DIFFERENTIAL AND INTEGRAL MODELS OF TOKAMAK

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Summary Modeling of 3D electromagnetic phenomena in TOKAMAK with typically distributed main and additional coils is not an easy business. Evaluated must be not only distribution of the magnetic field, but also forces acting in particular coils. Use of differential methods (such as FDM or FEM) for this purpose may be complicated because of geometrical incommensurability of particular subregions in the investigated area or problems with the boundary conditions. That is why integral formulation of the problem may sometimes be an advantage. The theoretical analysis is illustrated on an example processed by both methods, whose results are compared and discussed.

1. INTRODUCTION

TOKAMAK represents a thermonuclear reactor that is believed to become one of safe, environmentally friendly and powerful sources of electric energy, perhaps in the middle of this century. Its properties and behavior in various operation regimes have theoretically been investigated for about fifty last years. As the problems associated with utilization of thermonuclear energy are, however, extremely complicated, the first results in the area were achieved only after a lot of years of intensive work, at the beginning of nineties.

One of necessary conditions for functioning of such a device is generation of magnetic field of very specific properties. Its task is to keep hot plasma in the prescribed space. This field is usually realized by a system of special coils depicted in Figs. 1 and 2. The field may be, moreover, strengthened by a suitable magnetic circuit, of course, at the expense of its uniformity.

Computation of the magnetic field distribution and force effects in complex 3D systems of coils is not an easy business. Numerical methods based on differential algorithms require a lot of memory and relatively long time of computation. This is caused by necessity to include a lot of surrounding space into calculations, and problems may also appear with the boundary conditions that are not known in advance and must be only approximated.

On the other hand, methods working with integral expressions need less time, but their application is limited to linear problems.

The paper deals with comparison of both ways of numerical modeling of linear TOKAMAK. Investigated is also the influence of the iron core on the field distribution and force effects.

2. FORMULATION OF THE TECHNICAL PROBLEM

It is necessary to find the distribution of magnetic field generated by eight main coils (their simplified arrangement is depicted in Fig. 1) and its distortion caused by supplementary ring coils or magnetic circuit in one plane of symmetry (Fig. 2). In case of linear fields the computation are performed by means of FEM-based numerical analysis and integral expressions in order to evaluate the efficiency of both ways of calculation.

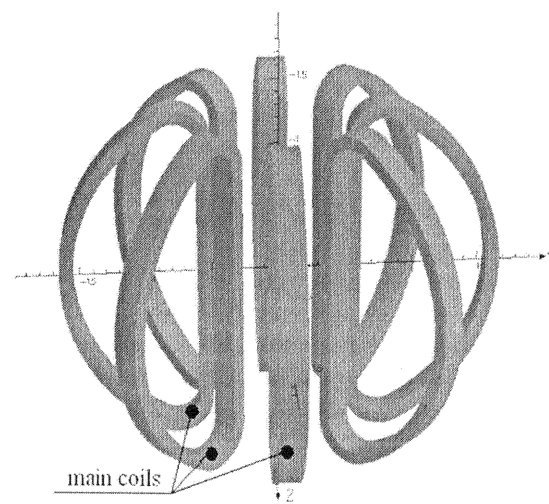


Fig. 1. The basic arrangement of main coil.

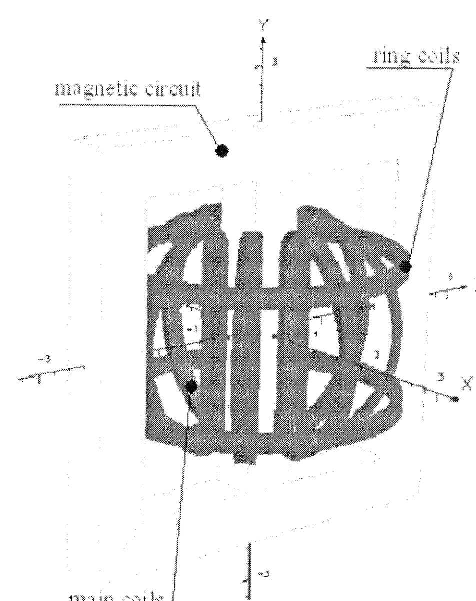


Fig. 2. Full arrangement with the supplementary ring coils and magnetic circuit.

