

# The auction algorithm for shortest path in the dynamic traffic network

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**Abstract** -Through modifying Auction Algorithm for the static shortest path, a new searching algorithm based on dynamic travel time was proposed. Taking the deviation between the searching in the static traffic network and the dynamic travel time related to the actual traffic into consideration, the searching of shortest paths in the dynamic network was chose. Firstly, the general spatial traffic network was enlarged to time-space network based on the definite expanding principles. Through this enlargement, the problem of searching dynamic shortest paths can be translated into the general shortest path problem. This translation can supply a basis for using the static shortest path searching methods in the dynamic traffic network. Then the new auction algorithm used for searching dynamic shortest paths was proposed that was concise without the help of the time-space network. Finally, the applicability of the new algorithm's was proved by a simple example.

**Keywords** - traffic engineering; Dynamic road network; Time-space network; Auction algorithm

## 1. Introduction

By the city road network traffic flow dynamic operating characteristics of the road network, road section travel time is changing all the time. However, in the most short-circuit research areas, most of the studies using the static route search, i.e. given within a certain period of time the traffic flow state is stable, the link travel time constant, use of existing algorithms in static network path search. This and the actual network dynamic existence certain deviation, therefore, this paper is dedicated to the construction of applied to dynamic path search algorithm.

In this paper, firstly, using the time-space network concepts, interpretations of static route search method in the dynamic network application, put forward to advance the auction algorithm from the time-space network simplicity. Then, the new algorithm is established based on the basic auction algorithm, and an example to verify the validity of the new algorithm.

## 2. Time-space network

The so-called time-space network, is determined by the time dimension and spatial dimension representation of dynamic road network .In the plane, transverse axis direction of said space distance, the direction of the longitudinal axis of said time period, so that to establish a correspondence between time and space, reflecting the road network traffic flow distribution.

### 2.1 Create a time-space network

Assuming a simple space transportation network, as shown in Figure 1,  $r$  is generating node,  $i$  is the middle node,  $s$  is attract node, the arrows indicate the sections. Firstly, time is divided into several periods, in each time period, this simple spatial network is repeatedly reproduced, corresponding to each node to expand the number of nodes,  $i_1, i_2, \dots, i_k$ , each period has its own travel time, set up as shown in Figure 2 time-space network. In the time space network, set up a virtual generation node  $r$ , a virtual attract node  $s$ , From virtual generation node  $r$  to each time interval generation node  $r_i$  virtual sections connected, in these

virtual road travel time is zero, similarly, each time attracting node  $s_i$  to the virtual node  $s$  with a virtual attract arc connection, and a virtual link the travel time is zero. If the time partition properly, OD may be better travel across several time section. For example, if the target at the second time slot at a time from a starting point, to the next node in the travel time is 3 time units. In the time space diagram, from  $r_2$  target cost 3 a unit of time travel time to reach the  $i_5$  node. Different time periods in various sections of the travel time based on the road resistance function to determine. Thus, simple spatial network is converted to reflect changes in the temporal network traffic flow. Through time and space concept of dynamic traffic network, the shortest path problem is converted into general shortest path problem: always from the virtual node  $r$  sets out, search the shortest path to virtual terminal  $s$ .

Therefore, in the dynamic network can also use static path search algorithm to solve the shortest path problem. However, the establishment of spatial network is a tedious work, need to know the time in various sections of the travel time. To solve this problem, this paper establishes a new algorithm, the path search, does not need to establish the time-space network, need only given sections of road impedance function, which will greatly reduce the search time.

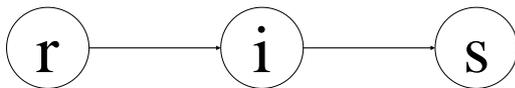


Figure1. A simple network

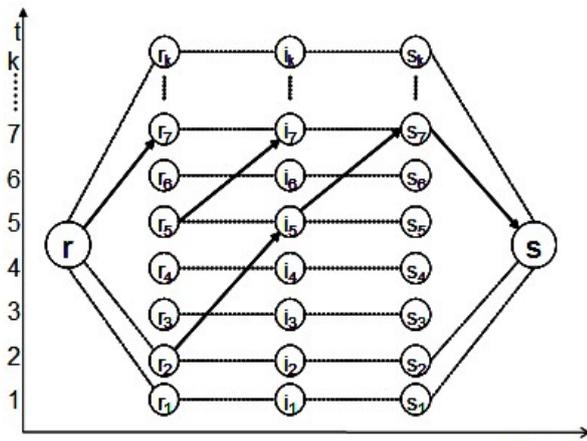


Figure2. Time-space network

### 3. Auction algorithm for the shortest paths

#### 3.1 auction algorithm

The auction algorithm for the shortest path problems derived from the auction algorithm for the assignment problem. Let be a directed network defined by a set of nodes and a set  $A$  of  $m$  directed arcs. Each arc  $(i, j) \in A$

