

# Temperature distribution in a cylindrical shell with heat source inside the cylinder

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**Abstract:** In recent years Karkac et al. [6] have described that a great variety of engineering problems involve the transient heat transfer which can be divide into two groups, non-periodic and periodic. Recently several authors [1, 7, 8] have discussed indetaie the method of solving many important problems in transient heat conduction. This paper deals to determining the temperature distribution in a cylindrical shell with heat source inside the cylinder is of radius a, b height h and symmetrical along z-axis, having a heat source inside which leads axially symmetrical temperature distribution. The problem of determining the temperature distribution in a cylindrical shell with heat source inside the cylinder.

**Key Words:** Temperature Distribution; Heat Source; Symmetrical Temperature Distribution; Cylindrical Shell.

## 1. INTRODUCTION

If a solid body is suddenly subjected to a change in environment, some time will take before an equilibrium temperature condition will Prevail in the body. In the transient heating or cooling Process of a solid body which takes place in the interim period before equilibrium is established, the analysis must be modified to take into account the change in internal energy the body with time and the boundary conditions must be adjusted to match the physical situation which is apparent in the unsteady-state heat transfer problems.

Transient conduction takes place in the heating or cooling of various blanks and articles, glass manufacture, brick burning, vulcanization of rubber and during and starting and stopping of various-heat exchangers, power installation etc.

Karkac, s. and Yener [6] have described that a great variety of engineering problems involve the transient heat transfer which can be divide into two groups, non-periodic and periodic. Luknon, A.V. [7], Balachandra, V.K. and Desmurchd, R.M. [1], D.Ramamurthy and J. Sairam [8] have discussed indetaie the method of solving many important problems in transient heat conduction.

### 1.1 MAIN PROBLEM:

#### Temperature distribution in a cylindrical shell with heat source inside the cylinder

The problem of determining the temperature distribution in a cylindrical shell with heat source inside the cylinder. Let us consider a cylinder of radii a, b height h and symmetrical along z-axis, having a heat source inside which leads axially symmetrical temperature distribution. Let  $(r, \theta, z)$  be the cylindrical coordinate system and the heat is conducted symmetrically with respect to z-axis. The temperature function  $\theta$  is the function of space and time.

The heat conduction equation is given as

$$\rho c \frac{\partial \theta}{\partial t} = k \nabla^2 \theta + \xi(r, z, t, \theta) \quad (1.1.1)$$

where

$\xi(r, z, t, \theta)$  is a source function.

The use of substitutions

$$\xi(r, z, t, \theta) = \theta(r, z, t) + \varepsilon(t)\theta(r, z, t) \quad (1.1.2)$$

$$u(r, z, t) = \theta(r, z, t) \exp \left\{ - \int_0^t \varepsilon(y) dy \right\} \quad (1.1.3)$$

$$\psi(r, z, t) = \theta(r, z, t) \exp \left\{ - \int_0^t \varepsilon(y) dy \right\} \quad (1.1.4)$$

The heat conduction equation (1.1.1) reduces to

$$\frac{\partial u}{\partial t} = k \nabla^2 u + \frac{\psi(r, z, t)}{\rho} \quad (1.1.5)$$

Where  $k = \frac{K}{\rho c}$ ,  $k$  is the diffusivity

$K$  the thermal conductivity,  $\rho$  the density and  $c$  is the specific heat.

Here we take the composite cylinder of variable density and suppose

$$\rho = \rho_0 e^{-c_1 z} \quad (1.1.6)$$

Where  $\rho_0$  and  $C_1$  are constant.

