

# Research on the Effects of Monetary Policy Shocks: an Empirical Analysis based on VAR Model

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**Abstract** –In this paper, we estimate a VAR model to present an empirical finding that an unexpected rise in the federal funds rate decreases the ratio of sales to stocks available for sales, while it increases finished goods inventories. In addition, dynamic responses of these variables reach their peaks several quarters after a monetary shock. In order to understand the observed relationship between monetary policy and finished goods inventories, we allow for the accumulation of finished goods inventories in an optimizing sticky price model, where prices are set in a staggered fashion. In our model, holding finished inventories helps firms to generate more sales at gave their prices. We then show that the model can generate the observed relationship between monetary shocks and finished goods inventories. Furthermore, we find that allowing for inventory holdings leads to a Phillips curve equation, which makes the inflation rate dependent on the expected present-value of future marginal cost as well as the current period's marginal cost and the expected rate of future inflation.

**Keywords** –Monetary Policy; Inventory Dynamics; Sticky Price; VAR model

## 1. Introduction

Much of business cycles literature has emphasized that inventory behavior is an important factor in understanding the character of the aggregate business fluctuations. An important research topic on inventory behavior is why the inventory investment is procyclical. Recent works in the literature on inventory behavior also have stressed that it is important to explain why inventory investment is not more procyclical over phases of business cycles. For example, Bils and Kahn (2000) shows that manufacturer's finished goods inventories are less cyclical than shipments.

In this paper, we analyze the role of monetary policy shocks in generating the observed sluggish adjustment of inventory stocks. To this end, we estimate a vector auto regression for a set of selected aggregate variables, which includes the ratio of sales to the stock available for sales as well as the real GDP, the inflation rate, and the federal funds rate. We then show that the real GDP and the ratio of sales to stocks increase in response to an expansionary monetary shock. Besides, the expansionary monetary shock decreases the stock of finished goods inventories measured at the end of each period, while its dynamic responses reach their minimum several quarters after the monetary shock. In an attempt to understand such behaviors of inventories in response to monetary shocks, we present a dynamic stochastic general equilibrium model. In this model, firms set prices as in the staggered price-setting model of Calvo (1983) and hold finished goods inventories to facilitate sales. Hence, our model incorporates the partial equilibrium model of Bils and

Kahn (2000) into a complete dynamic general equilibrium model with nominal price rigidity, which permits quantitative analysis on the effect of monetary shocks.

Our findings can be summarized as follows. First, it has been emphasized in the recent literature on the Phillips curve that the current period's inflation depends the aggregate real marginal cost as well as the expectation about the next period's inflation in a canonical closed-economy version of the classic Calvo model. In relation to this, we show that the current period's inflation rate depends on the expected present-value of the next period's marginal cost as well as the current period's marginal cost in the forward-looking Phillips curve equation when a fraction of firms make decisions on their prices and inventories at the same time. This is because when firms hold inventories, the opportunity cost of selling one unit in the current period is the expected present-value of the next period's marginal cost.

Second, we find that under both of a high depreciation of the inventory stock and a high elasticity of demand with respect to the stock available for sales, our model can generate the observed inventory dynamics in response to a monetary policy shock. The reason for this is because up to the first-order approximation, a sufficiently high depreciation rate helps to avoid excessive fluctuations of the finished-goods inventory in the model with only sale-expansion benefits from the inventory holdings.

Third, we take into account adjustment costs, which take place when a sales-stock ratio deviates from its fixed target ratio. We then demonstrate that the inclusion of

such adjustment costs in the model helps to match the observed variability of the finished goods inventories, if one wants to assume a small depreciation of the inventory stock. In particular, a key specification of the linear-quadratic cost function approach to the inventory behavior is the inclusion of the quadratic costs for deviations of sales-stock ratio from its fixed target ratio, as discussed in Ramey and West (1999). Besides, the quadratic adjustment costs described above reflect that holding inventories allows firms to satisfy their demands, which cannot be backlogged. Our findings therefore indicate that a joint specification of stock-out costs and sales-expansion benefits as incentives for holding finished goods inventories helps to understand the observed behavior of the finished goods inventories, for those goods that have small depreciation.

