

# Optimization of Piezoelectric Actuators for Vibration Control of Manipulators with Flexible Links

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**Abstract** – This work presents a tracking control model for a flexible robotic manipulator using motor torques and piezoelectric actuators. The dynamic model of the flexible manipulator is obtained in a closed form through the Lagrange Equations. The control uses the motor torques for the joints tracking control and also to reduce the low frequency vibration induced in the manipulator links. The stability of this control is guaranteed by the Lyapunov stability theory. Piezoelectric actuators and sensors are added for vibrations with frequencies beyond the reach of motor torque control. Robots' flexible links are built in complex geometries, which cannot be modeled by simple beam bending Equation. In this work we propose a methodology for accounting the complex geometry within the realm of the Euler-Bernoulli beam theory. The natural frequencies are calculated by the finite element method and the approximated Eigenfunctions are interpolated by polynomials. Three modes are used for the dynamics of the arm, while only two modes are used for the control. Additionally, this work introduces a formulation for simultaneous optimization of control and actuators and sensors through of dissipated energy maximization in the system by the control action with location and sizing optimization of piezoelectric actuators and sensors in the structures. Numerical experiments on Matlab/Simulink are used to verify the efficiency of the control model.

**Keywords** – Piezoelectric actuators; flexible links; tracking control; optimization; manipulators.

## 1. Introduction

The need of lightweight robots has attracted the attention of researchers everywhere to robotic manipulators with flexible links. These robots are essential in mobile applications, such as surface vehicles, aircrafts, and spacecrafts. The design of these manipulators requires a control system, which takes into account the interaction of the joint angles and the elastic modes. This complex task has the additional complication of the essential uncertainty that characterizes robotic manipulators, such as variable payload and joint frictional torques [1]. A flexible robot control design is composed by two steps: a robust tracking control, acting on the joint angles, and a stabilizer for the motion induced vibration suppression. Robotic systems can be considered linear in relation to some parameters, as mass, inertia, and damping factors, but this assumption is not valid for the state. Therefore, a position control law must be defined with an appropriate tracking error asymptotic stability, obtained with Lyapunov functions [2]. A stabilizer to damp the elastic oscillation is designed

based on a linearized model. However, the high frequency modes cannot be eliminated by the motor torque action alone, because their period is smaller than the torque control system can handle. Thus, the control of those vibrations must use higher frequency actuators like piezoelectric. Robots' flexible links are built in complex geometries, which cannot be modeled by simple beam bending Eq.s. In this work we propose a methodology for accounting the complex geometry within the realm of the Euler-Bernoulli beam theory.

The used of finite elements method for the determination of the eigenvectors is necessary since the analytical approach is cumbersome for complex non-prismatic beams. However, since we wish to retain the simplicity of the analytic derivation of the control, the eigenvectors are interpolated from the nodal values with polynomials [3]. The effectiveness of this interpolation is checked by the Rayleigh quotient [4]. In this work we propose a tracking control model for a robot arm with flexible links, where the motor torque controls the joint angle tracking and reduce the low frequency vibrations on the links. Piezoelectric

sensors and actuators are added to control the high frequency vibrations beyond the torque control. In the simulations it was used three modes and in the control two modes were used. A MATLAB code was created to assess the control model efficiency.

## 2. Dynamic Model

The physical system considered in this work is composed by a rigid and a flexible link, joints and motors, it is based on the robot design suggested by Bottega [5], but the geometry was generalized to allow non-prismatic designs. The second link was considered flexible and non-prismatic, therefore subject to motion induced vibration, which affects the trajectory of the endpoint. The sketch of a possible non-prismatic flexible link is shown in Figure 1. This link has a linearly varying cross section.

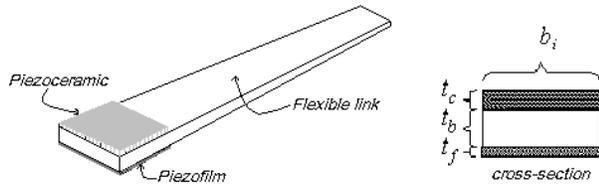


Figure 1. Link non-prismatic and flexible.

