# Auction algorithm is applied in the public transport network 

${ }^{1}$ Qiu-man Li, ${ }^{2}$ Sheng-xue He, ${ }^{3}$ Lu-wen Fei<br>${ }^{1}$ School of Management, University of Shanghai for Science and Technology, Shanghai, China<br>${ }^{2}$ School of Management, University of Shanghai for Science and Technology, Shanghai, China<br>${ }^{3}$ School of Management, University of Shanghai for Science and Technology, Shanghai, China

Email: lqmmakeit@126.com


#### Abstract

The selection of the bus lines is an important part of the urban public transport information query. In the choice of the best path, the passengers will be considering a variety of factors, such as time and cost. Therefore, it is uncertainly. In this paper, on the basis of summing up the public transport network characteristic, from the reality, to improve the auction algorithm based on a variety of passenger bus travel considerations and the trip routes considerations. There are so many different between public transport and travel by car, such as the possibility of direct, transfer, walking time, waiting time and many other factors, so the simple shortest path calculation method does not meet the actual requirements. Therefore, this article take the transfer time and waiting time quantitative, and as a considering factor added to the improvement of the auction algorithm, and in the last example is given to verify.


Keywords - Auction algorithm; Public transport network; Transfer program; The optimal path

## 1. Introduction

With the intensification of the urban transport problems, the status of public transport has become increasingly important. The level of development of public transport has been severely restricted the level of modernization of the entire city. Currently, the popularity of the public transport network is based in a public passenger bus system, its efficiency and convenience of a direct impact on people's travel choice. For passengers choose bus travel to consider many factors, these considerations are constraints for finding the optimal path algorithm. Therefore, the auction improved auction algorithm to adapt to select the optimal path of the bus.

## 2. Bus travel network analysis

A public transport road network $\mathrm{G}=(\mathrm{N}, \mathrm{A}), \mathrm{N}$ is a collection of nodes, collection represents all bus stops; A representative of the sections collections, sections including bus travel sections, passengers walk sections, waiting sections and transfer sections.

Transit network and the general road network are different, has its own particularity, select a suitable model of the travel route of the transit network. The most important factor to attract passengers to choose public transport travel is the establishment of effective and efficient public transport travel query system. The shortest path algorithm is the basic algorithm to calculate the shortest path between the
two points, is also solving the fundamental problem of the optimal public transport travel path.

Passenger psychology research is to determine the objectives of the model constraints. The passenger traveling psychological studies have shown that passenger primary consideration in the choice of public transport lines is the least transfer times, followed by the shortest travel distance, the lowest travel cost. However, as technology advances, the connectivity of the bus lines in the rise. Modern developed city encourage passengers to transfer and take advantage of the preferential transfer as a guide, to save travel time and traveling expenses as the ultimate goal. Accordingly, in a certain number of transfers Constraint, bus optimal path problem transform into a shortest path problem.

## 3. Shortest auction algorithms

### 3.1 The auction algorithm introduced

The auction algorithm for the shortest path problems derived from the auction algorithm for the assignment problem. Let be a directed network defined by a set of nodes and a set A of m directed arcs. Each arc $(i, j) \in A$ has an associated length (or arc cost) $t_{i j}$. The fundamental principle of the auction algorithm for the shortest paths was described
in its simplest form for the single origin and single destination case.

To simplify the presentation, we have the following assumptions:
(1) All cycles have positive lengths.
(2) Each node except for the destination has at least one outgoing incident arc. Any node not satisfying this condition can be connected to the destination with a large arc length without changing the problem and the subsequent algorithm materially.
(3) There is at most one arc between two nodes in each direction so that we can refer to an arc ( $i, j$ ) unambiguously. Again, this assumption is made for notational convenience and the algorithm can be trivially extended to the case where there are multiple arcs connecting a pair of nodes.

Let node $r$ be the origin node and let $S$ be the destination node. In the following assumptions, a path is denoted by a sequence of nodes $\left(r, i_{1}, i_{2}, \cdots, i_{k}\right)$ so $\left(i_{m}, i_{m+1}\right)$ is an arc for all $m=(1, \cdots, k-1)$.In addition, if the nodes $r, i_{1}, i_{2}, \cdots, i_{k}$ are distinct, the sequence $r, i_{1}, i_{2}, \cdots, i_{k}$ of is called a simple path. The length of a path is defined to be the sum of its arc lengths.

