CONVERSION OF DIELECTRIC DATA FROM THE TIME DOMAIN TO THE FREQUENCY DOMAIN

Vladimír Ďurman, Jaroslav Lelák

Slovak University of Technology, Faculty of Electrical Engineering and Information Technology,
Department of Electrotechnology
Ilkovičova 3, 81219 Bratislava, Slovak Republic
Phone:+421 260 291 612, fax:+421 265 425 822, E-mail: vladimir.durman@ stuba.sk

Summary Polarisation and conduction processes in dielectric systems can be identified by the time domain or the frequency domain measurements. If the system is a linear one, the results of the time domain measurements can be transformed into the frequency domain, and vice versa. Commonly, the time domain data of the absorption conductivity are transformed into the frequency domain data of the dielectric susceptibility. In practice, the relaxation processes are mainly evaluated by the frequency domain data. In the time domain, the absorption current measurements were preferred up to now. Recent methods are based on the recovery voltage measurements. In this paper a new method of the recovery voltage data conversion from the time domain into the frequency domain is proposed. The method is based on the analysis of the recovery voltage transient based on the Maxwell equation for the current density in a dielectric. Unlike the previous published solutions, the Laplace transform was used to derive a formula suitable for practical purposes. The proposed procedure allows also calculating of the insulation resistance and separating the polarisation and conduction losses.

1. INTRODUCTION

Failure-free operation of equipments for electric power production and distribution is nowadays of first-rate importance. Failures of power transformers, generators and cables caused by the insulation breakdown give rise to considerable financial losses. They also can directly or indirectly endanger a human life. Therefore researchers have developed methods for electric insulation condition assessment and prediction of insulation operation life. The general tendency in the field of diagnosis methods is the application of so-called non-destructive methods, i.e. methods that work with the voltage, which does not exceed the operational voltage of insulation systems [1]. These methods do not affect the insulation life. Frequently used is a category of methods known as the dielectric spectroscopy. The dielectric spectroscopy is based on current or voltage measurements in the time domain and also capacitance and tan δ measurements in the frequency domain. Insulation resistance is measured in parallel with application of the dielectric spectroscopy methods. The assessment of an insulating system is based obviously on evaluating its dielectric response during a long-term operation. Here, the well-known methods of absorption current and recovery voltage measurement can be used. The recovery voltage method is better resistant to the background noise, but there are still problems with evaluation of the results and their conversion to the frequency domain. This problem is treated in the next lines.

2. THEORETICAL BACKGROUND

The dynamic properties of dielectrics are usually represented by the impulse response $h(t)$ - response of the system resulting from Dirac impulse $\delta(t)$. Supposing that the electric field is the system input signal and the polarisation $P(t)$ is the system output signal, the system response to an arbitrary electric field $E(t)$ is

$$P(t) = \varepsilon_0 \int_0^t h(t-\lambda)E(\lambda)d\lambda. \qquad (1)$$

The electric field $E(t)$ generates a total current density $i(t)$ which is the sum of conduction, vacuum and displacement current densities [2]:

$$i(t) = \gamma_0 E(t) + \varepsilon_0 \frac{dE(t)}{dt} + \varepsilon_0 \frac{d}{dt}\int_0^t h(t-\lambda)E(\lambda)d\lambda, \qquad (2)$$

where γ_0 is the specimen conductivity which does not depend on time or frequency. If the input signal is a *step* electric field E_0, the current density is given

$$i(t) = \gamma_0 E_0 + \varepsilon_0 E_0 \delta(t) + \varepsilon_0 E_0 h(t). \qquad (3)$$

The function $h(t)$ is known from literature as the dielectric response of the system. As it is seen from (3), the dielectric response can be measured as a variable part of the current after application of a step voltage to the measured object. The complex dielectric susceptibility χ is simply the Fourier transform of the dielectric response

$$\chi(\omega) = \chi' - j\chi'' = \int_0^\infty h(t)\exp(-j\omega t)dt. \qquad (4)$$

As the measurements of polarisation currents in dielectrics are often influenced by noise, the measurement of recovery voltage is preferably used instead of it. During the recovery voltage measurement a constant voltage U_0 (electric field E_0) is applied to the test object for $0 \le t < t_1$. This time period must be long enough for settling of all the polarisation processes. In the time period $t_1 \le t < t_2$ is the object short-circuited and then it is left in open-circuit condition. The voltage on the test object is measured for $t_2 \leq t$ with a high input impedance voltmeter. Transformation of recovery voltage data from the time domain into the frequency domain can be accomplished by solving the transient of recovery voltage. Two problems arise from the recovery voltage measurements. The first is the forward problem, i.e. the calculation of the voltage from the known values of γ_0 and $h(t)$. The second is the inverse problem – calculation of γ_0 and $h(t)$ from the measured time dependence of recovery voltage [3]. In fact, the inverse problem is mostly solved by applying methods for the forward problem solution with optimisation of γ_0 and $h(t)$.