The equation (1.1.5) reduces to

$$K \left[ \frac{\partial^2 u(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, z, t)}{\partial r} + \frac{\partial^2 u(r, z, t)}{\partial z^2} \right] + \frac{\xi(r, z, t)}{\rho_0 e^{-c_1 z}} - \frac{\partial u(r, z, t)}{\partial t} = 0 \quad (1.1.7)$$

$$u(r, z, t) |_{r=a} = f_1(z, t) \quad (1.1.8)$$

$$u(r, z, t) |_{r=b} = f_2(z, t) \quad (1.1.9)$$

$$u(r, z, t) |_{z=0} = f_3(t) \quad (1.1.10)$$

$$u(r, z, t) |_{z=h} = f_4(t) \quad (1.1.11)$$

$$u(r, z, t) |_{t=0} = c_2 \quad (1.1.12)$$

Where  $c_2$  is constant

## SOLUTION OF PROBLEM

Using finite Hankel transform between the limit  $a$  to  $b$  with respect to  $r$ , given by

$$\bar{f}(\beta_i) = \int_a^b r f(r) (J_v(\beta_i r) y_v(a\beta_i) - y_v(\beta_i r) J_v(a\beta_i)) dr \quad (1.1.13)$$

with inversion series.

$$f(r) = \frac{\pi^2}{2} \sum_i \frac{\bar{f}(\beta_i) J_v^2(b\beta_i) \beta_i^2 [J_v(\beta_i r) y_v(a\beta_i) - y_v(\beta_i r) J_v(a\beta_i)]}{J_v^2(a\beta_i) - J_v^2(b\beta_i)} \quad (1.1.14)$$

Where summation over  $i$  extend over all the positive roots of the equation  $J_v(a\beta_i) y_v(b\beta_i) - y_v(a\beta_i) J_v(b\beta_i) = 0$  and the operation property is

$$\begin{aligned} & \int_a^b r \left( \frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} - \frac{v^2}{r^2} f \right) (J_v(\beta_i r) y_v(\beta_i a) - y_v(\beta_i r) J_v(\beta_i a)) dr \\ &= \frac{2}{\pi} \frac{J_v(a\beta_i)}{J_v(b\beta_i)} f(b) - \frac{2}{\pi} f(a) - \beta_i^2 \bar{f}(\beta_i) \end{aligned} \quad (1.1.15)$$

Now by equation (1.1.7), we get

$$\begin{aligned} & K \int_a^b r \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) (J_o(\beta_i r) y_o(\beta_i a) - y_o(\beta_i r) J_o(\beta_i a)) dr + K \frac{\partial^2 \bar{u}(\beta_i, z, t)}{\partial z^2} \\ &+ \frac{\bar{\xi}(\beta_i, z, t)}{\rho_0} e^{c_1 z} - \frac{\partial \bar{u}(\beta_i, z, t)}{\partial t} = 0 \end{aligned} \quad (1.1.16)$$

or

$$\begin{aligned} & K \left[ \frac{2}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} u(b, z, t) - \frac{2}{\pi} u(a, z, t) - \beta_i^2 \bar{u}(\beta_i, z, t) \right] + K \frac{\partial^2 \bar{u}(\beta_i, z, t)}{\partial z^2} \\ &+ \frac{\bar{\xi}(\beta_i, z, t)}{\rho_0} e^{c_1 z} - \frac{\partial \bar{u}(\beta_i, z, t)}{\partial t} = 0 \end{aligned} \quad (1.1.17)$$

Now using boundary conditions (1.1.8) and (1.1.9), we get

$$\begin{aligned} & K \left[ \frac{2}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} f_2(z, t) - \frac{2}{\pi} f_1(z, t) - \beta_i^2 \bar{u}(\beta_i, z, t) \right] + K \frac{\partial^2 \bar{u}(\beta_i, z, t)}{\partial z^2} \\ &+ \frac{\bar{\xi}(\beta_i, z, t)}{\rho_0} e^{c_1 z} - \frac{\partial \bar{u}(\beta_i, z, t)}{\partial t} = 0 \end{aligned} \quad (1.1.18)$$

The finite Fourier Sine transform of  $f(z)$ ,  $0 < z < h$  is defined as

$$\bar{f}_s(m) = \int_0^h f(z) \sin \frac{m\pi z}{h} dz \quad (1.1.19)$$

Where  $m$  is an integer.