3. MATHEMATICAL MODEL

The main coils carry direct currents I_1 while eventual auxiliary ring coils currents I_2. As far as the arrangement also includes magnetic circuit, it may be supposed linear, nonlinear isotropic or nonlinear anisotropic.

The *differential model* of magnetic field is given by equation [1]

$$\mathrm{rot}\left[\mu\right]^{-1}\mathrm{rot}\,\boldsymbol{A}=\boldsymbol{J} \quad , \qquad (1)$$

where $\boldsymbol{A}$ is the vector potential, $\boldsymbol{J}$ the density of the external field currents and μ the magnetic permeability (that is generally of tensorial character). In case that the arrangement does not include the magnetic circuit or its permeability μ is considered constant, equation (1) may be in a suitably chosen Cartesian coordinate system simplified into form

$$\Delta\boldsymbol{A}=-\mu\boldsymbol{J} \quad . \qquad (2)$$

Unambiguous solution of (1) or (2) is conditioned by imposing suitable boundary conditions that depend on the particular arrangement. Usually we are able to find one or more planes of symmetry, which significantly reduces the domain to be investigated and, consequently, the number of elements and nodes in discretization mesh.

Numerical solution of the above equations was realized by professional FEM-based code OPERA 9 (3D module TOSCA) [2].

As for the *integral model*, it can be used only in homogeneous media (in our case in the absence of the magnetic circuit). It is based on direct determination of the vector potential at a point P of radius vector $\boldsymbol{r}_P$ produced by current of density $\boldsymbol{J}$ in a field conductor (see Fig. 3).

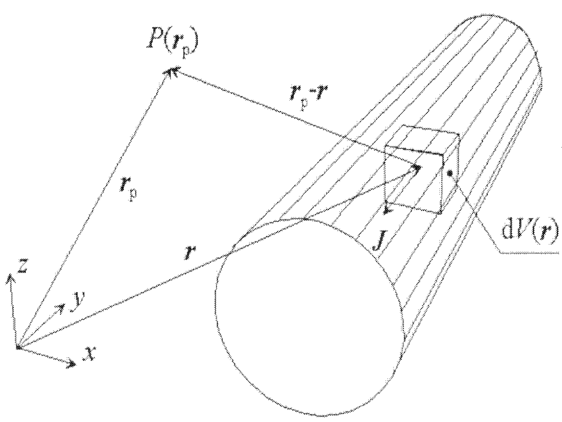


Fig. 3. To the formulation of the integral model.

The corresponding formula reads [3]

$$\boldsymbol{A}\left(\boldsymbol{r}_P\right)=\frac{\mu_0}{4\pi}\cdot\int_V\frac{\boldsymbol{J}\left(\boldsymbol{r}\right)}{\left|\boldsymbol{r}_P-\boldsymbol{r}\right|}\cdot\mathrm{d}V \qquad (3)$$

where vector $\boldsymbol{r}$ denotes the position of elementary volume $\mathrm{d}V$ of the field winding and

$$\left|\boldsymbol{r}_\mathrm{P}-\boldsymbol{r}\right|=\sqrt{\left(x_\mathrm{P}-x\right)^2+\left(y_\mathrm{P}-y\right)^2+\left(z_\mathrm{P}-z\right)^2}\ . \qquad (4)$$

Hence

$$\boldsymbol{B}\left(\boldsymbol{r}_P\right)=\frac{\mu_0}{4\pi}\cdot\int_V\frac{\left(\boldsymbol{r}_P-\boldsymbol{r}\right)\times\boldsymbol{J}\left(\boldsymbol{r}\right)}{\left|\boldsymbol{r}_P-\boldsymbol{r}\right|^3}\cdot\mathrm{d}V \qquad (5)$$

Numerical computation of discretized expressions (3) and (5) was performed by own program written in Borland Delphi [4].

