INSTANTANEOUS SWITCHING PROCESSES IN QUASI-LINEAR CIRCUITS

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Summary The paper considers instantaneous processes in electrical circuits produced by the stepwise change of the capacitance of the capacitor and the inductance of the inductor and by the switching on and switching off of the circuit. In order to determine the set of electrical circuits, for which it is possible to explicitly obtain the values of the currents and the voltages at the end of the instantaneous process, a classification of the networks with nonlinear elements is introduced in the paper.The instantaneous switching process in the moment t_0 is approximated when $T \to t_0$ with a sequence of processes in the interval $[t_0, T]$. For quasi-linear inductive and capacitive circuits, we present the type of the system satisfied by the currents and the voltages, the charges, as well as the fluxes in the interval $[t_0, T]$. From this system, after passage to the limit $T \to t_0$, we obtain the formulas for the values of the circuits at the end of the instantaneous process. The obtained results are applied for the analysis of particular processes.

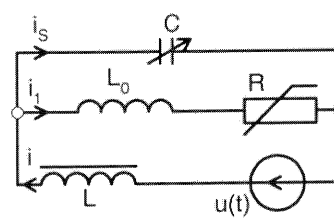
1. INTRODUCTION

The instantaneous switching processes in electric circuits are caused by the stepwise change of the parameters of some components, which are from now on called s-elements. One approach for analysis of such processes in circuits with nonlinear components is offered in [1]. This work studies an instantaneous process in the moment t_0 in a class of circuits with nonlinear components with the help of approximating sequences of processes in the interval $[t_0, T]$ when $T \to t_0$. For these processes it is possible to obtain explicit formulae for the values of the initial conditions of the process immediately following the instantaneous process. In [2] and [3] the above-mentioned approach is applicable for analysis of instantaneous switching processes respectively in linear circuits with concentrated and distributed parameters.

2. CLASSIFICATION OF THE CIRCUITS WITH ONE S-ELEMENT AND OF THE INSTANTANEOUS SWITCHING PROCESSES IN THEM

Further, the paper studies instantaneous switching processes in electric circuits caused by the instantaneous change of the state of a switch, a capacitor with stepwise changing capacitance, or an inductor with stepwise changing inductance. For this purpose we use the numeration of the types of components in a circuit introduced in [1]: 1. a voltage source; 2. a capacitor; 3. a resistor; 4. an inductor; and 5. a current source. With the help of this numeration, the closed contours through the s-element and the circuits with one s-element are separated in the following groups:

The closed contour through the s-element is called of type m, $1 \le m \le 5$, if besides an s-element it contains an element of number m and does not contain elements with numbers greater than m (if such numbers exist).



The circuit with one s-element is of type $1 \le m \le 5$ if among its closed contours through the s-element there is at least on of type m and none of types less than m.

Fig. 1.

Let in the moment t_0 the capacitance of the capacitor C (the s-element) from the circuit in Fig. 1 change from C_1 to C_2. With L_0 and L we denote respectively a linear and a nonlinear inductor, R is a nonlinear resistor, and $u(t)$ is the voltage of a source of continuously changing voltage. This circuit is inductive, the s-element in which is a capacitor.

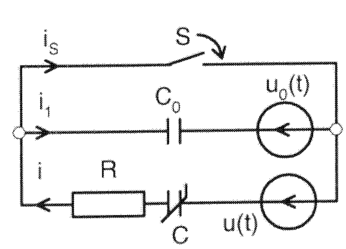


Fig. 2.

An example of a capacitive circuit is given in Fig. 2. In this circuit in the moment t_0 the switch S (the s-element) is switched on. With C_0 and C we denote respectively a linear and a nonlinear capacitor; R is a resistor with a linearly changing resistance; $u(t)$ and $u_0(t)$ are voltage sources.

If the instantaneous process happens in the moment t_0, then for $T \to t_0$ it is approximated with a sequence of processes, which occur in the interval $[t_0, T]$. In these intervals the s-element is considered as a component with a continuously changing parameter. In the case of a switch this parameter is the resistance between the contacts $R_T(t)$ that changes from infinity to zero when switching on and from zero to infinity when switching off. The capacitor with capacitance changing stepwise from C_1 to C_2 is replaced with a capacitor with capacitance $C_T(t)$ continuously changing in time, where $C_T(t_0) = C_1$ and $C_T(T) = C_2$. If the s-element is an inductor with inductance changing stepwise from L_1 to L_2, then we introduce an inductor with continuously changing inductance $L_T(t)$, for which $L_T(t_0) = L_1$ and $L_T(T) = L_2$.

Nonlinear capacitors, inductors and resistors are allowed as components of the circuit. For a nonlinear capacitor C we assume that its voltage u_C is a function of the charge q of type $u_C(q) = q\lambda(q)$, where $\lambda(q) > 0$ and $\frac{d(q\lambda(q))}{dq} > 0$ for $q \neq 0$. In the case of a nonlinear inductor L the current i_L through it is a function of the flux Ψ of type $i_L(\Psi) = \Psi\beta(\Psi)$, where $\beta(\Psi) > 0$ and $\frac{d(\Psi\beta(\Psi))}{d\Psi} > 0$ for $\Psi \neq 0$. The resistance of the nonlinear resistor is determined by the relevant function $R(i)$ of the current i through it.