The algorithm maintains a simple path $R=\left(r, i_{1}, i_{2}, \cdots, i_{k}\right)$ at all times. If $i_{k+1}$ is a node that does not belong to a path $R=\left(r, i_{1}, i_{2}, \cdots, i_{k}\right)$ and is an $\left(i_{k}, i_{k+1}\right)$ arc, extending $R$ by $i_{k+1}$ means replacing $R$ by the path ( $r, i_{1}, i_{2}, \cdots, i_{k}, i_{k+1}$ ), which is called the extension of $R$ by $i_{k+1}$.If $R$ does not consist of just the origin node $r$,contracting $R$ means replacing $R$ with the path $R=\left(r, i_{1}, i_{2}, \cdots, i_{k-1}\right)$

The algorithm also maintains a variable $p_{i}$ for each node $i$ (called price of $i$ ) satisfying (Complementary Slackness or CS for short):

$$
\begin{align*}
& p_{i} \leq t_{i j}+p_{j}, \forall(i, j) \in A  \tag{1a}\\
& p_{i}=t_{i j}+p_{j}, \forall(i, j) \in R \tag{1b}
\end{align*}
$$

It can be shown that if a pair $(R, p)$ satisfies the CS conditions, then the portion of $R$ between node 1 and any node $i \in R$ is the shortest path from $r$ to $i$, while $p_{r}-p_{i}$ is the corresponding shortest distance.

Assuming that an initial pair $(R, p)$ is satisfying CS is available. This is not a restrictive assumption when all arc lengths are nonnegative, since then one can use the default pair

$$
\begin{equation*}
R=R(r), p_{i}=0, \forall i \in R \tag{2}
\end{equation*}
$$

Initially, $(R, p)$ is any pair satisfying CS. The algorithm proceeds in iterations, transforming a pair
( $R, p$ ) satisfying CS into another pair satisfying CS.
At iteration, the path $R$ is either extended by a new node or contracted by deleting its terminal node. In the latter case the price of the terminal node is increased strictly. A degenerate case occurs when the path consists of just the origin node $r$; in this case the path is either extended or left unchanged with the price $p_{r}$ being strictly increased.

The iteration is as follows:
Let $i$ be the terminal node of $R$. If $p_{i}<\min _{(i, j) \in A}\left(t_{i j}+p_{j}\right)$ go to Step 1; otherwise, go to Step 2.

Step 1: Contract path: Set $p_{i}=\min _{(i, j) \in A}\left(t_{i j}+p_{j}\right)$, and if $i \neq r$, contract $R$ and go to the next iteration.

Step 2: Extend path: Extend $R$ by node $j_{i}$ where $j_{i}=\underset{(i, j) \in A}{\arg \min }\left(t_{i j}+p_{j}\right)$. If $j_{i}$ is the destination $S$, stop; $R$ is the desired shortest path. Otherwise, go to the next iteration.

### 3.2 The auction algorithm improvement

The CS determination of basic auction algorithm can seek the shortest path between two points in network, but the public transport network is particularity. Passengers walking between some sites, you should consider passengers walking between some sites when choosing a travel route. At this time, we just need to list the shortest distance matrix of the passengers to walk, together with the various bus lines correspond to the shortest distance matrix, which constitute the total distance of the shortest direct matrix. The bus travel of the passengers would not only consider the actual distance between two points, they will also consider whether you want to transfer, if the transfer, the transfer convenience, the total journey time and costs. Therefore, this is the considerations of passenger selecting bus lines travel. So the traditional auction algorithm CS judgment is not comprehensive, and its slight modifications. Increase a distance cost function.
$p_{i} \leq a_{i j}+p_{j}+p_{c t} \quad \forall(i, j) \in A$
$p_{i}=a_{i j}+p_{j}+p_{c t} \quad$ For all the continuous nodes
$p_{c}=0.5 c \quad c$ is the transfer costs
$p_{t}=0.5 t \quad t$ is thetransfer waiting time
$p_{\mathrm{ct}}=p_{c}+p_{t}$

## 4. Application example



Figure 1. A regional bus station

As shown in Figure1, the bus stops1,2,3,4,5,6, L-Line bus lines in sites 1,2,3,4 parking site, bus lines N -line site 1,6,4 parking stations, bus lines K-line site 2',5,4 parking sites, bus lines M-line docking site in site 1,3.

If you ride the bus L-line to take bus K-Line in Site 2, you need to walk some distance. At bus stop 3, bus

M-line and L-line same station transfer. Now a departure from the site 1 , the destination site 4 , find the optimal travel routes.

First, the public transport network is transformed into a transit network to figure 2:


Figure 2.Transit network digraph

Assume that all bus lines ticket price is 2 yuan not consider preferential transfer. Uplink and downlink are the same, therefore only consider the uplink route, simply anti multiplication can return. The letters on the arc for
the bus lines, the value to the distance between node to node.