The motion of the robot endpoint is a composition of the successive relative link motions. This movement is described using homogeneous matrix transformations. These transformations represent translations and rotations due to the joints angle change and the flexible link elastic deflections [6]. The deflections are obtained considering each link as a uniform beam with  $a_i$  length featuring a piezoceramic actuator bonded to the top face and piezofilm sensor bonded to the bottom face, as shown Figure 1. Links are modeled as Euler-Bernoulli beams, with deflection  $dy_i(x_i, t)$  satisfying the partial differential equation

$$(EI)_i \frac{\partial^4 dy_i(x_i, t)}{\partial x^4} + \rho_i \frac{\partial^2 dy_i(x_i, t)}{\partial t^2} = 0 \quad (1)$$

Where  $\rho_i$  is the uniform density and  $(EI)_i$  is the flexural rigidity constant of the link. Exploring the time and space separability on Eq.(1) by the modal analysis technique, the link deflection can be expressed as:

$$d_{y_i}(x_i, t) = \sum_{i=1}^{m_i} \varphi_{ij}(x_i) \delta_{ij}(t) \quad (2)$$

Where each term in the general solution of Eq.(1) is the product of a time harmonic function of the term  $\delta_{ij}(t) = e^{j\omega_{ij}t}$  and of a space eigenfunction of the form :

$$\varphi_{ij}(x_i) = c_1 \sin\left(\frac{\rho_i \omega_{ij}^2}{(EI)_i} x_i\right) + c_2 \cos\left(\frac{\rho_i \omega_{ij}^2}{(EI)_i} x_i\right) + c_3 \sinh\left(\frac{\rho_i \omega_{ij}^2}{(EI)_i} x_i\right) + c_4 \cosh\left(\frac{\rho_i \omega_{ij}^2}{(EI)_i} x_i\right) \quad (3)$$

Where  $\omega_{ij}$  is the  $j$ th natural angular frequency of the eigenvalue problem for link  $i$ . The determination of the

constant coefficients  $C_k$  uses clamped conditions at the link base and mass boundary conditions representing the balance of bending moment and shearing force at the link endpoint. This solution is possible when the link geometry is prismatic or slightly non-prismatic. If the link shape is irregular it is very difficult to obtain a closed form analytic solution. It is important to generalize the approach for other link shapes.

## 3. Approximating solutions for eigenfunctions

The links of the mechanic manipulators are modeled as beams, since their lengths are much larger than the cross-sectional height depth. Thereby, considering that the control can prevent large displacements, it is possible to apply the Euler-Bernoulli theory for a small displacements, where the equilibrium solution is given by

$$\int_0^l \frac{d^2}{dx^2} \left\{ EI \frac{d^2 dy(x)}{dx^2} \right\} dx - \int_0^l p(x) dx = 0, \quad 0 \leq x \leq l \quad (4)$$

Where  $p(x)$  are external forces actuating on the beam. For small displacements, the natural frequencies and modes can be considered independent of the external forces. Using the finite element method, it is possible to define the mass, stiffness and damping matrices, respectively by:

$$K_{ij} = \int_0^l EI \frac{d^2 \psi_i}{dx^2} \frac{d^2 \psi_j}{dx^2} dx \quad (5)$$

$$M_{ij} = \int_0^l \rho d \psi_i d \psi_j dx \quad (6)$$

$$C = \alpha M + \beta K \quad (7)$$

Where  $\psi$  are the element wise interpolation functions and  $\alpha, \beta$  are the Rayleigh damping constants. Usually four cubic Hermit Polynomials are as interpolation functions in each two-node finite element so the unknowns of the approximated problem are nodal displacements and its derivatives. The mass matrix can be further approximated by its lumped (diagonal) form. Then the natural modes and frequencies can be computed by the following eigenvalue problem.

