

$$\omega_v = \omega_k\sqrt{\xi^2 - 1} .$$

3. "ALL TIME" DESCRIPTION

From the eq. (10) and eq. (15) we can get the "all time" description (16) of the output voltage

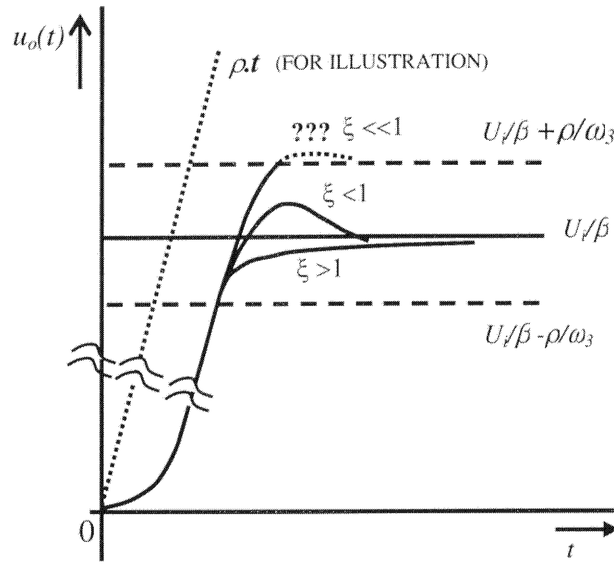

Fig.4. Depicting of the output voltage $u_o(t)$ - two pole model.

$$u_o(t, \omega_2 \to \infty) = \rho \cdot t \cdot [1 - \sigma(t - t_N)] + ...$$

$$... + \left[\frac{U_i}{\beta} - \frac{\rho}{\omega_3}\exp(-\omega_3(t - t_N))\right] \cdot \sigma(t - t_N) \qquad (17)$$

and from the eq.(11)

$$t_N(\omega_2 \to \infty) \cong (U_i / \beta) / \rho - \tau_3 . \qquad (18)$$

These equations describe the one-pole model [1, 2, 3, 4].

4. CONCLUSION

Although the essence of the problem of step response is nonlinear, the factors to be discussed in this paper pertain to the all-important linear-settling "tail", which applies to both small signals and large, too.

$$u_o(t) = \rho \cdot [t - \tau_2(1 - \exp(-t / \tau_2))] \cdot [1 - \sigma(t - t_N)] + ...$$

$$... + \left\{\frac{\rho}{\omega_k} \cdot \exp(-\omega_k\xi(t - t_N)) \cdot \left[\frac{1 - 2\xi^2}{\sqrt{\xi^2 - 1}} \cdot \sinh(\omega_v(t - t_N)) - 2\xi\cosh(\omega_v(t - t_N))\right] + \frac{U_i}{\beta}\right\} \cdot \sigma(t - t_N) \qquad (16)$$

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but for

$$u_o(t)_{MAX} \le U_i / \beta + \rho / \omega_3$$

only. It is valid for $\xi \ge 1$ always. For $\xi \le 1$ it should be most complicated story - see Fig.4.

If $\omega_2 \to \infty$ ($\tau_2 = 1/\omega_2 \to 0$; $\xi \to \infty$; see eq.(13)) we are able to determine from the eq.(16) that

In the our two pole model we must strive to have $\xi \ge 1$, thus (see eqs.(13) and (9))

$$\xi = \frac{1}{2}\sqrt{\frac{\omega_2}{\omega_3}} = \frac{1}{2}\sqrt{\frac{\omega_2}{\beta\omega_T}} \ge 1 .$$

From this we easy determine that we need

$$\omega_2 \geq 4\beta\omega_T = 4\omega_3 . \tag{19}$$

The amplifier used for fast settling to high accuracy should have a closed-loop response that is not much worse then critically-damped, as any oscillation or ringing may prolong settling time.

In practical circuits, which have stray capacitances, the added lags caused by the external loops elements will cause an amplifier having insufficient phase margin to ring. For this reason, designers (of OP AMP) strive to have the open-loop frequency characteristic be strongly dominated by a single time constant - it means:

- constant 90°phase shift
- -20dB/dec rolloff

see Fig.5. The phase margin is 90° or better, even for $\beta \rightarrow 1$.

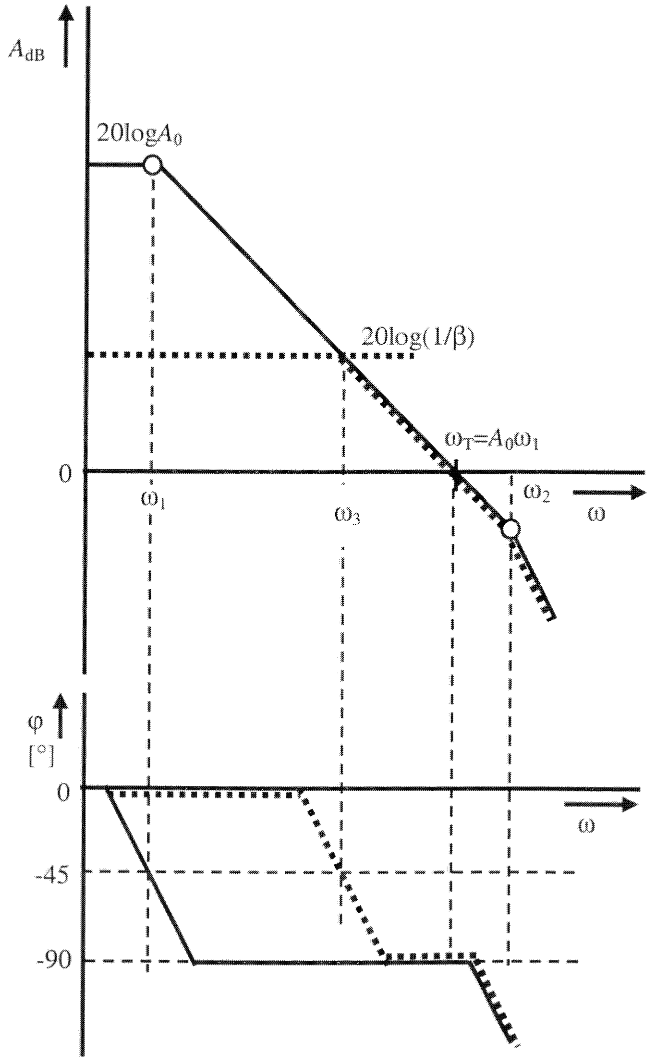

Fig.5. An appropriate OP AMP response (Bode plots); dotted lines - situation with feedback.

And the last, but very important, note: at the time t = 0 is u_d (0) = U_i , always - see Fig.3. It means it is suitable to save OP AMP inputs from an overvoltage for U_i large enough.

Another interesting knowledge about OP AMP dynamics you can get in [5, 6, 7, 8, 9].

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