

Fractional PID controller for a High performance Drilling Machine

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Abstract – This paper deals fractional order Proportional-Integral-Derivative (fractional order PID) controller used in a high performance drilling system for controlling the output obtained. The main objective is to obtain a stable, robust and controlled system by tuning the fractional order PID controller using minimization. The incurred value is compared with the traditional tuning techniques like Ziegler-Nichols and is proved better. Hence that tuning results establishes the tuning the fractional order PID controller using minimization technique gives less overshoot and better control performance.

Keywords- Ziegler- Nichols – Fractional order PID controller

1. Introduction

PID control is a generic feedback control technology and it makes up 90% of automatic controllers in industrial control systems. The PID control was first placed in the market in 1939 and has remained the most widely used controller in process control until today. The basic function of the controller is to execute an algorithm based on the control engineers input and hence to maintain the output at a level so that there is no difference between the process variable and the set point [1]. The popularity of PID controllers is due to their functional simplicity and reliability .They provide robust and reliable performance for most systems and the PID parameters are tuned to ensure a satisfactory closed loop performance [2]. A PID controller improves the transient response of a system by reducing the overshoot and by shortening the settling time of a system [3].

Standard methods for integer order tuning includes Ziegler-Nichols Ultimate-cycle tuning [4], Cohen-coons [6], Astrom and Hagglund [5] and many other techniques. In this paper we design fractional order PID controller for a High performance drilling machine. Fractional order PID controllers are variations of usual PID controllers:

$$C(s) = p + \frac{I}{s} + Ds \quad (1)$$

where the (first-order) integral and the (first-order) derivative of (1) are replaced by fractional derivatives like this:

$$C(s) = p + \frac{I}{s^\lambda} + Ds^\mu \quad (2)$$

(In principle, both λ and μ should be positive so that we still have an integration and a differentiation.) Fractional order PIDs have been increasingly used over the last years [7]. There are several analytical ways to tune them

[8, 9, 10]. This paper is organized as follows: Section 2 describes dynamic model of a high-performance drilling process. Section3 introduces the fractional order calculus.

Section4 describes an analytical method that lies behind the development of the rules. Sections5 and 6 describes tuning rules similar to those proposed by Ziegler and Nichols for (integer) PIDs. Section 7 describes fractional order PID controller design for drilling Machines using tuning rules. Finally results and comparison in the section 8

2. Dynamic model of a high-performance drilling process

The modeling of a high-performance drilling process [11] includes the modeling of the feed drive system, the spindle system and the cutting process. In this paper, the overall plant model is obtained by experimental identification using different step shaped disturbances in the command feed. The drilling force, F, is proportional to the machining feed, and the corresponding gain varies according to the work piece and drill diameter. The overall system of the feed drive, cutting process and dynamometric platform was modeled as a third-order system, and the experimental identification procedure yielded the transfer function as:

$$G(s) = \frac{1958}{s^3 + 17.89s^2 + 103.3s + 190.8} \quad (3)$$

where s is the Laplace operator. The model does have certain limits in representing the complexity and uncertainty of the drilling process. However, it provides a rough description of the process behavior that is essential for designing a network- based PID control system.

3. A brief introduction to fractional order calculus

A commonly used definition of the fractional differo-integral is the Riemann-Liouville definition

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} dt \quad (4)$$

For $m-1 < \alpha < m$; where, $\Gamma(0)$ is the well-known Euler,s gamma function. An alternative definition, based on the concept of fractional differentiation, is the Grunwald-Letnikov definition given by

$${}_a D_t^\alpha f(t) = \lim \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{(t-a)/h} \frac{\Gamma(\alpha+K)}{\Gamma(K+1)} f(t-kh) \quad (5)$$

One can observe that by introducing the notion of fractional order operator ${}_a D_t^\alpha f(t)$ the differentiator and integrator can be unified. Another useful tool is the Laplace transform. It is shown in [12] that the Laplace transform of an n-th derivative ($n \in R_+$) of a signal x(t) relaxed at t=0 is given by: $L\{D^n x(t)\} = s^n x(s)$ So, a fractional order differential equation, provided both the signals u(t) and y(t) are relaxed at $t=0$, can be expressed in a transfer function form:

$$G(s) = \frac{a_1 s^{\alpha_1} + a_2 s^{\alpha_2} + \dots + a_{mA} s^{\alpha_{mA}}}{b_1 s^{\beta_1} + b_2 s^{\beta_2} + \dots + b_{mB} s^{\beta_{mB}}} \quad (6)$$

Where $(a_m, b_m) \in R^2, (\alpha_m, \beta_m) \in R_+^2, \forall(m \in N)$

4. Tuning by minimization

In this tuning method for fractional PIDs by [13], we begin by devising a desirable behavior for our controlled system, described by five specifications (five, because the parameters to be tuned are five):