$$[K - \omega^2 M] = 0 \quad (8)$$

Where  $\omega^2$  are the characteristic values from the Eq.(8). The eigenvectors represent the vibration modes in nodal coordinates. Considering that the control algorithm requires twice differentiable eigenfunctions, it is necessary to create a continuous Interpolation from the discrete values. The natural choice would be using the same element wise Hermit polynomials used by the finite element approximation, but the eigenfunctions  $\varphi$  presents big oscillations, due to excessive sensitivity to the numerical Imprecision, specially of the derivatives. For computing the coefficients by least-squares it is necessary to considerate the pseudo inverse operator  $A^+$ . This operator has the following properties: if  $ATA$  is invertible, then  $A^+ = (A^T A)^{-1} A^T$ , if  $AA^T$  is invertible, then  $A^+ = A^T (A^T A)^{-1}$ . The coefficients from the polynomials are calculated from the results of the linear system  $Ax = y, x = A^+ y$ .

The matrix A comes from the mesh of finite element and y from the eigenvectors values and they are calculated from:

$$A = \begin{bmatrix} x_1^n & \cdots & x_1^1 & 1 \\ x_2^n & \cdots & x_2^1 & 1 \\ \vdots & \cdots & \vdots & \vdots \\ x_i^n & \cdots & x_i^1 & 1 \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \end{bmatrix} \quad (9)$$

Where n is the order of the polynomials and i is the number of points at the mesh. All three options can eliminate the oscillations on the eigenfunctions, but might be inadequate for use in the control solution, since the differentiation and the integration can generate different results. For this way, it is interesting adopt an error criterion. In this work it is adopted the Rayleigh Quotient as error criterion, which for analytic functions can be expressed from

$$\omega^2 = \frac{\int_0^l EI \left( \frac{d^2 \varphi}{dx^2} \right)^2 dx}{\int_0^l \rho a \varphi^2 dx} \quad (10)$$

Where a is the cross-section from the link. The discrete form is given by  $\omega^2 = \frac{\varphi^T K \varphi}{\varphi^T K \varphi}$ , where in this case  $\varphi$  are the eigenvectors.

#### 4. Equations of motion

The closed form equations of motion are derived using a Lagrangian approach, in the form of compact matrices, resulting

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + Kq + D\dot{q} + g(q) = u \quad (11)$$

Where  $q = [\theta, \delta]^T$  is the generalized coordinates vector,  $\theta$  is the  $n \times 1$  joint coordinates vector,  $\delta$  is the  $n \times 1$  elastic modes coordinates vector,  $B(q)$  is the positive definite symmetric inertia matrix,  $C(q, \dot{q})$  is the Coriolis and centrifugal forces vector,  $g(q)$  is the gravitational torque vector,  $K$  is the positive definite stiffness diagonal matrix,  $D$  is the positive semi definite link diagonal damping matrix and  $u$  is the joint input torque vector. The  $K$  matrix is calculate directly by the finite element method and can be diagonalizable by the eigenvalues from the Eq.(8) on the main diagonal. The values from  $\delta$  represent the amplitudes from the vibration modes and can be calculated from the equation of minimum potential energy:

$$[\delta_i]^T \left[ \int_0^l EI \left( \frac{d\varphi_i d\varphi_j}{dx^2} \right)^2 dx \right] [\beta_i] - [P_i]^T [\beta_i] = 0, \forall \beta_i, i, j = 1, 2, \dots, n \quad (12)$$

Where  $p_i$  are the forces apply about the link,  $n$  is the number of eigenfunctions  $\varphi$ , calculated according to showed above and  $\beta_i$  are virtual amplitudes introduced in the equation which can assumed any value.