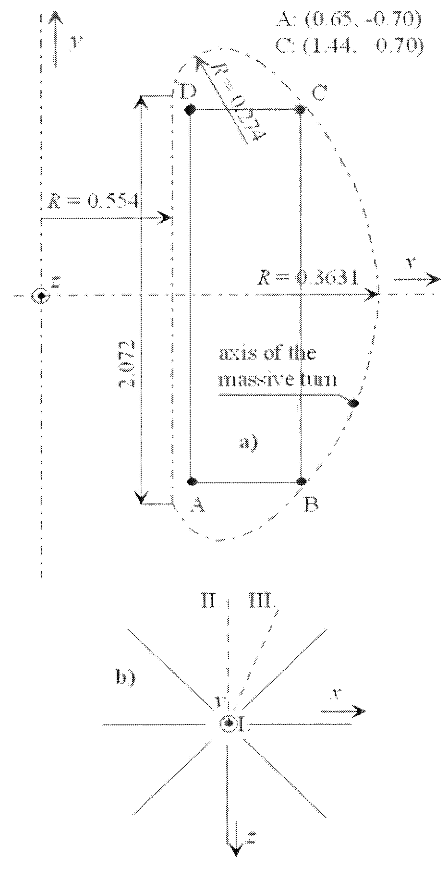
4. ILLUSTRATIVE EXAMPLES AND RESULTS

The methodology was used for solution of two different examples. Example **1** solves field distribution in fully homogeneous medium while example **2** in a medium that is homogeneous by parts.

Example **1**:

It is necessary to design an arrangement of the field coils that produce magnetic field of specific properties.

The simplest arrangement consists of eight main coils, each of them being represented by one massive conductor (see Fig. 1). Their dimensions follow from Fig. 4. The area of cross-section of one massive conductor $S_\mathrm{c}=0.2\times0.1=0.02\ \mathrm{m}^2$. The module $\left|\boldsymbol{J}\right|$ of the current density in each main coil is $5\cdot10^7\ \mathrm{A/m}^2$.



*Fig, 4. Basic dimensions of massive main turn **a)** together with the total schematic view **b)**.*

The FEM-based analysis of the field without magnetic circuit was calculated in 1/16 of the arrangement as is depicted in Fig. 5. The area was provided by correct boundary conditions and conditions of symmetry. The same model was created in the program based on the integral approach.

One of the first steps that are necessary to carry out is to validate the geometrical convergence of results in order to choose a sufficiently dense discretization mesh, particularly for the domain or conductors. The test was carried out for meshes ranging from about 20000 to 140000 nodes.

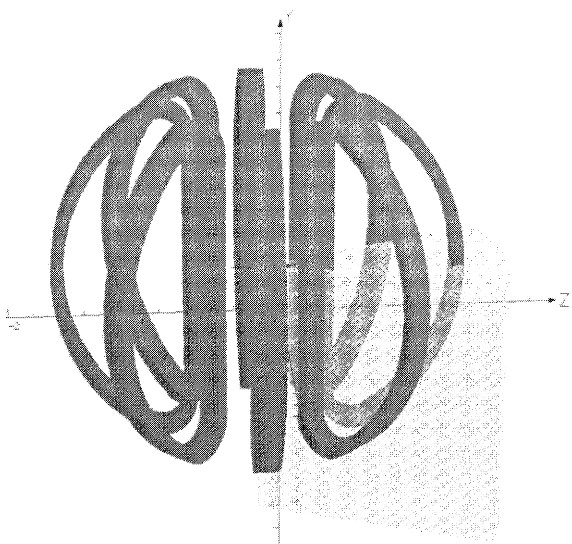


Fig. 5. Definition area of the linear problem.

Tables. 1 and 2 contain values of the module of magnetic flux density at selected points of the domain as functions of the number of nodes in the mesh.

Tab. 1. Convergence of the FEM-based model.

coordinate (m)			TOSCA 3D - $\|B\|$ (T)			
x	y	z	number of nodes			
			22000	42000	70000	139000
0.9	0.0	0.0	1.8265+0	1.8287+0	1.8293+0	1.8285+0
0.0	0.0	0.0	1.6250-6	1.6250-6	1.6250-6	1.6250-6
2.0	0.0	0.0	1.2570-1	1.2570-1	1.2570-1	1.1257-1
0.9	0.4	0.0	1.8401+0	1.8417+0	1.8401+0	1.8430+0
0.9	0.8	0.0	1.9476+0	1.9484+0	1.9477+0	1.9496+0
0.9	1.6	0.0	3.9638-2	3.9866-2	3.9598-2	3.9073-2

Tab. 2. Convergence of the integral model.