The process of auction algorithms to find the shortest path of node 1 to node 4:
(1)

Table 1. The calculation process of the program 1

| Iteration | Iterations | Iterations | Operating |
| :--- | :--- | :--- | :--- |


|  | before <br> the R | before the <br> $\mathbf{p}$ |  |
| :---: | :---: | :---: | :---: |
| 1 | $(1)$ | $(0,0,0,0,0,0)$ | Node 1 <br> contraction |
| 2 | $(1)$ | $(2,0,0,0,0,0)$ | Extending <br> to the node <br> 2 |
| 3 | $(1,2)$ | $(2,0,0,0,0,0)$ | Node 2 <br> contraction |
| 4 | $(1)$ | $(2,1,0,0,0,0)$ | Node 1 <br> contraction |
| 5 | $(1)$ | $(3,1,0,0,0,0)$ | Extending <br> to the node <br> 2 |
| 6 | $(1,2)$ | $(3,1,0,0,0,0)$ | Extending <br> to the node <br> 3 |
| 7 | $(1,2,3)$ | $(3,1,0,0,0,0)$ | Node 3 <br> contraction |
| 8 | $(1,2)$ | $(3,1,2,0,0,0)$ | Node 2 <br> contraction |
| 9 | $(1)$ | $(3,3,2,0,0,0)$ | Node 1 <br> contraction |
| 10 | $(1)$ | $(5,3,2,0,0,0)$ | Extending <br> to the node <br> 2 |
| 11 | $(1,2)$ | $(5,3,2,0,0,0)$ | Extending <br> to the node <br> 3 |
| 12 | $(1,2,3)$ | $(5,3,2,0,0,0)$ | Extending <br> to the node <br> 4 |
| 13 | $(1,2,3,4)$ | $(5,3,2,0,0,0)$ | End |
| 510 | 0,2 |  |  |

Line: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$, Distance: 5 , Transfer: 0 , Fare: 2 yuan. Total distance: 6
(2)

| Table 2. The calculation process of the program2 |  |  |  |
| :---: | :---: | :---: | :---: |
| Iteration | Iterations <br> before the <br> $\mathbf{R}$ | Iterations before <br> the p | Operating |
| 1 | $(1)$ | $(0,0,0,0,0,0)$ | Node 1 <br> contraction |
| 2 | $(1)$ | $(2,0,0,0,0,0)$ | Extending <br> to the node <br> 2 |
| 3 | $(1,2)$ | $(2,0,0,0,0,0)$ | Node 2 <br> contraction |
| 4 | $(1)$ | $(2,(0.5,0), 0,0,0,0)$ | Node 1 <br> contraction |
| 5 | $(1)$ | $(2.5,(0.5,0), 0,0,0,0)$ | Extending <br> to the node <br> 2 |
| 6 | $(1,2)$ | $(2.5,(0.5,0), 0,0,0,0)$ | Extending <br> to the node <br> $2^{\prime}$ |
| 7 | $\left(1,2,2^{\prime}\right)$ | $(2.5,(0.5,1), 0,0,0,0)$ | Node 2' <br> contraction |
| 8 | $(1,2)$ | $(2.5,(1.5,1), 0,0,0,0)$ | Node 2 <br> contraction |
| 9 | $(1)$ | $(3.5,(1.5,1), 0,0,0,0)$ | Node 1 <br> contraction |
| 2 |  |  |  |


| 10 | (1) | (3.5,(1.5,1),0,0,0,0) | Extending to the node 2 |
| :---: | :---: | :---: | :---: |
| 11 | $(1,2)$ | (3.5,(1.5,1),0,0,0,0) | Extending to the node $2^{\prime}$ |
| 12 | (1,2,2') | (3.5,(1.5,1),0,0,0,0) | Extending to the node 5 |
| 13 | (1,2,2',5) | (3.5,(1.5,1),0,0,0,0) | Node 5 contraction |
| 14 | (1,2,2') | (3.5,(1.5,1),0,0,1,0) | Node 2' contraction |
| 15 | $(1,2)$ | (3.5,(1.5,2),0,0,1,0) | Node 2 contraction |
| 16 | (1) | (3.5,(2.5,2),0,0,1,0) | Node 1 contraction |
| 17 | (1) | (4.5,(2.5,2),0,0,1,0) | Extending to the node 2 |
| 18 | $(1,2)$ | (4.5,(2.5,2),0,0,1,0) | Extending to the node 2' |
| 19 | (1,2,2') | (4.5,(2.5,2),0,0,1,0) | Extending to the node 5 |
| 20 | (1,2,2',5) | (4.5,(2.5,2),0,0,1,0) | Extending to the node 4 |
| 21 | (1,2,2',5,4) | (4.5,(2.5,2),0,0,1,0) | End |