#### 5. Tracking control

This section introduces the flexible robot arm tracking control, based on an adaptive controller and a robust control law to reduce the elastic vibrations of the arms. The improved tracking controller using nominal compensation of dynamic nonlinearities of system Eq.(11) is given by:

$$u = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + Kq_d + D\dot{q}_r + g(q) - K_p s \quad (13)$$

Where  $K_p$  is the positive definite diagonal gain matrix,  $\dot{q}_r = \dot{q} - \Lambda \tilde{q}$  is the reference velocity vector, with tracking error  $\tilde{q} = q - q_d$ ,  $q$  is the robot path,  $q_d$  is the desired path and  $s = \dot{q}_d - \dot{q}_r = \dot{\tilde{q}} + \Lambda \tilde{q}$  is the reference error We can prove using Lyapanov stability theory (11) that with control law (13) the tracking error tend to zero and the deflection modes remain bounded. However, the damping may be too small. In this case, we can add a control law  $D'_\Delta \delta'_d$ , depend on tracking dynamics obtained from the  $q_d$  variable:

$$D'_\Delta \equiv D_\Delta - \text{diag}(f_{11}, \dots, f_{nn}) \quad (14)$$

Where  $D_\Delta$  is the positive definite diagonal gain matrix and  $f_{ij}$  are functions that depend on the deflection velocities. The Eq. (14) is added to Eq. (13) to obtain the control law of the system Eq. (11) expressed as:

$$u = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + Kq_d + D\dot{q}_r + g(q) - K_p s + (D'_\Delta \delta'_d)^T \quad (15)$$

Where  $D'_\Delta \delta'_d$  is a robust control action that damps the system and eliminate the steady vibrations.

#### 6. Vibration control

We propose a feedback control voltage to the piezoceramic actuator, expressed as

$$P(t) = -c_a K c_a^T \dot{P}_f(t) \quad (16)$$

With

$$c_a = \frac{E_b E_c t_c t_f d_{31}}{\rho_b A_b (E_b t_b + 6E_c t_c)} (\dot{\varphi}(x_a + a_{pc}) - \dot{\varphi}(x_a)) \quad (17)$$

Where  $K_c$  is the feedback gain,  $E_c$  and  $E_b$  are the elastic modulus of the piezoceramic and the link, respectively,  $t_c$ ,  $t_f$  and  $t_b$  are the piezoceramic, piezofilm, and link thicknesses, respectively,  $A_b$  is the cross section of the link,  $d_{31}$  is the piezoelectric constant and  $\rho_b$  is the mass density,  $a_{pc}$  is the size of the actuator,  $x_a$  is the localization from the actuator on the link,  $\dot{P}_f(t)$  is the voltage generated by the piezofilm sensor, obtained by integrating the electric charge developed at a point on the piezofilm, expressed as:

$$P_f(t) = C_s \delta = \frac{K_{31}^2 b_f}{c g_{31}} d_{ni} \delta \quad (18)$$

Where  $K_{31}^2$  is the electromechanical coupling factor,  $C$  is the capacitance of the film sensor,  $d_{ni}$  is the distance from the bottom of the piezofilm sensor to the neutral axis and  $g_{31}$  is the piezoelectric stress constant [13]. This additional controller Eq.(16) is combined to the original one. The resulting control law for the system Eq. (15) is expressed as

$$u = B(q)\dot{q}_r + C(q, \dot{q})\dot{q}_r + Kq_d + D\dot{q}_r + g(q) - K_p s + (D_\Delta \delta_d + C_a p(t))^T \quad (19)$$

## 7. Location and sizing actuators optimization

Controlling structural vibration depends not only on the control law, but also on the selection and location of the actuators and Sensors. In this work we propose a methodology for the actuator and sensor position and sizing optimization, based on maximizing the control energy dissipation. This procedure takes into account the actuators and sensors mass and stiffness, and their effect on the mechanical behavior of the structure. This influence is combined to the control characteristics to obtain an objective function that depends on the actuators location and sizing and the control gain. The deflections are obtained considering each link as a beam with  $a_i$  length featuring, a piezoceramic actuator bonded to the top face and a piezofilm sensor bonded to the bottom face as shown Figure 1. The dynamic of the flexible link with  $m$  piezoelectric sensors and actuators in terms of modal coordinates can be expressed as

$$B_{\delta\delta}\ddot{\delta}_d + C_a\dot{\delta}_d + K\delta_d + g(\delta) = C_a p(t) \quad (20)$$