Then the inverse finite Fourier Sine transform of  $\bar{f}(m)$  is given by

$$f(z) = \frac{2}{h} \sum_{m=1}^{\infty} \bar{f}(m) \sin \frac{m\pi z}{h} \quad (1.1.20)$$

And operational property is given by

$$\int_0^h \frac{\partial^2 u}{\partial z^2} \sin \left( \frac{m\pi z}{h} \right) dz = \frac{m\pi}{h} [(-1)^{m+1} u(\beta_i, h, t) + u(\beta_i, 0, t)] - \frac{m^2 \pi^2}{h^2} \bar{u}(\beta_i, m, t) \quad (1.1.21)$$

Now using finite Fourier Sine transform between the limit 0 to  $h$  with respect to  $z$  given by the equation (1.1.19) in equation (1.1.18) we obtain

$$K \int_0^h \frac{\partial^2 \bar{u}}{\partial z^2} \sin \frac{m\pi z}{h} dz + \int_0^h \frac{\bar{\xi}(\beta_i, z, t)}{\rho_0} e^{c_1 z} \sin \frac{m\pi z}{h} dz - \frac{\partial \bar{u}}{\partial t} + K \left[ \frac{2}{\pi} \frac{J_0(a\beta_i)}{J_0(b\beta_i)} \bar{f}_2(m, t) - \frac{2}{\pi} \bar{f}_1(m, t) - \beta_i^2 \bar{u}(\beta_i, m, t) \right] = 0 \quad (1.1.22)$$

On using boundary conditions (1.1.10), (1.1.11) and operational property (1.1.21) in equation (1.1.22), we get

$$K \left[ \frac{m\pi}{h} (-1)^{m+1} f_4(t) + f_3(t) \right] - \frac{m^2 \pi^2}{h^2} K \bar{u}(\beta_i, m, t) + \bar{G}(\beta_i, m, t) - \frac{\partial \bar{u}}{\partial t} + K \left[ \frac{2}{\pi} \frac{J_0(a\beta_i)}{J_0(b\beta_i)} \bar{f}_2(m, t) - \frac{2}{\pi} \bar{f}_1(m, t) - \beta_i^2 \bar{u}(\beta_i, m, t) \right] = 0 \quad (1.1.23)$$

Where

$$\bar{G}(\beta_i, m, t) = \int_0^h \frac{\bar{\xi}(\beta_i, z, t)}{\rho_0} e^{c_1 z} \sin \frac{m\pi z}{h} dz \quad (1.1.24)$$

Now using Laplace transform between the limit 0 to  $\infty$  with respect to  $t$  in equation (1.1.23), we get

$$K \left[ \frac{m\pi}{h} (-1)^{m+1} \bar{f}_4(p) + \bar{f}_3(p) \right] - \frac{m^2 \pi^2}{h^2} K \bar{u}(\beta_i, m, p) + \bar{G}(\beta_i, m, p) - \int_0^\infty e^{-pt} \frac{\partial \bar{u}}{\partial t} dt + K \left[ \frac{2}{\pi} \frac{J_0(a\beta_i)}{J_0(b\beta_i)} \bar{f}_2(m, p) - \frac{2}{\pi} \bar{f}_1(m, p) - \beta_i^2 \bar{u}(\beta_i, m, p) \right] = 0 \quad (1.1.25)$$

$$K \left[ \frac{m\pi}{h} (-1)^{m+1} \bar{f}_4(p) + \bar{f}_3(p) \right] - \frac{m^2 \pi^2}{h^2} K \bar{u}(\beta_i, m, p) + \bar{G}(\beta_i, m, p) - [p \bar{u}(\beta_i, m, p) + \bar{u}(\beta_i, m, 0)] + K \left[ \frac{2}{\pi} \frac{J_0(a\beta_i)}{J_0(b\beta_i)} \bar{f}_2(m, p) - \frac{2}{\pi} \bar{f}_1(m, p) - \beta_i^2 \bar{u}(\beta_i, m, p) \right] = 0 \quad (1.1.26)$$