Line: $1 \rightarrow 2 \rightarrow 2, \rightarrow 5 \rightarrow 4$, Distance: 4.5, Transfer:1, Fare:4 yuan. Total distance: 6
(3)

Table 3. The calculation process of the program3

| Iteration | Iterations <br> before <br> the R | Iterations <br> before the $\mathbf{p}$ | Operating |
| :---: | :---: | :---: | :---: |
| 1 | $(1)$ | $(0,0,0,0,0,0)$ | Node 1 <br> contraction |
| 2 | $(1)$ | $(4,0,0,0,0,0)$ | Extending <br> to the node <br> 3 |
| 3 | $(1,3)$ | $(4,0,0,0,0,0)$ | Node 3 <br> contraction |
| 4 | $(1)$ | $(4,0,2,0,0,0)$ | Node 1 <br> contraction |
| 5 | $(1)$ | $(6,0,2,0,0,0)$ | Extending <br> to the node <br> 3 |
| 6 | $(1,3)$ | $(6,0,2,0,0,0)$ | Extending <br> to the node <br> 4 |
| 7 | $(1,3,4)$ | $(6,0,2,0,0,0)$ | End |

Line: $1 \rightarrow 3 \rightarrow 4$, Distance: 6 , Transfer:1, Fare:4 yuan. Total distance: 8
(4)

Table 4. The calculation process of the program4

| Iteration | Iterations <br> before <br> the $\mathbf{R}$ | Iterations <br> before the <br> $\mathbf{p}$ | Operating |
| :---: | :---: | :---: | :---: |
| 1 | $(1)$ | $(0,0,0,0,0,0)$ | Node 1 |


|  |  |  | contraction |
| :---: | :---: | :---: | :---: |
| 2 | $(1)$ | $(3,0,0,0,0,0)$ | Extending to <br> the node 6 |
| 3 | $(1,6)$ | $(3,0,0,0,0,0)$ | Node 6 <br> contraction |
| 4 | $(1)$ | $(3,0,0,0,0,5)$ | Node 1 <br> contraction |
| 5 | $(1)$ | $(8,0,0,0,0,5)$ | Extending to <br> the node 6 |
| 6 | $(1,6)$ | $(8,0,0,0,0,5)$ | Extending to <br> the node 4 |
| 7 | $(1,6,4)$ | $(8,0,0,0,0,5)$ | End |

Line: $1 \rightarrow 6 \rightarrow 4$, Distance: 8 , Transfer:0, Fare: 2 yuan. Total distance: 9

It can be found by calculating the shortest line is (2), take bus L-line to the site 2 get off and walk some distance to transfer bus K-line to reach the destination site 4, this trip have to transfer to a bus. However, after taking the fares factors into account, we can found the route (1) is the optimal solution. The bus L line to the destination, although the distance is farther away, but do not transfer, and total fare is cheaper than the line (1). The route (3) and (4) total distance is too long, the passengers obviously not choose. So to sum up, the Optimal Route is line (1).

## 5. Conclusion

According to the characteristics of the public transport network, add the passenger decision-making demand as constraints into the objective function, to give the scheduled transfer constraints, the bus optimal path convert to the shortest path problem.

## References

[1] Bertsekas D P. Auction algorithms for transportation problems. Annals of Operation Researth,1989,20: 67-96.
[2] Oday Ibraheem Abdullah,Josef Schlattmann.Vibration Analysis of the Friction cluth Disc Using Finite Element

Method, Advances in Mechanical Engineering and its Applications,2021,1(4):86-91.
[3] Cheng Lin. Urban traffic network flow theory. Nanjing: Southeast University Press, 2010.
[4] Rekha Maradugu, M.Pushpa, K. Vamsikrishnareddy,
A. Sudhakar. A novel method of optimized Resource Allocation for Software Release Planning. Advances in Mechanical Engineering and its Applications, 2012,1(2):29-32.
[5] Xiao-Jun Yang. Local fractional Fourier analysis. Advances in Mechanical Engineering and its Applications, 2012, 1(1):12-16.
[6] Bertsekas D P. An auction algorithm for shortest paths. SIAM J. for Optimization, 1991,1: 425-447.
[7] Shahin Mohammad pour,Elham Shamekhi.An Innovative View to Cultural Infrastructures of Investment. Advances in Applied Economics and Finance, Vol. 1, No. 1, 2012: 29-31.
[8] Wang Jingyuan, Lin Cheng. Auction algorithm for the shortest paths in the application of traffic assignment, Journal of transportation systems engineering and information, 2006, 6(6):79-82.
[9] Saeed Balochian, Soheil Vosoughi. Design and Simulation of turbine speed control system based on Adaptive Fuzzy PID controller. Advances in Mechanical Engineering and its Applications, 2012.1(3): 37-42.