The total energy stored in the system can be expressed as

$$W = T + U = \frac{1}{2} \delta^T B(\delta)\dot{\delta} + \frac{1}{2} \delta^T K\delta \quad (21)$$

Differentiating the Eq.(21) with respect to the time we obtain

$$\dot{W} = \dot{T} + \dot{U} = \frac{1}{2} \delta^T B(\delta)\ddot{\delta} + \delta^T B(\delta)\dot{\delta} + \frac{1}{2} \delta^T K\dot{\delta} \quad (22)$$

Using the Eq.(20) and Eq.(22) with the control law Eq.(16), we obtain

$$\dot{W} = \dot{T} + \dot{U} = -\dot{\delta}^T D\dot{\delta} - \dot{\delta}^T (c_a K c_a^T c_s)\dot{\delta} < 0 \quad (23)$$

where the first and the second terms describe the energy rates removed from the system by the internal damping and by the control feedback, respectively. Integrating the Eq.(22) we obtain

$$W_{t_0} = W_f + W_c = \int_{t_0}^{\infty} \dot{\delta}^T D\dot{\delta} dt + \int_{t_0}^{\infty} \dot{\delta}^T (c_a K c_a^T c_s)\dot{\delta} dt \quad (24)$$

Where  $W_f$  represent the energies dissipated by internal damping and  $W_c$  represent the energies dissipated by the

control action. For effective vibration suppression, it is reasonable to derive a method to increase the energy dissipated by the control. We observe that  $W_c$  depends on the locations and the sizing of the actuators and feedback matrix gain  $K_c$ . Therefore,  $W_c$  can be used as an optimization criterion to determine location and sizing of actuator and feedback gains. For determining  $W_c$ , we write the Eq.(20) in state-space form as:

$$\dot{z} = \tilde{H}z \quad (25)$$

Where  $z = [\delta, \dot{\delta}]^T$  and

$$H = \begin{bmatrix} 0 & 1 \\ -B_{\delta\delta}^{-1}K & -B_{\delta\delta}^{-1}(C_{\delta\delta} + D + c_a K c_a^T c_s) \end{bmatrix} \quad (26)$$

The induced control dissipation energy by the active damping control  $W_c$  can be written as

$$W_c = \int z^T Q z dt \quad (27)$$

Where

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & c_a K c_a^T c_s \end{bmatrix} \quad (28)$$

it is a  $2m \times 2m$  matrix. Applying standard state transformation techniques to the Eq.(27) we obtain

$$W_0 = z_0^T P z_0 \quad (29)$$

Where  $P$  is symmetric positive definite matrix, solution of the Lyapanov equation

$$\tilde{H}^T P + P\tilde{H} = -Q \quad (30)$$

It is important that know the  $W_c$  depends on the initial conditions of the flexible structure. In order to eliminate this dependence, we assume that initial state of  $z$  satisfies  $W_a^{-1}z_0$  where  $W_a = \text{diag}(\lambda_i)$ , with random value of  $\lambda_i > 0$ . Therefore, we obtain an objective function  $J_0 = \text{tr}(W_a P W_a)$  for energy dissipated by the control which depends on the location and the size of the piezoelectric actuators and the gain  $K_c$ .