Now using boundary conditions (1.1.12), we get

$$\bar{u}(\beta_i, m, p) \left[ \frac{m^2 \pi^2}{h^2} K + p + \beta_i^2 K \right] = \frac{Km\pi}{h} \bar{f}_4(p) (-1)^{m+1} + \bar{f}_3(p) + \bar{G}(\beta_i, m, p) + \frac{2K}{\pi} \frac{J_0(a\beta_i)}{J_0(b\beta_i)} \bar{f}_2(m, p) - \frac{2K}{\pi} \bar{f}_1(m, p) + c_2 \quad (1.1.27)$$

then

$$\bar{u}(\beta_i, m, p) = \frac{Km\pi}{h} (-1)^{m+1} \frac{\bar{f}_4(p)}{\left[ \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K + p \right]} + K \frac{\bar{f}_3(p)}{\left[ \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K + p \right]} - \frac{2K}{\pi} \frac{\bar{f}_1(m, p)}{\left[ \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K + p \right]} + \frac{C_2}{\left[ \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K + p \right]} \quad (1.1.28)$$

Now using inverse Laplace transform and convolution theorem in equation (1.1.28), we get

$$\bar{u}(\beta_i, m, t) = \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K(t-s)} ds + K \int_0^t f_3(s) e^{-\left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K(t-s)} ds + \int_0^t \bar{G}(\beta_i, m, s) e^{-\left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K(t-s)} ds + \frac{2K}{\pi} \frac{J_0(a\beta_i)}{J_0(b\beta_i)} \int_0^t \bar{f}_2(m, s) e^{-\left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K(t-s)} ds - \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) e^{-\left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) K(t-s)} ds$$

$$e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + c_2 e^{-K\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)t} \quad (1.129)$$

Now using inverse Fourier Sine transform given by the equation (1.1.20), we get

$$\bar{u}(\beta_i, z, t) = \frac{2}{h} \sum_{m=1}^{\infty} \bar{u}(\beta_i, m, t) \sin \frac{m\pi z}{h} \quad (1.130)$$

Where  $\bar{u}(\beta_i, m, t)$  is given by equation (1.1.29), substituting the value of  $\bar{u}(\beta_i, m, t)$  from equation (1.1.29) in equation (1.1.30), we obtain

$$\begin{aligned} \bar{u}(\beta_i, z, t) = & \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + K \int_0^t f_3(s) \right. \\ & e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + \int_0^t \bar{G}(\beta_i, m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + \frac{2K}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} \\ & \int_0^t \bar{f}_2(m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds - \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds \\ & \left. + c_2 e^{-K\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)t} \right] \sin \frac{m\pi z}{h} \end{aligned} \quad (1.131)$$

On using Inversion theorem of finite Hankel transform given by the equation (1.1.14), we get

$$\begin{aligned} u(r, z, t) = & \frac{\pi^2}{2} \sum_i \left[ \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + K \right. \right. \\ & \int_0^t f_3(s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + \int_0^t \bar{G}(\beta_i, m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + \frac{2K}{\pi} \\ & \frac{J_o(a\beta_i)}{J_o(b\beta_i)} \int_0^t \bar{f}_2(m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds - \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) \\ & \left. e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + c_2 e^{-K\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)t} \right] \sin \frac{m\pi z}{h} \Bigg] \\ & \frac{J_o^2(b\beta_i)\beta_i^2[J_o(\beta_i r)y_o(a\beta_i) - y_o(\beta_i r)J_o(a\beta_i)]}{J_o^2(a\beta_i) - J_o^2(b\beta_i)} \end{aligned} \quad (1.132)$$

This gives temperature distribution in cylindrical shell.

## 1.2 PARTICULAR CASE

**Case I :-** Let us consider that the density  $\rho$  of the cylinder is constant I.e  $\rho = \rho_o$  then the equation (1.1.32) reduces to

$$\begin{aligned} u(r, z, t) = & \frac{\pi^2}{2} \sum_i \left[ \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + K \right. \right. \\ & \int_0^t f_3(s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + \int_0^t \left( \int_0^h \frac{\bar{\xi}(\beta_i, z, s)}{\rho_o} e^{c_1 z} \sin \frac{m\pi z}{h} dz \right) \\ & e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + \frac{2K}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} \int_0^t \bar{f}_2(m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds \\ & \left. - \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) e^{-\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)K(t-s)} ds + c_2 e^{-K\left(\frac{m^2\pi^2}{h^2}+\beta_i^2\right)t} \right] \sin \frac{m\pi z}{h} \Bigg] \\ & \frac{J_o^2(b\beta_i)\beta_i^2[J_o(\beta_i r)y_o(a\beta_i) - y_o(\beta_i r)J_o(a\beta_i)]}{J_o^2(a\beta_i) - J_o^2(b\beta_i)} \end{aligned} \quad (1.2.1)$$