The known solutions of the forward problem can be divided into the two categories:

- the solution based on modelling the recovery voltage transient by means of an equivalent circuit,
- the solution based on solving equation (2) for the current flowing through the dielectric during the test.

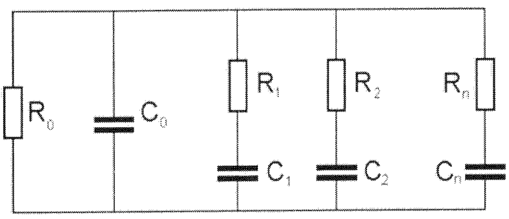

Fig. 1. Equivalent circuit for modelling the recovery voltage

The equivalent circuit used for modelling is in Fig 1. Here C_i, R_i are the individual elements of the circuit, n is the number of elements, C_0 is the capacitance corresponding to the optical permittivity and R_0 is the resistance corresponding to the steady state conductivity. Let $u_i(t)$ ($i = 1, 2, \ldots, n$) are voltages across the capacitances C_i and $u_r(t)$ is the voltage across the capacitance C_0. Then the next set of differential equations is valid for $t_2 < t$

$$\frac{du_i}{dt} = \frac{1}{R_i C_i}(u_r - u_i) \qquad i = 1, 2, \ldots, n \tag{5}$$

$$\frac{du_r}{dt} = \frac{1}{C_0}\left[-\frac{u_r}{R_0} - \sum_{i=1}^{n}\frac{u_r - u_i}{R_i}\right]. \tag{6}$$

The initial value of u_i (assigned as U_i) depend on the charging and discharging period

$$U_i = U_0\left[1 - \exp\left(-\frac{t_1}{R_i C_i}\right)\right]\exp\left(-\frac{t_2 - t_1}{R_i C_i}\right). \tag{7}$$

The system of equations (5, 6) can be solved by using the Laplace transform. Denoting the Laplace transform of $u_r(t)$ by $u_r(p)$ we have

$$u_r(p)\left(\frac{1}{R_0} + pC_0\right) = \sum_{i=1}^{n}(U_i - u_r(p))\frac{pC_i}{1 + p\tau_i}, \tag{8}$$

where $\tau_i = R_i C_i$.

We can calculate the inverse Laplace transform of $u_r(p)$ from equation (8) by using procedure of partial fraction expansion. In practice the values of C_0 and R_0 are known from some additional measurements. As the values of $u_r(t)$ are measured during the recovery voltage test, the unknown parameters in equation (8) are only the pairs (R_i, C_i). For finding the set of unknown parameters, optimisation methods are preferably used. A problem here is to find a good starting guess of parameters. If the starting guess is poor the calculation can be sensitive to round-off errors and the method does not converge. According to [4] a good approximation of the recovery voltage data was found already for the set with two pairs (R_i, C_i). A method similar to fitting absorption conductivity by a linear combination of exponentials modified for recovery voltage tests was also published [5].

For calculation of the recovery voltage transient, we can modify equation (2) to the form

$$I(t) = \frac{C_G}{\varepsilon_0}\left[\gamma_0 U(t) + \varepsilon_0\varepsilon_\infty\frac{dU(t)}{dt} + \right.$$

$$\left. + \varepsilon_0\frac{d}{dt}\int_0^t h(t-\lambda)U(\lambda)d\lambda\right], \tag{9}$$

where $I(t)$ is the current flowing through the dielectric during the recovery voltage test, $U(t)$ is the voltage on the object, ε_∞ is the optical permittivity of the dielectric and C_G is geometric or vacuum capacitance. Calculation of the dielectric response from equation (9) is not easy. Some authors use series of basis functions to solve it [6]. After applying the basis functions, a set of linear equations for parameters of the basis functions can be formatted. This is also difficult to solve as the matrix is badly conditioned.

Another way of solving equation (9) is numerical integration [3, 7]. First, the measured values of recovery voltage are approximated by a suitable function on both sides of the time interval. The convolution integral is calculated using e.g. trapezoidal rule. The whole equation is than integrated once again to eliminate the derivative $dU(t)/dt$.