coordinate (m)			integral model - $\|B\|$ (T)			
x	y	z	number of nodes			
			19400	25800	44000	70000
0.9	0.0	0.0	3.5381+0	1.8604+0	1.8305+0	1.8290+0
0.0	0.0	0.0	1.0592-3	1.3863-3	1.2670-3	1.0591-6
2.0	0.0	0.0	5.5325-2	1.1241-1	1.1621-1	1.1625-1
0.9	0.4	0.0	1.9650+0	1.8821+0	1.8424+0	1.8434+0
0.9	0.8	0.0	1.4576+0	1.9691+0	1.9262+0	1.9270+0
0.9	1.6	0.0	1.3748-1	6.1101-2	5.0926-2	5.1238-2

Note: signs + and – in the values denote E+ and E- (exponents)

Next computations were realized on meshes with 70000 numbers of elements.

Fig. 6 shows distribution of component B_z along axis x of the turn (see Fig. 4) while Fig. 7 along axis y.

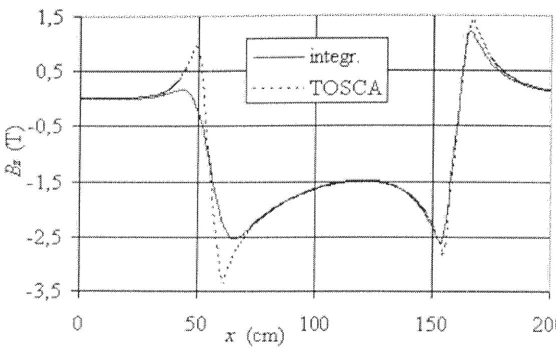


Fig. 6. Distribution of component B_z along axis x for $y = 0\,m$.

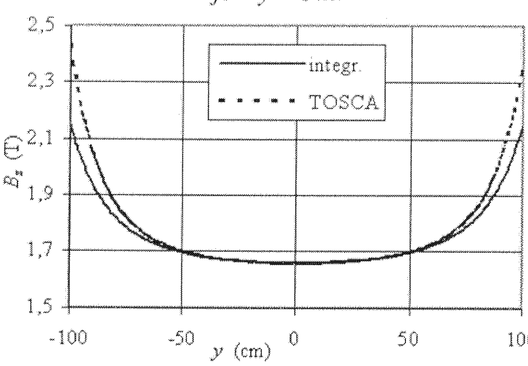


Fig. 7. Distribution of component B_z along axis y for $x = 0.9\,m$.

Differences between both methods within the massive conductors follow from the realization of the integral method that works not with the massive conductor but with a lot of thin conductors that substitute the massive.

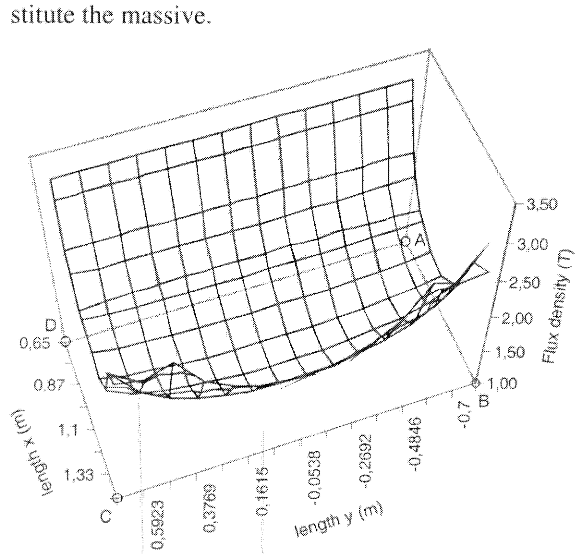


*Fig. 8. Distribution of the module $|\boldsymbol{B}|$ in the central plane ABCDA of the turn (indicated in Fig. 4**b** by letters I-II); its minimal and maximal values are 1.42 T and 3.1 T, respectively.*

Fig. 8 shows distribution of the module $|\boldsymbol{B}|$ in the central plane ABCDA (see Fig. 4**b**, letters I-II) of the massive turn calculated by the integral method. It is apparent that the field in the central part of the turn (represented by the light points) is highly uniform, even when near the conductor its values grow.

An analogous field distribution in the plane ABCDA between two neighboring coils (indicated also in Fig. 4**b** by letters I-III) calculated again by the integral method is depicted in Fig. 9. Here the values of magnetic flux density are obviously lower and level of its uniformity is not as high as in the previous case.