**Case II:-** when source function is taken as

$\xi(r, z, t) = e^{-zt}$  then by equation (1.1.24) we obtain

$$\bar{G}(\beta_i, m, t) = \int_0^h \frac{e^{-zt}}{\rho_o} e^{c_1 z} \sin \frac{m\pi z}{h} dz$$

or

$$\bar{G}(\beta_i, m, t) = \frac{1}{\rho_o} \left[ \frac{e^{(c_1-t)z}}{(c_1-t)^2 + \frac{m^2\pi^2}{h^2}} \left( (c-t) \sin \frac{m\pi z}{h} - \frac{m\pi}{h} \cos \frac{m\pi z}{h} \right) \right]_0^h$$

Finally it becomes

$$\bar{G}(\beta_i, m, t) = \frac{1}{\rho_o} \left[ \frac{m\pi}{h} \frac{1}{(c_1 - t)^2 + \frac{m^2\pi^2}{h^2}} (1 + (-1)^{m+1} e^{(c_1-t)h}) \right] \quad (1.2.2)$$

Then on substituting the value of  $\bar{G}(\beta_i, m, t)$  from equation (1.2.2) in equation (1.1.29), we obtain

$$\begin{aligned} \bar{u}(\beta_i, m, t) &= \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + K \int_0^t f_3(s) \\ &e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \int_0^t \frac{1}{\rho_o} \frac{m\pi}{h} \frac{1}{(c_1 - s)^2 + \frac{m^2\pi^2}{h^2}} (1 + (-1)^{m+1} e^{(c_1-s)h}) \\ &\times e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \frac{2K}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} \int_0^t \bar{f}_2(m, s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds \\ &- \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + c_2 e^{-K\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)t} \end{aligned} \quad (1.2.3)$$

Now using inverse Fourier Sine transform given by the equation (1.1.20), we get

$$\begin{aligned} \bar{u}(\beta_i, z, t) &= \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + K \int_0^t f_3(s) \right. \\ &e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \int_0^t \frac{1}{\rho_o} \frac{m\pi}{h} \frac{1}{(c_1 - s)^2 + \frac{m^2\pi^2}{h^2}} (1 + (-1)^{m+1} e^{(c_1-s)h}) \\ &\times e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \frac{2K}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} \int_0^t \bar{f}_2(m, s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds \\ &\left. - \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + c_2 e^{-K\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)t} \right] \sin \frac{m\pi z}{h} \end{aligned} \quad (1.2.4)$$

Now using inversion theorem of finite Hankel transform given by (1.1.14) in equation (1.2.4), we get

$$\begin{aligned} u(r, z, t) &= \frac{\pi^2}{2} \sum_i \left[ \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{Km\pi}{h} (-1)^{m+1} \int_0^t f_4(s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \right. \right. \\ &K \int_0^t f_3(s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \int_0^t \frac{1}{\rho_o} \frac{m\pi}{h} \frac{1}{(c_1 - s)^2 + \frac{m^2\pi^2}{h^2}} \\ &(1 + (-1)^{m+1} e^{(c_1-s)h}) \times e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + \frac{2K}{\pi} \frac{J_o(a\beta_i)}{J_o(b\beta_i)} \int_0^t \bar{f}_2(m, s) \\ &e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds - \frac{2K}{\pi} \int_0^t \bar{f}_1(m, s) e^{-\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)K(t-s)} ds + c_2 \\ &\left. \left. e^{-K\left(\frac{m^2\pi^2}{h^2} + \beta_i^2\right)t} \right] \sin \frac{m\pi z}{h} \right] \frac{J_o^2(b\beta_i)\beta_i^2 [J_o(\beta_i r) y_o(a\beta_i) - y_o(\beta_i r) J_o(a\beta_i)]}{J_o^2(a\beta_i) - J_o^2(b\beta_i)} \end{aligned} \quad (1.2.5)$$