## 2. Effects of a Monetary Policy Shock on Inventory Dynamics

We begin our analysis by estimating the effects of a monetary policy shock on inventory dynamics. In order to identify a monetary policy shock, we estimate a unrestricted VAR for selected aggregate variables and then impose a set of structural restrictions on the variance-covariance matrix of its residual vector, which has been widely used in the literature since the work of Sims (1980).

The choice of variables included in the VAR is made to reflect the requirement that one can see the effects of monetary policy on the key aggregate variables and inventory dynamics at the same time. Hence, the sample we use in this paper consists on the real GDP, the GDP deflator inflation rate, the ratio of sales to stocks, the finished goods inventory stock measured at the end of period, and the federal funds rate. The unrestricted VAR we estimated in this paper can be then written as:

$$X_t = \Gamma_0 + \sum_{k=1}^4 \Gamma_k X_{t-k} + \mu_t$$

In order to identify monetary policy shocks, we follow the identification strategy employed in Christiano, Eichenbaum and Evans (2001). More explicitly, the

monetary policy of the central bank is described as follows:

$$r_t = f(\Omega_t) + \ell_{rt}$$

Where  $r_t$  is the federal funds rate,  $f$  is a linear function,  $\Omega_t$  is the information set at period  $t$ , and  $\ell_{rt}$  is the monetary policy shock. Here, we identify the federal funds rate as the monetary policy instrument. As an example of such an identification scheme, we can choose an ordering of the variables in  $X_t$  of the form:

$$X_t = \left[ \pi_t, \log Y_t, \log \frac{S_t}{A_t}, \log L_t, r_t \right]'$$

Where  $\pi$  is the inflation rate,  $Y$  is the output,  $S/A$  is the sales to stock ratio, and  $L$  is the inventory stock. The relationship between  $X_t$  and the vector of true shocks, denoted by  $e_t$ , is assumed to satisfy

$$D^{-1}X_t = \tilde{\Gamma}_0 + \sum_{k=1}^4 \tilde{\Gamma}_k X_{t-k} + De_t$$

For a concrete example of the identification of elements in the matrix  $D$ , we can use the variance-covariance matrix of the residual vector  $u$ . Suppose that variances of elements of  $e_t$  are assumed to be one. Comparing with the formulas above, we can see that the following relationship holds:

$$\Omega = DD'$$

Figure 1 reports impulse responses to an expansionary monetary shock. An expansionary monetary shock increases the rate of inflation, real GDP, and the ratio of sales to stocks, while it decreases the inventory stock of finished goods measured at the end of period. Besides, one can see that the sales-stock ratio as well as the inflation rate and real GDP show hump-shaped responses to an expansionary monetary policy shock, while the finished goods inventory stock displays U-shaped responses to the same shock.