The procedures described here in connection with the recovery voltage evaluation are relative complicated. They require some special routines and optimisation methods, which are sometimes not reliable. That is why we try to find an easier solution of the problem with help of the standard transformation methods.

3. DATA CONVERSION WITH HELP OF THE LAPLACE TRANSFORM

To start the data conversion with help of the Laplace transform, let us define the unit-step function $\nu(t)$ as: $\nu(t) = 1$ for $t \geq 0$, $\nu(t) = 0$ for $t < 0$. It is clear from the course of recovery voltage test, that the current through the measured object is zero in the time interval $t_2 \leq t$. This can be written mathematically as: $I(t)\ \nu(t-t_2) = 0$. The voltage $U(t)$ on the object during the test is

$$U(t)=U_0[v(t)-v(t-t_1)]+u_r(t), \qquad (10)$$

where $u_r(t)$ is the recovery voltage and U_0 is the charging voltage. The recovery voltage equals to zero for $t \leq t_2$. Under these assumptions, we can rewrite equation (9) in the form

$$0=v(t-t_2)\left[\gamma_0 U(t)+\varepsilon_0\varepsilon_\infty\frac{dU(t)}{dt}+\right.$$

$$\left.+\varepsilon_0\frac{d}{dt}\int_0^t h(t-\lambda)U(\lambda)d\lambda\right]. \qquad (11)$$

Before executing the Laplace transform, we must substitute the expression for $U(t)$ into equation (11) and also partially modify the convolution integral in it:

$$\frac{d}{dt}\int_0^t h(t-\lambda)U(\lambda)d\lambda=U_0[h(t)-h(t-t_1)]+$$

$$+\frac{d}{dt}\int_{t_2}^t h(t-\lambda)u_r(\lambda)d\lambda. \qquad (12)$$

For calculating the Laplace transform of a product of an arbitrary function $f(t)$ and $v(t-t_2)$ we use formula

$$L[f(t)]=\int_0^{t_2} f(t)\exp(-pt)dt+L[f(t)v(t-t_2)], \qquad (13)$$

where L denotes the Laplace transform. By using equations (11), (12) and (13) we have

$$u_r(p)=-\frac{U_0X(p)}{\frac{\gamma_0}{\varepsilon_0}+p\varepsilon_\infty+ph(p)}, \qquad (14)$$

where $X(p)$ is given by

$$X(p)=[1-\exp(-t_1p)]h(p)-\int_0^{t_2} h(t)\exp(-pt)dt+$$

$$+\exp(-t_1p)\int_0^{t_2-t_1} h(t)\exp(-pt)dt. \qquad (15)$$

If the dielectric response $h(t)$ is of exponential type $h(t)$=A $\exp(-t/\tau)$, then the function $X(p)$ has the form

$$X(p)=\frac{A\exp(-pt_2)}{1+p\tau}[\exp(-\frac{t_2}{\tau})-\exp(-\frac{t_2-t_1}{\tau})]. \qquad (16)$$

The forward problem of computing the recovery voltage from the known values of γ_0 and $h(t)$ is resolved in this way by application of some standard procedure for the inverse Laplace transform to equation (14) with substitution (16).

If the dielectric response is of a complicated form, we can use an expansion into the sum of exponential functions and then perform the inverse Laplace transform as it was suggested above. This approximation is not necessary if the charging interval t_1 of the recovery voltage test is very long and the period when the object is short-circuited is negligible. Under these assumptions the formula (15) for $X(p)$ is reduced to

$$X(p)=-\exp(-t_1p)h(p), \qquad (17)$$

as the terms for $h(p)$ and the Laplace integral with upper limit t_2 are the same. Finally, we get for the recovery voltage

$$u_r(p)=\frac{U_0h(p)}{\frac{\gamma_0}{\varepsilon_0}+p\varepsilon_\infty+ph(p)}. \qquad (18)$$

The exponential term in (17) need not be considered as it causes only a shift of the t-axis origin.

Although the formula (18) is very simple, the solution of recovery voltage *inverse* problem requires more complicated procedure. The theoretical dielectric response has about 3 – 4 parameters. The total count of unknown parameters is larger, as we must include the steady state conductivity.