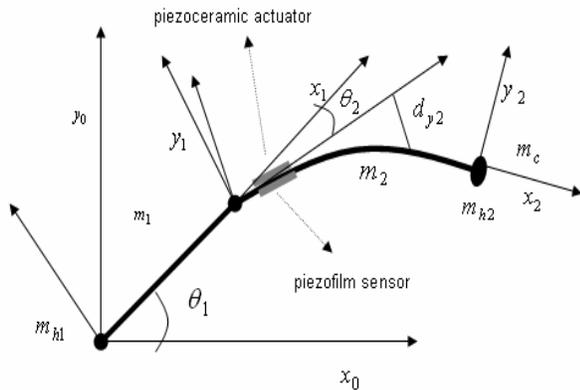
To design a precise and agile manipulator, it is reasonable to take it as light as possible. This is accomplished by adding a Function of the actuator and sensor masses to the objective function shown above. We added a quadratic term dependent on the size of the actuator  $a_{pc}$  to  $J_0$  resulting in the following composite objective function

$$\begin{aligned} J_0 &= \alpha a_{pc}^2 - W_c \\ 0 &\leq x_a \leq a_i \\ 0 &< a_{pc} + x_a \leq a_i \\ K_c &\leq K_{max} \end{aligned} \quad (31)$$

Where  $\alpha$  depends on the piezoelectric material cost and  $K_{max}$  depends on the actuator power limitation.

### 8. Result

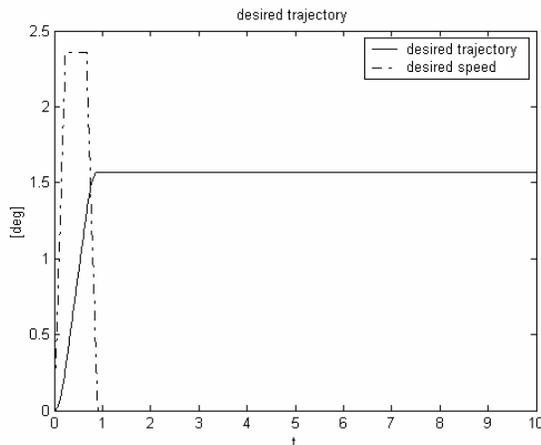
This work three vibration modes are used in the simulation, instead of two, while the control is still derived with two modes. The main reason behind this difference is to test if the control is robust enough to damp the additional mode. The control laws were tested on a simplified model robot with a rigid first link and a flexible second link as shown in Figure 2. Gravitational effects were ignored. The Lagrangian coordinate vector is  $q = [\theta_1 \ \theta_2 \ \delta_1 \ \delta_2 \ \delta_3]^T$ .



**Figure2.** Model of planar one-link flexible manipulator featuring piezoelectric actuators and sensors

The results were obtained using a block-diagram implemented in MatLab, where the fourth-order Runge-Kutta method with  $\Delta t = 1 \text{ ms}$  was used to integrate the equations for a five-second simulation.

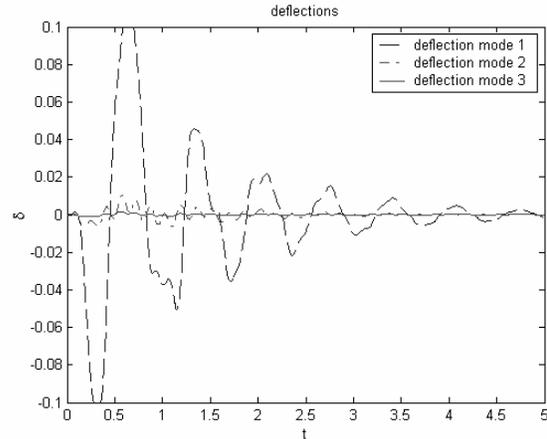
Figure 3 shows the trapezoidal speed trajectory tracking used with amplitude  $\frac{\pi}{2}$  for the joint angles 1 and 2 without initial Tracking Error.