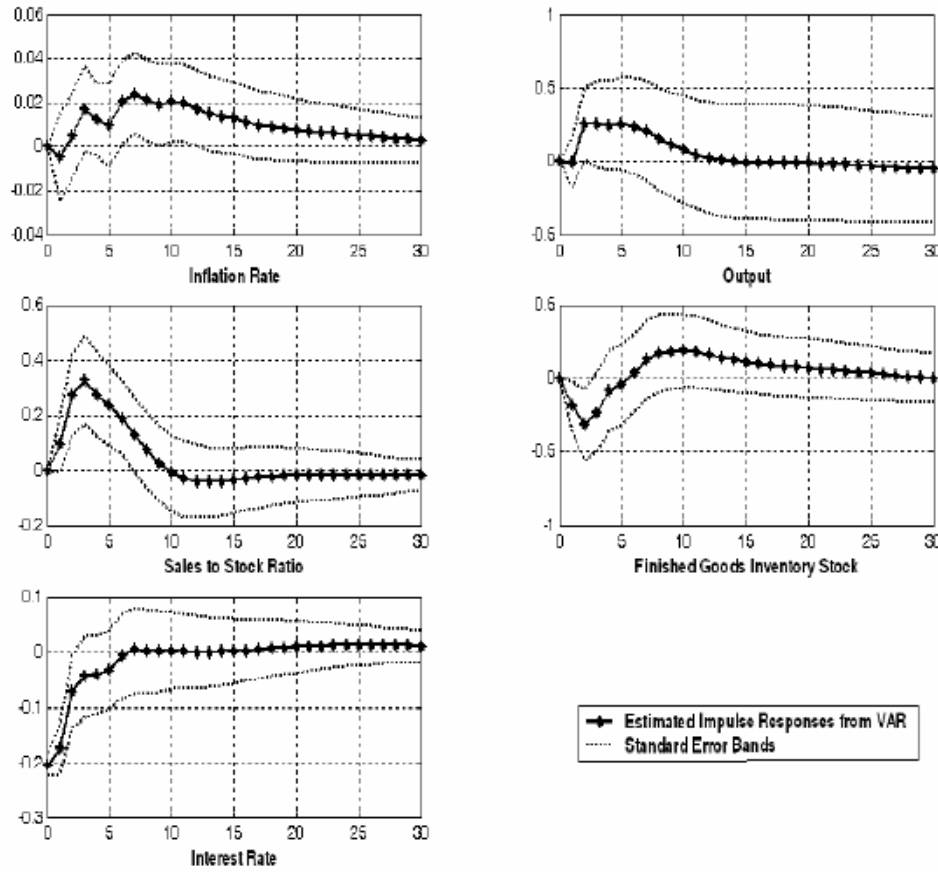


Figure 1. Estimated impulse responses from a VAR model

### 3. Model

This section presents a dynamic stochastic general equilibrium model with nominal price rigidity and inventory holding. Money is assumed to play only a role of unit of account, following recent literature on sticky price models. It is also assumed that it takes one period for private agents to observe monetary shocks. We do this to make the information set of households and firms consistent with the identification strategy for monetary shocks described in the previous section. Specifically, when  $I_t$  denotes the information set at period  $t$ , we assume that  $I_t$  includes all the past monetary policy shocks other than the monetary policy shock at period  $t$ . Hence, private agents do not observe the realization at period  $t$  of the monetary policy shock when they form their expectation about  $X_{t+1}$  based on  $I_t$ , which is denoted by  $E_t[X_{t+1}]$ .

#### 3.1 Firms

We assume that there are two classes of goods, depending upon whether to accumulate finished goods inventories. In what follows, goods that require accumulating their inventories are called "inventory goods", while goods that do not hold inventories are called "non-inventory goods".

Households and government purchase both two classes of goods for their consumption in each period  $t=0, 1, \dots$ . Specifically, an index of the two classes of goods is defined as:

$$S_t = \left( \gamma^{\frac{1}{\phi}} S_t^{\frac{\phi-1}{\phi}} + (1-\gamma)^{\frac{1}{\phi}} S_t^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, \phi > 0$$

Where  $S_t$  denotes the aggregate sales at period  $t$  for firms that hold inventories and  $S_t$  denotes the aggregate sales at period  $t$  for firms that do not require inventories. During each period, households minimize the total cost of obtaining  $S_t$ , which in turn leads to the following demand curves:

$$\bar{S}_t = \gamma \left( \frac{\bar{P}_t}{P_t} \right)^{-\phi} \times S_t; \quad \tilde{S}_t = (1-\gamma) \left( \frac{\tilde{P}_t}{P_t} \right)^{-\phi} \times S_t$$

Where  $\bar{P}_t$  denotes the price index for goods holding their inventories and  $\tilde{P}_t$  denotes the price index for goods that do not hold inventories. The aggregate price index, denoted by  $P_t$ , is now given by

$$P_t = \left( \gamma P_t^{1-\phi} + (1-\gamma) \tilde{P}_t^{1-\phi} \right)^{\frac{1}{1-\phi}}$$