### Case III

when

$$\xi(r, z, t) = c_5 \quad (1.2.6)$$

than by equation (1.1.24), we get

$$\bar{G}(\beta_i, m, t) = \frac{c_5}{\rho_o} \left[ \frac{e^{c_1 h}}{\left(c_1^2 + \frac{m^2\pi^2}{h^2}\right)} \left(0 - \frac{m\pi}{h} \cos m\pi\right) - \frac{1}{\left(c_1^2 + \frac{m^2\pi^2}{h^2}\right)} \left(0 - \frac{m\pi}{h} 1\right) \right]$$

Finally it becomes

$$\bar{G}(\beta_i, m, t) = \frac{c_5}{\rho_o} \left[ \frac{e^{c_1 h}}{\left(c_1^2 + \frac{m^2\pi^2}{h^2}\right)} \left(-\frac{m\pi}{h} (-1)^m\right) + \frac{m\pi}{h} \frac{1}{\left(c_1^2 + \frac{m^2\pi^2}{h^2}\right)} \right]$$

$$\bar{G}(\beta_i, m, t) = \frac{c_5}{\rho_o} \frac{m\pi}{h \left( c_1^2 + \frac{m^2 \pi^2}{h^2} \right)} [1 + (-1)^{m+1} e^{c_1 h}] \quad (1.2.7)$$

and

$$f_1(t) = e^{-c_6 t} \quad (1.2.8)$$

$$f_2(t) = e^{-c_7 t} \quad (1.2.9)$$

$$f_3(t) = e^{-c_8 t} \quad (1.2.10)$$

$$f_4(t) = e^{-c_9 t} \quad (1.2.11)$$

Where  $c_6, c_7, c_8, c_9$  are constant now using equation (1.2.7) to (1.2.11) in equation (1.1.29) we get

$$\begin{aligned} \bar{u}(\beta_i, m, t) = & \frac{K m \pi}{h} (-1)^{m+1} \int_0^t e^{-c_9 s} e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K (t-s)} ds + K \int_0^t e^{-c_8 s} \\ & e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K (t-s)} ds + \int_0^t \frac{c_5}{\rho_o} \frac{m\pi}{h \left( c_1^2 + \frac{m^2 \pi^2}{h^2} \right)} (1 + (-1)^{m+1} e^{c_1 h}) \times \\ & e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K (t-s)} ds + \frac{2K}{\pi} \frac{J_o(a \beta_i)}{J_o(b \beta_i)} \int_0^t e^{-c_7 s} e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K (t-s)} ds - \frac{2K}{\pi} \\ & \int_0^t e^{-c_6 s} e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K (t-s)} ds + c_2 e^{-K \left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) t} \end{aligned} \quad (1.2.12)$$

or

$$\begin{aligned} \bar{u}(\beta_i, m, t) = & \frac{\frac{K m \pi}{h} (-1)^{m+1} \left( e^{-c_9 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_9 \right)} + \frac{1}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_8 \right)} \\ & \left[ e^{-c_8 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right] + \frac{c_5}{\rho_o} \frac{m\pi}{h \left( c_1^2 + \frac{m^2 \pi^2}{h^2} \right)} (1 + (-1)^{m+1} e^{c_1 h}) \\ & \times \frac{\left( 1 - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right)} + \frac{2K}{\pi} \frac{J_o(a \beta_i)}{J_o(b \beta_i)} \frac{\left( e^{-c_7 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_7 \right)} \\ & - \frac{2K}{\pi} \frac{\left( e^{-c_6 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_6 \right)} + c_2 e^{-K \left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) t} \end{aligned} \quad (1.2.13)$$