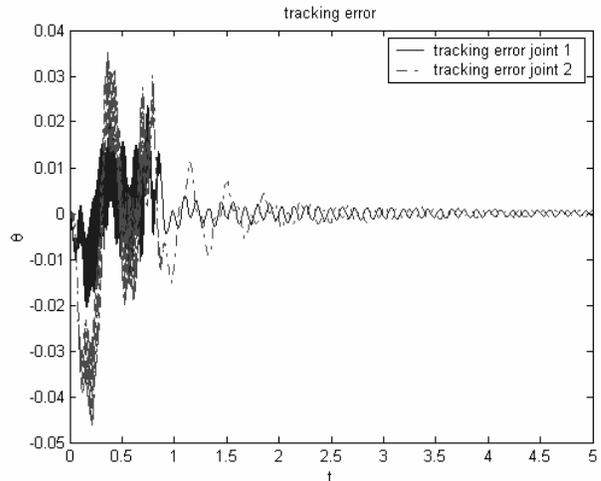
**Figure3.** Desired trajectory and speed of the joint angle 1 and 2.

### 9. Simulations

In this work, we use the mechanical and geometrical properties of the piezoelectric materials presented in [19]. Firstly, we simulated a damped system with a control law, Eq. (13). Figure 4 shows that the elastic deflections tend to zero and they are limited due to natural damping of the system. Figure 5 shows that the system tracking error also tends to zero.



**Figure4.** Deflection of first, second and third modes with damping



**Figure5.** Tracking error of the trapezoidal trajectory tracking

In the second simulation, we used the control law given by Eq. (17) in the same system used before. Figure 7 shows an increase in the system damping and a zero convergence faster than the deflections. This is a result of the addition of  $D_{\Delta} \delta_d$  controller.

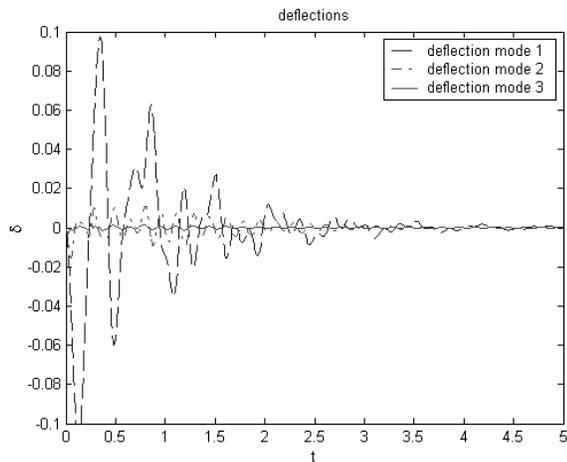


Figure6. Deflections of first, second and third modes for damping system with robust control.

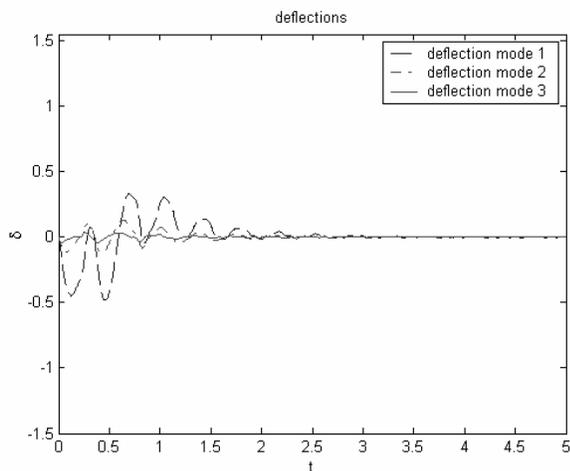


Figure6. Deflections of first, second and third modes for damping system with piezoelectric actuator and sensor.

## 10. Conclusions and considerations

In this work we introduced a technique for tracking and vibration control of a robot with flexible links. This technique uses the motor torque for the joint angle control and also for control the low frequency vibrations in the robot links. Piezoelectric actuators and sensors are added to the system to control the high frequency vibrations that cannot be reduced by the motor alone. We also introduced an optimization procedure for the size and position of the piezoelectric actuator and sensor, using the energy dissipated by the control in the objective function. This technique can be developed to build light manipulators with flexible links, while preserving the force and precision. It also reduces the energy consumption and suits the needs for aerospace systems or for tasks that demand lightness, precision and agility.