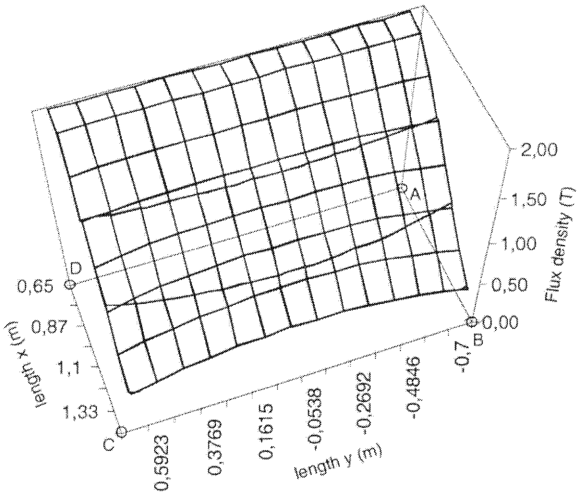


*Fig. 9. Distribution of the module $|\boldsymbol{B}|$ in the plane ABCDA between two neighboring turns(indicated in Fig. 4**b** by letters I-III); its minimal and maximal values are 0.415 T and 2.08 T, respectively.*

Example **2**:

In this example we solved the arrangement depicted in Fig. 2 without and with the magnetic circuit, respectively.

Both ring coils are placed symmetrically with respect to axis y, at the distance $y = \pm 0.8$ m. Their cross-section is rectangular, of area $S_r = 0.2 \times 0.2$ m. Their average radius $R_r = 1.65$ m. While each main coil carries current $I_1 = 10^6$ A, both ring coils carry current $I_2 = 2 \cdot 10^6$ A, but of opposite signs.

The cross-section of all parts of the magnetic circuit $S_c = 0.6 \times 0.6$ m and internal dimensions of each window are 1.5×3.6 m. Its relative permeability is $\mu_r = 3000$.

The field distribution was calculated in several planes and along some lines. Some results are depicted in the following figures.

Fig. 10 shows distribution of the module of magnetic flux density along a circle of radius 0.9 m for $y = 0.4$ m. The presence of magnetic circuit influences particularly the space in each window, where $|\boldsymbol{B}|$ reaches somewhat higher value.

Analogously, Fig. 11 shows distribution of the module of magnetic flux density along a circle of radius 0.9 m for $y = 0.8$ m.

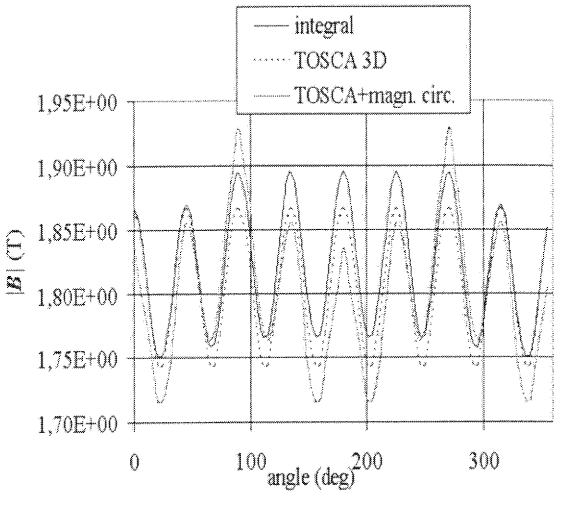


Fig. 10. Distribution of $|\boldsymbol{B}|$ along a circle of radius 0.9 m for $y = 0.4$ m ($I_1 = 10^6$ A, $I_2 = 2 \cdot 10^6$ A).

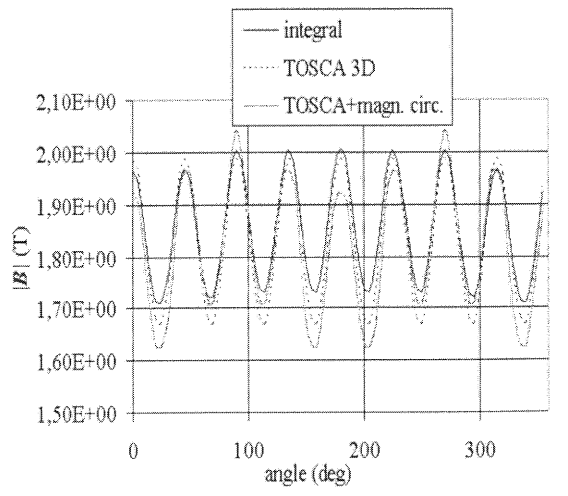


Fig. 11. Distribution of $|\boldsymbol{B}|$ along a circle of radius 0.9 m for $y = 0.8$ m ($I_1 = 10^6$ A, $I_2 = 2 \cdot 10^6$ A).