1. The open loop is to have some specified crossover frequency w_{cg} :

$$\left|C(w_{cg})G(w_{cg})\right| = 0db \quad (7)$$

2. The phase margin ϕ_m is to have some specified value:

$$-\pi + \phi_m = \arg[c(w_{cg})G(w_{cg})] \quad (8)$$

3. To reject high-frequency noise, the closed loop transfer function must have a small magnitude at high frequencies; hence, at some specified frequency w_h , its magnitude is to be less than some specified gain H:

$$\left|\frac{C(w_h)G(w_h)}{1+C(w_h)G(w_h)}\right| < H \quad (9)$$

4. To reject output disturbances and closely follow references, the sensitivity function must have a small magnitude at low frequencies; hence, at some specified frequency w_l , its magnitude is to be less than some specified gain N:

$$\left|\frac{1}{1+C(w_l)G(w_l)}\right| < N \quad (10)$$

5. To be robust when gain variations of the plant occur, the phase of the open loop transfer function is to be (at least roughly) constant around the gain-crossover frequency:

$$\frac{d}{dw} \arg [C(w)G(w)] \Big|_{w=w_{cg}} = 0 \quad (11)$$

Then the five parameters of the Fractional PID are to be chosen using the Nelder-Mead direct search simplex minimization method. This derivative free method is used to minimize the difference between the desired performance specified as above and the performance achieved by the controller. Of course this allows for local minima to be found: so it is always good to use several initial guesses and check all results (also because sometimes unfeasible solutions are found).

5. The set of s-shaped response based tuning rules

The set of rules proposed by Ziegler and Nichols apply to systems with an S- shaped unit-step response, such as the one seen in Fig 1. From the response an apparent delay L and a characteristic time – constant T may be determined (graphically, for instance). A simple plant with such a response is

$$G = \frac{k}{1+sT} e^{-Ls} \quad (12)$$

The specifications used were

$$w_{cg} = 0.5rad / s \quad (13)$$

$$\phi_m = 2.3rad \approx 38^\circ \quad (14)$$

$$w_h = 10rad / s \quad (15)$$

$$w_l = 0.01rad / s \quad (16)$$

$$H = -10db \quad (17)$$

$$N = 20db \quad (18)$$

Matlab,s implementation of the simplex search in function fmincon was used (7) was considered the function to minimize, and (8) to (11) accounted for as constraints.

Obtained parameters P, I, λ, D and μ very regularly with L and T. using a least-squares fit, it was possible to adjust a polynomial to the data, allowing (approximate) values for the parameters to be found from a simple algebraic calculation [14.15]. The parameters of the polynomials involved are given in Table 1. This means that

$$P = -0.0048 + 0.2664L + 0.4982T + 9.0232L^2 - 0.0720T^2 - 0.0348TL \quad (19)$$

And so on. These rules may be used if

$$0.1 \leq T \leq 50 \text{ and } L \leq 2 \quad (20)$$

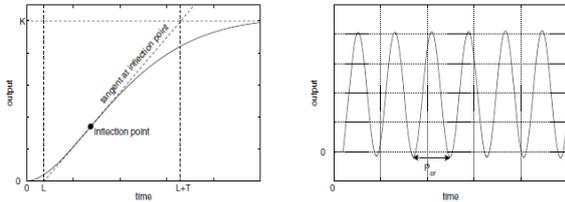


Fig. 1. Left: S-shaped unit-step response; right: plant output with critical gain control.

Table 1. Parameters for the first set of tuning rules for S-shaped response plants

Parameters to use when $0.1 \leq T \leq 5$					
	P	I	λ	D	μ
1	-0.0048	0.3254	1.5766	0.0662	0.8736
L	0.2664	0.2478	-0.2098	-0.2528	0.2746
T	0.4982	0.1429	-0.1313	0.1081	0.1489
L^2	0.0232	-0.1330	0.0713	0.0702	-0.1557
T^2	-0.0720	0.0258	0.0016	0.0328	-0.0250
LT	-0.0348	-0.0171	0.0114	0.2202	-0.0323
Parameters to use when $5 \leq T \leq 50$					
	P	I	λ	D	μ
1	2.1187	-0.5201	1.0645	1.1421	1.2902
L	-3.5207	2.6643	-0.3268	-1.3707	-0.5371
T	-0.1563	0.3453	-0.0229	0.0357	-0.0381
L^2	1.5827	-1.0944	0.2018	0.5552	0.2208
T^2	0.0025	0.0002	0.0003	-0.0002	0.0007
LT	0.1824	-0.1054	0.0028	0.2630	-0.0014

6. The set of critical gain based tuning rules

The second set of rules proposed by Ziegler and Nichols apply to systems that, inserted into a feedback control-loop with proportional gain, show, for a particular gain, sustained oscillations, that is, oscillations that do not decrease or increase with time, as shown in Fig1. The period of such oscillations is the critical period P_{cr} , and the gain causing them is the critical gain K_{cr} . Plants given by (12) have such a behavior. Reusing the data collected for finding the rules in section 5, obtained with specifications (13) to (18), it is seen that parameters P, I, λ, D and μ obtained vary regularly with K_{cr} and P_{cr} . The regularity was again translated into formulas (which are no longer polynomial) using a least- squares fit [16]. The parameters involved are given in Table 2. This means that

$$p = 0.4139 + 0.014sK_{cr} + 0.1584P_{cr} \frac{0.4384}{K_{cr}} - \frac{0.0855}{P_{cr}} \quad (21)$$