## References

1. V. Bottega, R. Pergher, A. Molter and J. S. O. Fonseca, "Modelagem, controle e simulação de manipuladores robóticos com braços flexíveis de geometria irregular". In: CMNE/CILAMCE Iberian Latin American Congress on Computational Methods in Engineering, Porto, 2007.
2. S. Arimoto, *Control Theory of Non-Linear Mechanical Systems*, Oxford Clarendon Press, London, 1996.
3. J. K. Bathe and E. L. Wilson, *Numerical Methods in Finite Element Analysis*. Prentice Hall, 1976.
4. D. L. Clive and I. H. Shames, *Solid Mechanics. A variational approach*, McGraw-Hill, 1973.
5. V. Bottega. "Controle de sistemas mecânicos não-lineares aplicado a um manipulador robótico", Dissertação de Mestrado, UFRGS/CPGMAP, Porto Alegre, 1998.
6. W. J. Book, "Recursive lagrangian dynamics of flexible manipulator arms", *Int. Journal Robotics Res.*, vol. 3, no. 3, pp. 87-101, 1984.
7. Meirovitch, *Analytical Methods in Vibration*, Macmillan, New York, 1967.
8. A. de Luca, P. Lucibello and F. Nicolo, "Automatic symbolic modeling and nonlinear control of robots with flexible links". In *Proc. IEEE Work on Robot Control*, Oxford, UK, pp. 62-70, 1988.
9. D. Luenberger, "Algorithmic analysis in constrained optimization". *SIAM-AMS Proceedings*, 9, pp.39-51, 1976 .
10. M. A. Arteaga, "On the properties of a dynamic model of flexible robot manipulators", *ASME Journal of Dynamics Systems, Measurement, and control*, vol. 120, pp. 8-14, 1998.
11. J. P. La Salle and S. Lefschetz, "Stability by Lyapunov's direct method", Academic Press, New York, 1961.
12. E. F. Crawley and J. De Luis, "Use of piezoelectric actuators as elements of intelligent structures", *AIAA Journal*, vol. 25, pp. 1373-1385, 1987.
13. H. T. Banks, R. C. Smith and Y. Wang, *Smart material structures: modeling, estimation and control*, John Wiley & Sons, Paris, 1996.
14. Denoyer, K. K. and Kwak, M. K., 1996, "Dynamic Modelling and Vibration Suppression of a Slewing Structure Utilizing Piezoelectric Sensor and Actuators", *Journal of Sound and Vibration*, vol. 189, no. 1, pp. 13-31.
15. Li, Y., Onoda, J. e Minesugi, K., 2002, "Simultaneous Optimization of Piezoelectric Actuator Placement and Feedback for Vibration Suppression", *Acta Astronautica*, vol. 56, no. 6, pp. 335-341.
16. De Luca, A., Lucibello, P. and Nicolo, F., 1988, "Automatic Symbolic Modeling and Nonlinear Control of Robots with Flexible Links". In *Proc. IEEE Work on Robot Control*, Oxford, UK, pp. 62-70.
17. V. Bottega, "Controle e otimização estrutural de manipuladores robóticos com elementos flexíveis usando

atuadores e sensores piezelétricos”. Tese de Doutorado, UFRGS/CPGMAP, Porto Alegre, 2005.

18. A. de Luca, L. Lanari S. Lucibello and Panzoeri, “Control experiments on a two-link robot with a flexible forearm”, 29th IEEE Conf. Decision and Control, Honolulu, HI, pp 5-7, 1990.

19. S. B. Cho, S. S. Cho, H. C. Shin and H. K. Kim, “Quantitative feedback theory control of a single-link flexible manipulator featuring piezoelectric actuator and sensor”, Smart Material Structures, vol. 8, pp. 338–349, 1999.