4. EXPERIMENTAL PART

The approach described in the preceding part was applied to the recovery voltage data measured on a PVC cable in the course of an accelerating ageing. A combine voltage-temperature ageing test was performed on 3-core PVC cable with the maximum temperature 80 °C and the ac voltage 12.5 kV. The whole test lasted 150 days. The recovery voltage measurements were made on 5 m length samples cut from the cable after 10, 50, 100 and 150 days of ageing. During the recovery voltage test a charging voltage of 100 V was applied to the sample for period of 5000 s. Next, the sample was short-circuited for 0.1 s by means of a computer-controlled relay and then connected to the high input impedance voltmeter. The recovery voltage was measured in period of 5000 s. For the recovery voltage measurements a Keithley 617 electrometer was used with build in source of charging voltage. The value of C_0 needed for calculations was measured by the HIOKI Z HiTESTER 3531 at 1 kHz.

The recovery voltage data were processed under the assumption that the equation (18) is valid. The dielectric response was approximated by the well-known Havriliak-Negami formula

$$h(p)=\frac{\Delta\varepsilon}{[1+(p\tau)^{1-\alpha}]^{\beta}}. \qquad (19)$$

As it was mentioned above, we have to reduce the number of unknown parameters as much as possible, so we use the measured value of $C_0=\varepsilon_\infty C_G$. In order to include the capacitance C_0 into equation (18), we must multiply both the numerator and denominator of (18) by the value of C_G. The unknown parameter γ_0 is then changed to R_0:

$$\frac{1}{R_0}=\frac{\gamma_0}{\varepsilon_0}C_G \qquad (20)$$

and also the parameter $\Delta\varepsilon$ is changed to $C_G\Delta\varepsilon$. The resistance R_0 and capacitance C_0 have already been introduced in connection with modelling the recovery voltage by an equivalent circuit.

The whole procedure of finding the unknown parameters R_0, $C_G\Delta\varepsilon$, τ, β and α is based on the inverse Laplace transform

$$u_r(t) = L^{-1}\left[\frac{U_0 C_G h(p)}{\frac{1}{R_0} + pC_0 + pC_G h(p)}\right]. \qquad (21)$$

The unknown parameters were optimised with help of a Nelder-Mead type simplex search method. The inverse Laplace transform was performed by algorithm published in [8]. It can be proved that the function (19) is strictly proper, i.e. after converting to the rational function, the degree of its denominator is higher then the one of the numerator. The function has no poles in the right-half complex plane. Therefore we can calculate the values of dielectric susceptibility by replacing the Laplace parameter p with $j\omega$ in (19). In fact, the procedure described here allows data conversion from the time domain into the frequency domain.

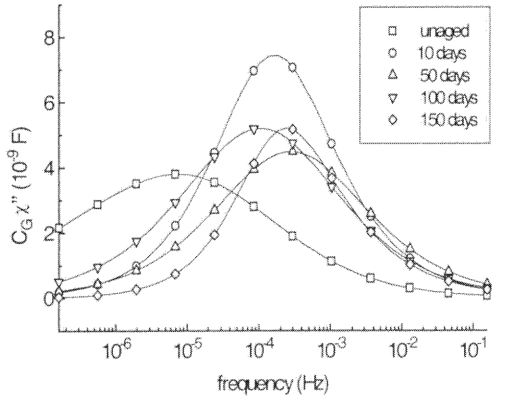

Fig. 2. Frequency dependence of $C_G\chi''$ with ageing time as parameter

Typical results of the experimental data conversion are shown in Fig. 2. As the primary objective of this paper is the presentation of a new conversion method, we will not accomplish a physical analysis of the cable ageing phenomena. We can just state, that a polarisation process was observed in all the specimens. This process seems to be weak in non-aged sample. On the other hand, an unusually high increment of $C_G\Delta\varepsilon$ occurs in the aged samples. The relaxation time of the process depends slightly on the ageing duration. A probable explanation of this situation is formation of a migration polarisation following a non-uniform temperature distribution in the bulk of insulating system during ageing.

5. CONCLUSION

A new method of the recovery voltage data conversion from the time domain into the frequency domain was developed and verified. The method is based on the analysis of the recovery voltage transient based on the equation for the current density in a dielectric. Unlike the previous published solutions, the Laplace transform was used to derive a formula suitable for practical purposes. The formula can be applied in the exact form (14, 15) or in the simplified form (18). The only necessary condition for using it, is the knowledge of an analytical form of the dielectric response. According to our tests, the formula works well with various types of dielectric responses. The proposed procedure allows also calculation of the insulation resistance (conductivity) and separation of the polarisation and conduction losses. Last but not least, the procedure works with the Laplace transform of an analytical formula. We need not perform any transformation of the measured data influenced by an instrument error.

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