Furthermore, there is a continuum of differentiated goods for each type of goods classes. Households purchase differentiated goods in the retail market and

combine them into composite goods using a Dixit-Stiglitz (1977) aggregator. More explicitly,  $S_t$  is defined as an index of differentiated goods:

$$S_t = \left( \int_0^1 \left( \frac{A_{jt}}{A_t} \right)^{\frac{\theta}{\varepsilon_i}} (S_{jt})^{\frac{\varepsilon_i-1}{\varepsilon_i}} dj \right)^{\frac{\varepsilon_i}{\varepsilon_i-1}}; \varepsilon_i > 1; 0 \leq \theta \leq 1$$

Where  $S_{jt}$  denotes differentiated goods of type  $j$  in the inventory goods class and  $A_{jt}$  is the stock of firm  $j$  available for sales at period  $t$ . In addition, the parameter  $\mu$  measures the elasticity of demand with respect to the amount of the stock available for sales and  $\varepsilon_i$  is the elasticity of demand for an individual firm with respect to its own price. In particular, as the parameter  $\theta$  takes a higher value between 0 and 1, holding inventory stock creates a larger effect on sales at given prices of goods.

Households minimize the total cost of obtaining differentiated goods indexed by a unit interval  $[0, 1]$ , taking as given their nominal prices  $P_{jt}$ . The cost-minimization then gives a demand curve of the form:

$$S_{jt} = \left( \frac{A_{jt}}{A_t} \right)^{\theta} \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_i} S_t$$

Where the price index for differentiated goods in non-inventory goods class, denoted by  $P_t$ , is defined to be

$$P_t = \left( \int_0^1 \left( \frac{A_{jt}}{A_t} \right)^{\theta} (P_{jt})^{1-\varepsilon_i} dj \right)^{\frac{1}{1-\varepsilon_i}}$$

Similarly, demand curves of differentiated goods in the non-inventory goods class is

$$S_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_n} S_t$$

Where output and price indices of the non-inventory goods class, respectively, are defined as

$$S_t = \left( \int_0^1 (S_{jt})^{\frac{\varepsilon_n-1}{n}} dj \right)^{\frac{\varepsilon_n}{\varepsilon_n-1}}; P_t = \left( \int_0^1 (P_{jt})^{1-\varepsilon_n} dj \right)^{\frac{1}{1-\varepsilon_n}}, \varepsilon_n > 1$$

### 3.2 Phillips Curve Equation

In this section, we consider a Phillips curve equation for the aggregate inflation rate on the basis of log-linear approximations to the pricing equations of firms. In doing so, we first log-linearize around the steady state with constant prices to yield

$$\bar{P}_t - \bar{P}_t = \frac{\alpha i}{1 - \alpha i} (\bar{\pi}_t - \xi \bar{\pi}_t - 1)$$

Where  $\pi_t (= \log P_t - \log P_{t-1})$  denotes the inflation rate

of the inventory goods sector. In addition, log-linearizing around the steady state with constant prices leads to

$$\sum_{k=0}^{\infty} (\alpha_i \beta)^k E_t \left[ \bar{P}_{t+k} - \bar{P}_t + k + \xi (\bar{P}_{t+k-1} - \bar{P}_{t-1}) \right] = 0$$

It then follows from that we can obtain a linear difference equation of the form:

$$\bar{P}_t - \bar{P}_t = -\alpha i \beta \xi \pi_t + (1 - \alpha i \beta) m s_t + \alpha i \beta E_t \times [\pi_t + t + (\bar{P}_{t+1} - \bar{P}_{t+1})]$$

Where  $m s_t$  denotes the log-deviation of  $MS_t$  from its steady state value. Thus, we can obtain a linear difference equation for the inflation rate of the inventory goods sector of the form:

$$\bar{\pi}_t - \xi \bar{\pi}_{t-1} = k m s_t + \beta E_t [\bar{\pi}_{t+1} - \xi \bar{\pi}_t]$$

Next, we turn to the derivation of the Phillips curve for the inflation rate for the non-inventory goods. In the similar way as we did above, log-linearizing these formulas and then rearranging leads to a linear difference equation for the inflation rate of the non-inventory goods sector:

$$\tilde{\pi}_t - \xi \tilde{\pi}_{t-1} = k \tilde{m} c_t + \beta E_t [\tilde{\pi}_{t+1} - \xi \tilde{\pi}_t]$$