Now using inverse Fourier Sine transform given by the equation (1.1.20), we get

$$\begin{aligned} \bar{u}(\beta_i, z, t) = & \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{\frac{K m \pi}{h} (-1)^{m+1} \left( e^{-c_9 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_9 \right)} + \right. \\ & \frac{1}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_8 \right)} \left[ e^{-c_8 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right] + \frac{c_5}{\rho_o} \frac{m\pi}{h \left( c_1^2 + \frac{m^2 \pi^2}{h^2} \right)} \\ & (1 + (-1)^{m+1} e^{c_1 h}) \times \frac{\left( 1 - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right)} + \frac{2K}{\pi} \frac{J_o(a \beta_i)}{J_o(b \beta_i)} \\ & \left. \frac{\left( e^{-c_7 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_7 \right)} - \frac{2K}{\pi} \frac{\left( e^{-c_6 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_6 \right)} + c_2 e^{-K \left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) t} \right] \\ & \sin \frac{m \pi z}{h} \end{aligned} \quad (1.2.14)$$

On using inverse Hankel transform given by the equation (1.1.14), we get

$$\begin{aligned}
u(r, z, t) = & \frac{\pi^2}{2} \sum_i \left[ \frac{2}{h} \sum_{m=1}^{\infty} \left[ \frac{K m \pi}{h} (-1)^{m+1} \left( e^{-c_9 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right) \right. \right. \\
& \left. \left. \frac{1}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_9 \right)} \right] + \frac{c_5}{\rho_o} \frac{m \pi}{h} \frac{1}{\left( c_1^2 + \frac{m^2 \pi^2}{h^2} \right)} \left( 1 + \right. \right. \\
& \left. \left. (-1)^{m+1} e^{c_1 h} \right) \times \frac{\left( 1 - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right)} + \frac{2K}{\pi} \frac{J_o(a \beta_i)}{J_o(b \beta_i)} \frac{\left( e^{-c_7 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_7 \right)} \right. \\
& \left. - \frac{2K}{\pi} \frac{\left( e^{-c_6 t} - e^{-\left(\frac{m^2 \pi^2}{h^2} + \beta_i^2\right) K t} \right)}{\left( K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) - c_6 \right)} + c_2 e^{-K \left( \frac{m^2 \pi^2}{h^2} + \beta_i^2 \right) t} \right] \sin \frac{m \pi z}{h} \left. \right] \\
& \frac{J_o^2(b \beta_i) \beta_i^2 [J_o(\beta_i r) y_o(a \beta_i) - y_o(\beta_i r) J_o(a \beta_i)]}{J_o^2(a \beta_i) - J_o^2(b \beta_i)} \quad (1.2.14)
\end{aligned}$$

## REFERENCES

- [1] Bhalchand, V.K. and Desmochd, R.M. 'Heat Transfer'. Prentice Hall of India P. Ltd., New Delhi, (1939).
- [2] Birkhoof, G. and Rota, G.C., Ordinary differential Equations, Bhaiddell Publishing CP., Waltham, Mass, (1969).
- [3] Desh Pande, V.L., On conduction of Heat in a semi infinite circular cylinder kyung pook Math Journal Volume 13, Number-1, June-1973.
- [4] Erdelyi, A., Tables of integral transforms, volume I. McGraw Hill, New York (1954).
- [5] Grandohtein, I.S. and Ryzhik, I.W., Tables of integrals, series and Product, Academic Press, New-York (1965).
- [6] Kakac, S and Yener, "Heat conduction", Middle East Technical University Publications, Number 60, Ankara, (1979).
- [7] Lyknow, A.V., Theory of Heat conduction Vysshaya Shkola, MOSCOW, (1972).
- [8] Ramamurthy D. and Sairam J., The mathematics education, Vol. XXXI, No.2, June (1997).
- [9] Siddique, A., A Class of Generalized Transform and their Application in the boundary value problem of Heat Conduction. Kyung pook mathematical Journal volume 30, Number 2, December, (1990).
- [10] Sneddon, I.N., Special functions of Mathematical Physics and chemistry, oliver and Boyd Ltd. (1966).
- [11] Sneddon, I.N., The use of integral transform by Mc-Graw-Hill. Inc (1972).
- [12] Sneddon, I.N., Fourier Transforms, MC-Graw-Hill, Book company, Inc. New-York (1951).

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