has an associated length (or arc cost)  $t_{ij}$ . The fundamental principle of the auction algorithm for the shortest paths was described in its simplest form for the single origin and single destination case.

To simplify the presentation, we have the following assumptions:

- (1) All cycles have positive lengths.
- (2) Each node except for the destination has at least one outgoing incident arc. Any node not satisfying this condition can be connected to the destination with a large arc length without changing the problem and the subsequent algorithm materially.
- (3) There is at most one arc between two nodes in each direction so that we can refer to an arc  $(i, j)$  unambiguously. Again, this assumption is made for notational convenience and the algorithm can be trivially extended to the case where there are multiple arcs connecting a pair of nodes.

Let node  $r$  be the origin node and let  $s$  be the destination node. In the following assumptions, a path is denoted by a sequence of nodes  $(r, i_1, i_2, \dots, i_k)$  so  $(i_m, i_{m+1})$  is an arc for all  $(m = 1, \dots, k - 1)$ . In addition, if the nodes  $r, i_1, i_2, \dots, i_k$  are distinct, the sequence  $r, i_1, i_2, \dots, i_k$  of is called a simple path. The length of a path is defined to be the sum of its arc lengths.

The algorithm maintains a simple path  $R = (r, i_1, i_2, \dots, i_k)$  at all times. If  $i_{k+1}$  is a node that does not belong to a path  $R = (r, i_1, i_2, \dots, i_k)$  and is an  $(i_k, i_{k+1})$  arc, extending  $R$  by  $i_{k+1}$  means replacing  $R$  by the path  $(r, i_1, i_2, \dots, i_k, i_{k+1})$ , which is called the extension of  $R$  by  $i_{k+1}$ . If  $R$  does not consist of just the origin node  $r$ , contracting  $R$  means replacing  $R$  with the path  $R = (r, i_1, i_2, \dots, i_{k-1})$

The algorithm also maintains a variable  $p_i$  for each node  $i$  (called price of  $i$ ) satisfying (Complementary Slackness or CS for short):

$$p_i \leq t_{ij} + p_j, \quad \forall (i, j) \in A \quad (1a)$$

$$p_i = t_{ij} + p_j, \quad \forall (i, j) \in R \quad (1b)$$

It can be shown that if a pair  $(R, p)$  satisfies the CS conditions, then the portion of  $R$  between node 1 and any node  $i \in R$  is the shortest path from  $r$  to  $i$ , while  $p_r - p_i$  is the corresponding shortest distance.

Assuming that an initial pair  $(R, p)$  satisfying CS is available. This is not a restrictive assumption when all arc lengths are nonnegative, since then one can use the default pair

$$R = R(r), p_i = 0, \forall i \in R \quad (2)$$

Initially,  $(R, p)$  is any pair satisfying CS. The algorithm proceeds in iterations, transforming a pair  $(R, p)$  satisfying CS into another pair satisfying CS. At each iteration, the path  $R$  is either extended by a new node or contracted by deleting its terminal node. In the latter case the price of the terminal node is increased strictly. A degenerate case occurs when the path consists of just the origin node  $r$ ; in this case the path is either

extended or left unchanged with the price  $p_r$  being strictly increased.

The iteration is as follows:

Let  $i$  be the terminal node of  $R$ . If  $p_i < \min_{(i,j) \in A} (t_{ij} + p_j)$  go to Step 1; otherwise, go to Step 2.

Step 1: Contract path: Set  $p_i = \min_{(i,j) \in A} (t_{ij} + p_j)$ , and if  $i \neq r$ , contract  $R$  and go to the next iteration.