In order to obtain a Phillips curve equation for the aggregate inflation rate, we add up two linear difference equations above. So that the Phillips curve can be written as

$$\pi_t = \frac{\xi}{1 + \xi \beta} \pi_{t-1} + \frac{\gamma k}{1 + \xi \beta} m s_t + \frac{(1 - \gamma) k}{1 + \xi \beta} \tilde{m} c_t + \frac{\beta}{1 + \xi \beta} E_t [\pi_{t+1}]$$

Where the aggregate inflation rate, denoted by  $\pi_t$ , is defined as a weighted average of inflation rates of inventory and non-inventory goods:

$$\pi_t = \gamma \bar{\pi}_t + (1 - \gamma) \tilde{\pi}_t$$

### 3.3 Inventory Dynamics

In order to derive a linearized law of motion for the inventory stock, we first consider log-linear approximations to the optimization condition for the stock-sales ratio. Specifically, log-linearizing formula leads to

$$S_t - a_t = \frac{1}{1 - (1 - \delta_a)\beta} \tilde{m} c_t - m s_t \times \frac{(1 - \delta_a)\beta(\mu - 1)}{(1 - (1 - \delta_a)\beta)(\mu - (1 - \delta_a)\beta)}$$

We can express the ratio of sales to stocks in terms of the aggregate real unit cost and changes in the marginal utility of consumption:

$$\bar{S}_t - \alpha_t = b_0 v_t - b_1 E_t[v_t + 1] - b_2 E_t[\lambda_{t+1} - \lambda_t]$$

Furthermore, log-linearizing this equation leads one to express the aggregate value-added output of the inventory goods sector in terms of the aggregate value-added output and relative price:

$$\bar{y}_t = y_t - (1 - \gamma)\phi p_{rt}$$

Log-linearizing this equation and then substituting formula into the resulting equation leads to a linear difference equation for the inventory stock of the form:

$$l_t = \rho l_{t-1} + \frac{1}{(1 - sM)sa} y_t - \frac{(1 - \gamma)\phi}{(1 - sM)sa} p_{rt}$$

It is then clear from this formula that it is necessary to have a law of motion for  $P_{rt}$ . Hence, substituting  $P_{rt}$  into the definition of  $P_t$ , we have:

$$p_{rt} = p_{rt-1} + \frac{1}{\gamma} (\pi_t - \tilde{\pi}_t)$$

## 4. Simulation Results

### 4.1 Calibration and Estimation

The computation of a numerical solution to the model requires assigning numbers to parameters of the model. Specifically, parameters of the model are partitioned into three classes. For the first class of parameters, we simply choose their values. For example, we set  $\sigma = 1$  and  $\chi = 1$ , which imply a logarithmic utility function for consumption and a quadratic function for the hours worked. We also set  $\beta = 0.99$ , which assumes that the average yearly real interest rate is 4 %. The other parameter values that belong to the first class are specified in Table 1.