Further, we consider the instantaneous switching processes in circuits with one s-element of type 2 (capacitive circuits) and of type 4 (inductive circuits) that satisfy the following supplementary conditions:

a) capacitive circuits with an inductor with stepwise changing inductance as an s-element;
b) inductive circuits with a capacitor with stepwise changing capacitance as an s-element (e.g. the circuit in Fig. 1);
c) switching in capacitive circuits, all of whose closed capacitive contours through the s-element contain only linear capacitors (e.g. the circuit in Fig. 2);
d) switching in inductive circuits, all of whose closed inductive contours through the s-element contain only linear inductors.

The possibility too obtain explicit formulae for the currents and the voltages at the end of the instantaneous processes, as in the case of linear circuits [2], allows us to call the above-mentioned circuits with nonlinear components quasi-linear.

3. MATHEMATICAL MODELS OF INSTANTANEOUS SWITCHING PROCESSES IN QUASI-LINEAR CIRCUITS

Let us consider the instantaneous process caused by the stepwise change in the moment t_0 of the capacitance of the capacitor C in the inductive circuit in Fig. 1. This process is approximated for $T \to t_0$ with sequences of processes in intervals $[t_0, T]$, for which the capacitance C is the continuous function $C_T(t)$ for which $C_T(t_0) = C_1$ and $C_T(T) = C_2$. Let u_S be the voltage of the capacitor C. The Weber-Ampere characteristic of the nonlinear inductor L is a function of the flux of type $i(\Psi) = a\Psi + b\Psi^3$. With i, i_1, and i_S we denote the currents in the separate branches of the circuit, and $R(i_1)$ is the resistance of the nonlinear resistor. In the interval $[t_0, T]$ the following system for the process in the circuit holds:

$$\begin{aligned}
&\frac{du_S}{dt} = -u_S\frac{d(\ln(C_T(t))}{dt} + \frac{i_S}{C_T(t)} \\
&\frac{d\Psi}{dt} = -u_S + u(t) \qquad (1) \\
&L_0\frac{di_S}{dt} = \left(a\Psi + b\Psi^3 - i_S\right)R(a\Psi + b\Psi^3 - i_S) - \\
&\qquad - u_S\left(L_0\left(a + 3b\Psi^2\right) + 1\right) + L_0\left(a + 3b\Psi^2\right)u(t)
\end{aligned}$$

As a second example, let us consider the switching on in the moment t_0 of the circuit in Fig. 2. This process is approximated for $T \to t_0$ with sequences of processes in intervals $[t_0, T]$. Then the switch is replaced by a resistor with resistance $R_T(t)$, which changes from infinity to zero. The voltage of the nonlinear capacitor C is determined depending on the charge q by the formula $u_C(q) = aq + bq^3$ and the voltage of the linear capacitor C_0 is u_{C0}. The currents in the separate branches are denoted with i, i_1, and i_S; the resistance of the resistor R is denoted by R. Let u_S be the voltage of the switch S. With this notation the following system describes the process in the circuit for $t \in [t_0, T]$:

$$\begin{aligned}
&\frac{du_S}{dt} = -\frac{u_S}{C_0 R_T(t)} + \frac{u(t) - u_C(q) - u_S}{RC_0} - \frac{du_0(t)}{dt} \\
&\frac{dq}{dt} = \frac{-u_S + u(t) - u_C(q)}{R}. \qquad (2)
\end{aligned}$$

The general form of the systems (1) and (2) for $t \in [t_0, T]$ when $T \to t_0$ is

$$\begin{aligned}
&\frac{dx}{dt} = Ax\,\alpha_T(t) + g_T(x, y, t); \\
&\frac{dy}{dt} = A_1 x\,\alpha_T(t) + h_T(x, y, t), \qquad (3)
\end{aligned}$$

where x is a scalar and y is an n-dimensional variable. The function $g_T(x,y,t)$ is scalar, and the function $h_T(x,y,t)$ is a column matrix with n rows. With A we denote a nonzero constant, and A_1 is a constant column matrix with n rows. The scalar function $\alpha_T(t)$ is defined and continuous in the interval (t_0, T), which for $t \to t_0$ or $t \to T$ can grow infinitely. In the system (1) $\alpha_T(t) = \frac{d\ln C_T(t)}{dt}$, and in the system (2) $\alpha_T(t) = \frac{1}{R_T(t)}$.