5. EVALUATION OF BOTH PROGRAMS

As both mentioned numerical methods are based on quite different principles, we compared some interesting parameters of the calculation.

1. Velocity and accuracy of computations:

The comparison is carried out for computation of the linear case according to Fig. 1. Solution was realized on a computer P4, 2.6 GHz, 512 MB RAM. Parameters of the discretization mesh were in both cases comparable, about 70000 nodes. Other values are given in Tab. 3.

Tab. 3. Selected characteristics of the solution.

	OPERA	integr.
time of calculation	3.19 min	0.25 s
memory requirements (MB)	20 (300 HDD)	14

It is obvious that computation of the field by means of integral expression is faster by two orders. Another advantage of the integral approach consists in the fact that the field can be calculated directly at the selected points (in case of the FEM-based approach the field is calculated only at the nodes and at points not identical with these nodes the field quantities have to be interpolated. On the other hand, the integral approach is in our case based on substitution of massive conductors by a lot of thin filaments, which may lead to certain errors when mapping the field inside them.

2. *Environments of both programs:*

OPERA 3D is a top-quality professional code consisting of several modules. Creation of the model is relatively easy, offered is a number of predefined objects. The definition area may often be simplified by using various symmetries or periodicity of the investigated device. Powerful is also postprocessor that makes it possible to map various quantities in a lot of forms.

Field computation by means of the integral expressions is realized by own program ElmagPole written in Borland Delphi (author Pavel Karban). Its environment can be seen in Fig. 12.

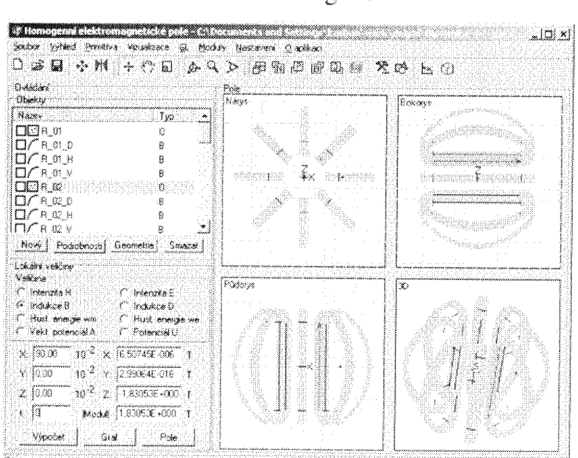


Fig. 12. Environment of program ElmagPole.

Building models is here more complicated. Predefined are several entities (lines, arcs). In the present version the program does not consider periodicity, so that the complete model has to be processed. As mentioned, the program works with line conductors, so that massive conductors have to be simulated by several thin conductors. This results in certain errors when mapping fields inside them or in their neighborhood.

3. *Advantages and drawbacks of both methods*

FEM-based SW (such as OPERA) is generally not suitable for solution of problems characterized by geometrically incommensurable subregions (which leads to serious problems with mesh generation). On the other hand, method of integral expressions (particularly in combination with BEM) is much more convenient for solution of such tasks. Its drawback is, however, that its advantages may be fully used only in linear media. Its application in nonlinear problems is still accompanied by many complications.

6. CONCLUSION

As is obvious from the previous text, solution of magnetic fields even in very complicated 3D arrangements may be realized by means of user programs that work with high velocity and may be considered sufficiently reliable and accurate. The reason is that the structure of professional programs is general and, moreover, in linear tasks the approach based on integral expressions is much faster.

The authors developed (in a relatively short time) a quite comprehensive program that can solve electromagnetic fields in suitable arrangements not only by means of integral expressions, but also by the first and second-kind Fredholm integral equations. Planned is further development of the program in the direction to even more complicated structures with eventual nonhomogenities and implementation of the higher-order methods.

Acknowledgment

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