Table 1. Parameter Values

| Parameter                                       | Values         |                |                  | Description and definitions                            |
|-------------------------------------------------|----------------|----------------|------------------|--------------------------------------------------------|
|                                                 | Fig. 1         | Fig. 2         | Fig. 3           |                                                        |
| Subset of parameters whose values are chosen    |                |                |                  |                                                        |
| $\sigma$                                        |                | 1              |                  | Inverse of inter-temporal substitution                 |
| $\chi$                                          |                | 1              |                  | Inverse of elasticity of labor supply                  |
| $\beta$                                         |                | 0.99           |                  | Time discount factor                                   |
| $s_M$                                           |                | 0.5            |                  | Share of material inputs in gross output               |
| $\xi$                                           |                | 0.99           |                  | Degree of indexation                                   |
| $\gamma$                                        |                | 0.5            |                  | Output share of inventory goods sector                 |
| $\phi$                                          |                | 0.5            |                  | Substitution elasticity of two sectors                 |
| $\frac{s}{\bar{\lambda}}$                       |                | 0.63           |                  | Average sales-stock ratio                              |
| $\bar{\mu}$                                     | 1.8            | 1.8            | 1.04             | Markup of non-inventory goods                          |
| $b$                                             | 0              | 0              | 0.76             | Degree of habit persistence                            |
| Subset of parameters whose values are estimated |                |                |                  |                                                        |
| $\delta_a$                                      | 0.58<br>(0.03) | 0.01<br>(0.02) | 0                | Depreciation rate of inventory                         |
| $\tau$                                          | 0              | 0.66<br>(0.03) | 5.80<br>(0.0002) | Share of adjustment costs in total costs               |
| $\rho_r$                                        | 0.85<br>(0.01) | 0.85<br>(0.01) | 0.91<br>(0.03)   | Partial adjustment coefficient in interest rate rule   |
| $\phi_\pi$                                      | 1.50<br>(0.04) | 1.00<br>(0.04) | 1.69<br>(0.44)   | Responsiveness to inflation rate in interest rate rule |
| $\phi_y$                                        | 0.60<br>(0.01) | 0.70<br>(0.02) | 0.61<br>(0.27)   | Responsiveness to output gap in interest rate rule     |
| $\alpha$                                        | 0.93<br>(0.01) | 0.97<br>(0.01) | 0.96             | Fraction of firms that re-optimize in each period      |
| $\theta$                                        | 0.89<br>(0.03) | 0.37<br>(0.03) | 0.80<br>(0.0002) | Elasticity of demand for goods with respect to stocks  |

### 4.2 Results

In this section, we report quantitative implications of the model we described. In particular, we ask if the model can generate observed dynamic responses of the sales-stock ratio and the finished goods inventories in response

to a monetary policy shock. Furthermore, we do this for models with and without adjustment costs.

Figure 2 demonstrates impulse responses to an expansionary monetary policy shock from the model without adjustment costs and compares them with estimated impulse responses from the VAR discussed

before. Figure 2 indicates that one needs a high level of depreciation in order to match the estimated impulse responses when there are no adjustment costs. It also shows that an exogenous fall in the interest rate increases

real output, the inflation rate and the ratio of sales to stocks but decreases the inventory stock of finished goods. This is consistent with the observed impulse responses from the VAR.