Step 2: Extend path: Extend  $R$  by node  $j_i$  where  $j_i = \arg \min_{(i,j) \in A} (t_{ij} + p_j)$ . If  $j_i$  is the destination  $s$ , stop;  $R$  is the desired shortest path. Otherwise, go to the next iteration.

### 3.2 The auction algorithm in dynamic network

Given  $t \in T$ ,  $T$  expressed a certain period of time. Given road impedance function  $C$ .  $C_{ij}^t$  is in  $t$  time, travel time of section  $(i, j)$ . Set another parameter  $U_i^t$ , is in  $t$ , before starting from before node of  $i$ , travel time of node  $r$  to node  $i$ . Different from the basic algorithm, Nodes in different time has a corresponding price  $p_i^t$ , that at the moment  $t$  of arrival in the net before the node, node  $i$  price. Different time each parameter has a corresponding parameter values, showing dynamic road network time-varying, reflect the traffic flow state changes.

New algorithm for the selection of the initial value of CS meets conditions:

$$R = R(r); p_i^t = 0, \forall i \in R, \forall t \in T \quad (3)$$

$$U_i^t = \begin{cases} 0 & i = r \\ C_{ri}^t & (r, i) \in A \\ \infty & (r, i) \notin A \end{cases} \quad (4)$$

Hypothesis  $i$  is the end of path 2, if  $i$  no arc and  $i = r$ , end the iteration, no solution; or else turn step, iterative.

Step one:

if there is no outflow arc from  $i$ , path  $R$  contraction, turn the next iteration.

if there is an outflow arc from  $i$ , and  $i = r$ , step two;

if there is an outflow arc from  $i$ , and  $i \neq r$ , for each piece of outflow arc  $(i, j) \in A$ , definition:

$$U_j^{t+U_i^t} = U_i^t + C_{ij}^{t+U_i^t} \quad (5)$$

Wherein  $U_j^{t+U_i^t}$  said at the moment  $t+U_i^t$ , the travel time from  $i$  to  $j$ , go to step two;

Step two:

If  $p_i^t < \min_{(i,j) \in A} (C_{ij}^{t+U_i^t} + p_j^{t+U_i^t})$ , go to step three, otherwise the step four;

Step three:

$p_i^t := \min_{(i,j) \in A} (C_{ij}^{t+U_i^t} + p_j^{t+U_i^t})$ ,  $R$  contraction, into the next iteration;

Step four:

extension path  $R$  to node  $j_i$ . Turn to the next iteration.

Finally, termination, path is the shortest path.

### 4. Example

In this example, to find the most short circuit from  $r$  to  $s$ , wherein,  $i, j$  is intermediate nodes. The various sections of the road impedance function are  $C_{rj} = (t-7)^2 + 4$ ;  $C_{ri} = (t-7)^2 + 1$ ;  $C_{ij} = (t-8)^2 + 5$ ;  $C_{js} = (t-14)^2 + 3$ ;  $C_{is} = (t-8)^2 + 10$ ,  $t$  is the moment of into sections.

Let  $t_0 = 7$ , start from  $r$ .

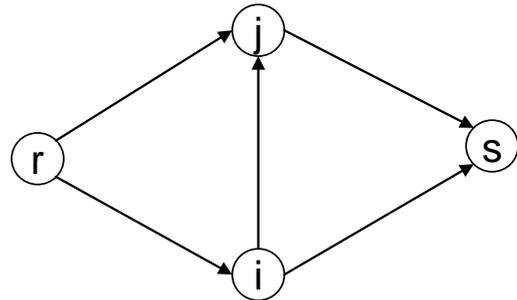


Figure 3. Example of the network map

Using MATLAB, get the shortest distance is 10, the shortest path for  $(r, i, j, s)$ . During the process of iteration, the new algorithm can get by, starting point to any node of the shortest distance, then according to the superscript, can trace its previous nodes, and then easily track the path.

### 5. Conclusion

Comparison of the static path in the search for the shortest path, based on the dynamic travel time dynamic path search more practical significance. New algorithm for the basic algorithm based on index with time, introducing new parameters, time out network dynamics. New algorithm of iterative steps are simple and clear, according to the iteration process in propelling a distance parameter, can accurately calculate the starting point to each node in the network travel time, and can easily keep track of the shortest path, and can test the iterative results, make the results more accurate. Finally, through examples, that the new algorithm in dynamic road network is feasible and effective.

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