The system (3) is considered for $T \in (t_0, T_0)$ and $t \in (t_0, T)$. Let us denote by $x_T(t)$, $y_T(t)$ the solution of the system (3) with initial conditions $x(t_0)=x_0$, $y(t_0)=y_0$. For the function

$$\Phi_T(t,\tau)=\exp\left(A\int_\tau^t \alpha_T(\sigma)d\sigma\right)$$

let us assume that there exist the limits

$$\Phi_T(T,\tau)=\lim_{t\to T}\Phi_T(t,\tau) \qquad \Phi_T(t,t_0)=\lim_{\tau\to t_0}\Phi_T(t,\tau)$$

$$\Phi^+=\lim_{T\to t_0}\Phi_T(T,t_0).$$

With some additional assumptions the solutions of the above mentioned problem are bounded when $T\to t_0$ and the following equalities are valid:

$$x^+=\lim_{T\to t_0}x_T(T)=\Phi^+x_0;$$

$$y^+=\lim_{T\to t_0}y_T(T)=y_0+A_1A^{-1}\left(\Phi^+-1\right)x_0, \qquad (4)$$

which determine the states of the variables at the end of the instantaneous process.

4. ANALYSIS OF THE GIVEN EXAMPLES

Let us use the formulae (4) to determine the values of the variables in the circuit in Fig. 1 after the stepwise change of the capacitance of the capacitor C. Let the initial conditions of the process be $u_S(t_0)=u_S^0$, $\Psi(t_0)=\Psi^0$, $i_S(t_0)=i_S^0$. In this case

$$\Phi_T(t,\tau)=\exp\left(-\int_\tau^t\frac{d(\ln C_T(\sigma))}{d\sigma}d\sigma\right)=\exp\left(\ln\frac{C_T(\tau)}{C_T(t)}\right)=\frac{C_T(\tau)}{C_T(t)}$$

$$\Phi_T(T,t_0)=\Phi^+=\frac{C_1}{C_2}.$$

Then from the system (1) we obtain the following results for the values of the variables u_S, Ψ, and i_S at the end of the instantaneous process:

$$u_S^+=\frac{C_1}{C_2}u_S^0; \qquad \Psi^+=\Psi^0; \qquad i_S^+=i_S^0.$$

Let the initial conditions for the instantaneous switching on of the circuit in Fig. 2 be $u_S(t_0)=u_{C_0}(t_0)-u_0(t_0)$, $q(t_0)=q^0$. In this case,

$$\Phi_T(t,\tau)=\exp\left(-\int_\tau^t\frac{1}{C_0R_T(\sigma)}d\sigma\right).$$

Additionally, let us assume that the following relationship holds:

$$\Phi^+=\lim_{T\to t_0}\lim_{\tau\to t_0}\lim_{t\to T}\Phi_T(t,\tau)=0,$$

which corresponds to the changing from infinity to zero function: resistance $R_T(t)$. Then from the system (2) and the formulae (4) we obtain

$$u_S^+=0; \qquad q^+=q^0.$$

From these equalities it follows that at the end of the instantaneous process for the voltages u_{C_0} and u_C respectively of the capacitors C_0 and C it is true that

$$u_{C_0}^+=u(t_0), \qquad u_C^+=u_C^0=u_C(t_0).$$

5. ENERGY OF THE INSTANTANEOUS SWITCHING

For capacitive and inductive circuits with one s-element let us denote by i_S and u_S the current and voltage of the s-element. If i_S^0 and u_S^0 are the values of the current i_S and the voltage u_S right before the instantaneous switching, and i_S^+ and u_S^+ are the values of these variables at the end of the instantaneous process, then let us denote $\Delta i_S=i_S^+-i_S^0$ and $\Delta u_S=u_S^+-u_S^0$. Using the analogy with the instantaneous switching processes in linear circuits, let us introduce the term proper instantaneous switching. The instantaneous switching is proper if $\Delta u_S=0$ in a capacitive circuit and if $\Delta i_S=0$ in an inductive circuit. In the first two of the specified cases of quasi-linear circuits the instantaneous switching is always proper.

If $i_{S_T}(t)$ and $u_{S_T}(t)$ are the current and the voltage of the s-element for the interval $[t_0, T]$ in the approximating sequence of processes, then the limit (in case it exists)

$$W^+=\lim_{T\to t_0}\int_t^T i_{S_T}(t)\,u_{S_T}(t)dt$$

determines the energy of the instantaneous switching. In the third and fourth cases, for the energy of the instantaneous switching we obtain, respectively, the formulae:

$$W^+=\frac{C}{2}\left(\left(u_S^+\right)^2-\left(u_S^0\right)^2\right)$$

$$W^+=\frac{L}{2}\left(\left(i_S^+\right)^2-\left(i_S^0\right)^2\right), \qquad (5)$$

where: C is a constant that depends on the capacitance of the capacitors in the capacitive contours closed through the s-element; L is a constant that depends on the inductance of the inductors in the inductive contours closed through the s-element.

Since in the first two cases of instantaneous switching processes in quasi-linear circuits we have that $W^+=0$, we can presume that in these cases too the formulae (5) are valid. From (5) we obtain that the equality $W^+=0$ is true for the proper instantaneous switching processes in quasi-linear circuits.

6. CONCLUSION

In the presented cases of instantaneous switching processes in quasi-linear circuits with nonlinear components we obtain explicit formulae for the values of the variables at the end of the instantaneous process, as in the cases of linear circuits. In the cases of instantaneous switching processes in circuits with nonlinear components, which do not belong to set of the defined quasi-linear circuits, we obtain systems of nonlinear equations for the values at the end of the process.

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