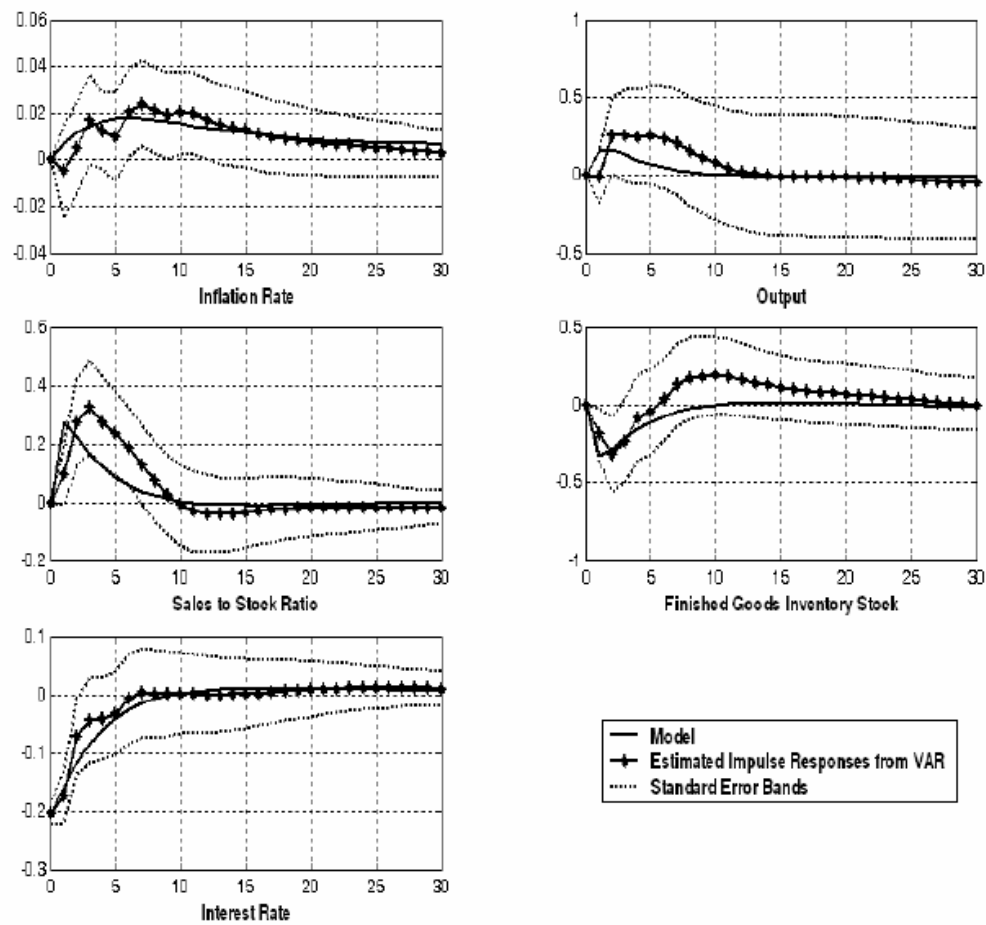


Figure 2. Impulse Responses: High Depreciation Rate without Adjustment costs

Figures 3 and 4 report impulse responses of models with adjustment cost that responding to an expansionary monetary policy shock. Figures 3 and 4 are constructed under the constraint that the depreciation rates of inventories are small. Specifically, we restrict the depreciation rate in an interval between 0 and 0.01 for Figure 2 and set the depreciation rate equal to zero for Figure 4. Figure 3, in particular, does not show hump-shaped responses of real output to an expansionary

monetary shock. Hence, we assume a certain degree of consumption habit persistence when we generate model impulse responses in Figure 4.

Figure 4 then indicates that the model with adjustment costs can generate dynamic responses of the selected variables consistent with the estimated ones from the VAR.