And so on. These rules may be used if

$$P_{cr} \leq 8 \text{ and } K_{cr}P_{cr} \leq 640 \quad (22)$$

Table 2. Parameters for the first set of tuning rules for plants with critical gain and period

	Parameters to use when $0.1 \leq T \leq 5$				
	P	I	λ	D	μ
1	0.4139	0.7067	1.3240	0.2293	0.8804
K_{cr}	0.0145	0.0101	-0.0081	0.0153	-0.0048
P_{cr}	0.1584	-0.0049	-0.0163	0.0936	0.0061
$1/K_{cr}$	-0.4384	-0.2951	0.1393	-0.5293	0.0749
$1/P_{cr}$	-0.0855	-0.1001	0.0791	-0.0440	0.0810
	Parameters to use when $5 \leq T \leq 50$				
	P	I	λ	D	μ
1	-1.4405	5.7800	0.4712	1.3190	0.5425
K_{cr}	0.0000	0.0238	-0.0003	-0.0024	-0.0023
P_{cr}	0.4795	0.2783	-0.0029	2.6251	-0.0281
$1/K_{cr}$	32.2516	-56.2373	7.0519	-138.9333	5.0073
$1/P_{cr}$	0.6893	-2.5917	0.1355	0.1941	0.2873

7. Fractional PID controller design for drilling machine

The model of the drilling machine is a third-order transfer function as:

$$G(s) = \frac{1958}{s^3 + 17.89s^2 + 103.3s + 190.8} \quad (23)$$

The unit -step responses of the Drilling machine model is selected in Fig 2. To design on fractional PID controller, the model(23) should be approximated by a first order lag plus time delay system which is give in the following

$$G = \frac{10.3}{1 + 0.387s} e^{-0.197s} \quad (24)$$

Then using tuning rules in the paper, obtained parameters and p, I, λ, D and μ :

$$P=0.2079, I= 0.4969, \lambda =1.4883, D= 0.08965, \mu =0.9694$$

The transfer function for the fractional PID controller is:

$$c(s) = 0.2079 + \frac{0.4969}{s^{1.4883}} + 0.08965s^{0.9694} \quad (25)$$

The unit-step responses of the Drilling machine with fractional PID controller and integer PID controller are shown in Fig3

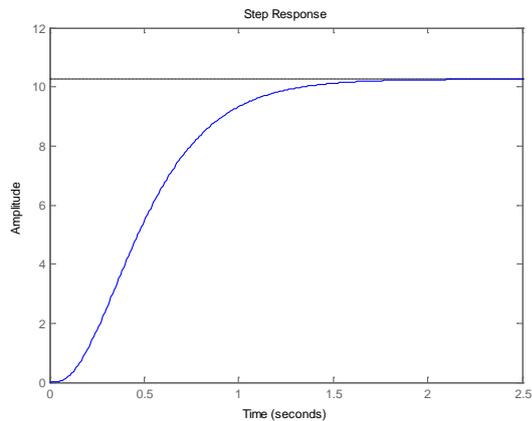


Fig2. Unit step response of drilling machine

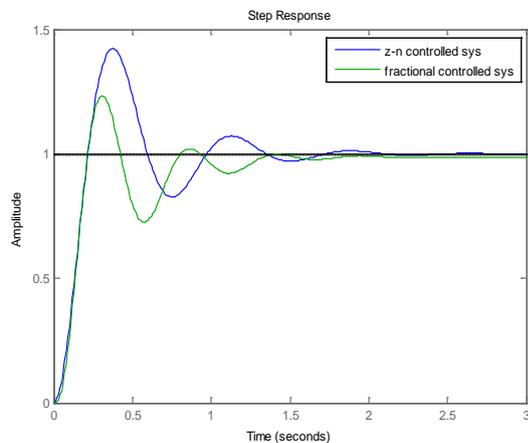


Fig3. Unit-step responses of the Drilling machine with Fractional PID controller and Integer PID controller

8. Results and conclusion

Using fractional order PID controller we have significantly reduced percentage overshoot and rise time and settling time. A comparison of time domain specifications peak overshoot, peak time ,rise time and settling time are tabulated as given in Table 3.It is found very clearly that fractional order PID controller reduce the overshoot by a large value. Settling time, rise time and peak time have also improved.

In this paper tuning rules (inspired by those proposed by Ziegler and Nichols for integer PID s) are given to tune Fractional PID s. Two different sets of fixed performance specifications are used.

Table 3. comparison

Type of controller	Integer(z-n)	Fractional
Peak time (sec)	0.37	0.31
Peak overshoot (%)	42.3	23.7
Rise time (sec)	0.15	0.11
Settling time (sec)	1.61	1.3

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