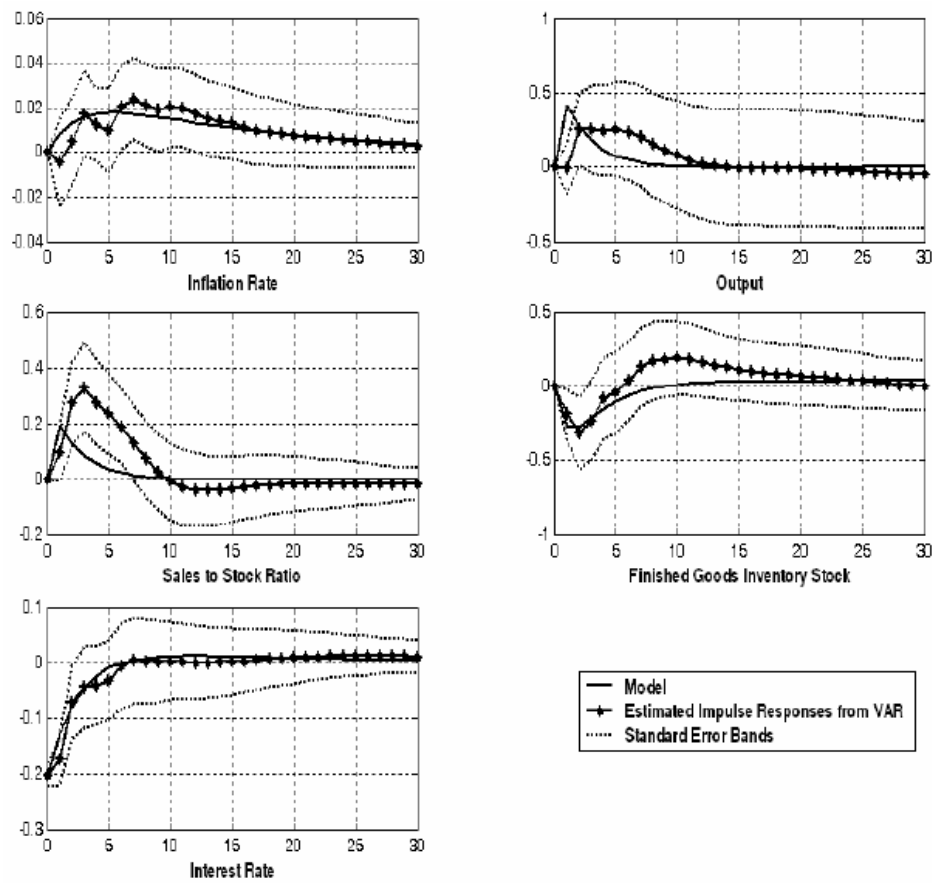


Figure 3. Impulse Responses: Quadratic Adjustment costs and Low Depreciation Rate

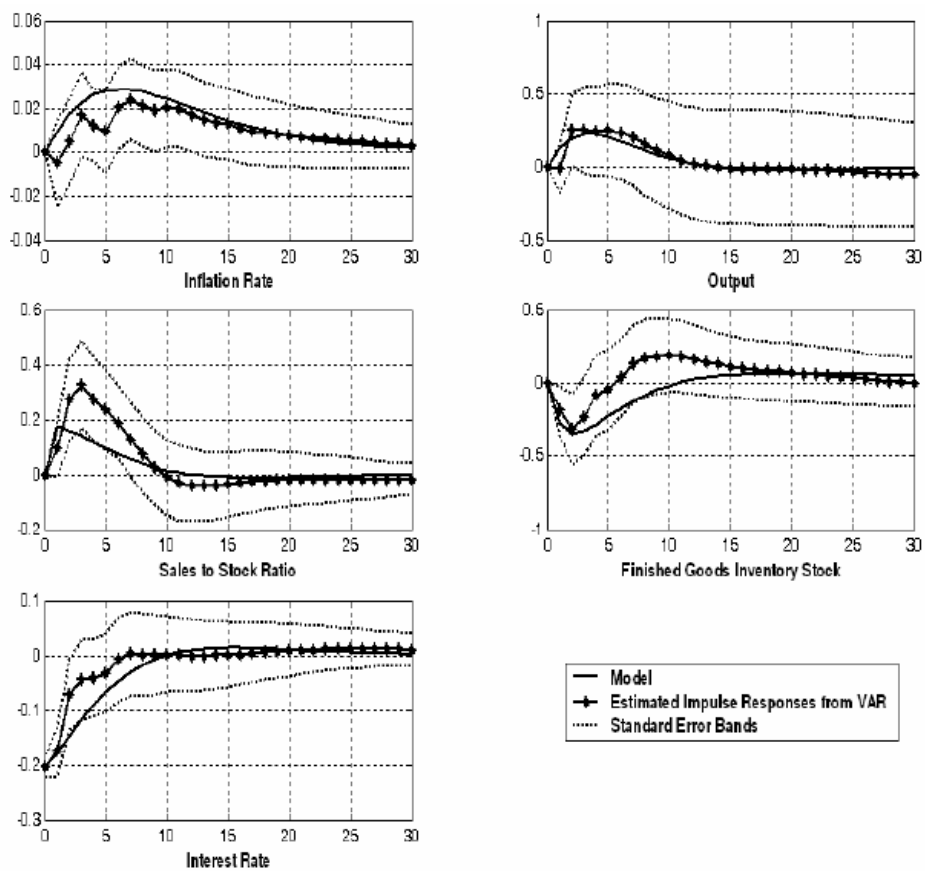


Figure 4. Impulse Responses: Quadratic Adjustment costs and Zero Depreciation Rate

## 5. Conclusion

We have investigated if a sticky price model with inventories can explain the observed dynamic responses of finished goods inventories in response to a monetary policy shock. We have demonstrated two alternative modeling strategies to match the observed variability of finished goods inventory dynamics in response to monetary policy shocks. One is to assume a large depreciation of the inventory stock in the absence of any additional mechanism to avoid excessive responses of inventories. The other is to assume a combination of a small depreciation rate and adjustment costs from deviations of sale-stock ratio from its target.

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## Vitae

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