

# Analysing Constrained Machining Conditions in Turning Operations by Differential Evolution

<sup>1</sup> Yograj Singh, <sup>2</sup> Pinkey Chauhan

Dept. of Mathematics, University of Delhi, Delhi, India

\* Dept. of Mathematics, Indian Institute of Technology Roorkee, Roorkee, India

Email: [yograjchauhan26@gmail.com](mailto:yograjchauhan26@gmail.com), [pinkeychauhan030@gmail.com](mailto:pinkeychauhan030@gmail.com)

**Abstract** – Machining processes are the core of manufacturing industry, where raw material is shaped into a desired product by removing unwanted material. The determination of optimal machining conditions is very crucial while producing high quality products at minimal costs. This paper proposes the process parameter optimization of multi pass turning operations. The multi-pass turning includes several rough passes and single finish pass with several constraints around twenty, imposed during roughing and finishing operations on tool life, surface finish, cutting forces, machining power and chip-tool interface. In this paper Differential Evolution (DE), a potential global optimizer, is employed to obtain the optimal set of parameters that minimize the total unit production cost under several constraints. Results obtained using DE are compared with results obtained using PSO, GA and ACO as recorded in literature.

**Keywords** – Multi pass turning operation; Parameter optimization; Unit Production cost; Differential Evolution

## 1. Introduction

In manufacturing industry, economic machining is a prime need due to high capital involved in these large scale industries. These days the notion of computer numerically controlled (CNC) machining is introduced to deal the problem of economic machining. During machining process, metals or wood parts are shaped to obtain desired product by removing unwanted material. The machining process of any desired product must satisfy the specified quality restrictions such as surface finish, accuracy and surface integrity. The objective is to minimize the unit cost or machining time according to available conditions. To meet these objectives we have to determine the optimal machining conditions by optimizing machining parameters such as cutting speed, feed rate depth of cut and number of passes under several constraints imposed on machining processes. Although handbooks are available to provide recommended machining parameters but these handbooks does not satisfy the need of economic machining. To overcome this drawback, sufficient research work has been done and still going on, in this field to provide a reliable method that produces efficient solution. The analysis of single and multi pass turning operations under practical constraints starts nearly three decades ago by [2] who considered production cost or machining time as minimization criteria. Some conventional optimization techniques such as scatter searches [10], dynamic programming [4], geometric programming [2] and Hook-Jeeves method [6] were also employed to optimize machining models. Some studies also concentrated on

refining multi pass machining models by adding some more real constraints and parameters that makes simulated models more realistic[4],[17].

The complexity of machining models increases with increase of no of practical constraints and parameters. Conventional optimization techniques produce local optimal solution and are not appropriate for these highly constrained problems. Therefore heuristic methods such as PSO [1], [7], DE [14], ABC [15-16] etc. are preferred to deal with such complex problems as they provide near global optimum solution. Recently many non-traditional optimization techniques like Simulated annealing (SA) [9], Genetic algorithm [3], Ant colony optimization [6] and Particle Swarm optimization [5] has been employed to determine the optimal conditions for constrained machining models. More recently, Yusup et al. [11], [12] performed a comprehensive review on machining parameter optimization for both traditional and advanced machining methods using various evolutionary techniques.

In this study we have presented the application of Differential Evolution (DE) for optimizing machining parameters in multi pass turning operations. Here we have considered the model proposed by [9] and considered in [5]. This model includes several constraints on tool life, surface finish, cutting forces machining power, chip tool interface along with parameter bounds. The objective function is to minimize the total unit production cost by determining optimal number of passes, cutting speed, feed rate and depth of cut during roughing and finishing passes.

The remaining paper is organized as follows. Section 2 describes the considered machining model along with notations used in model. Section 3 gives a brief introduction of employed methodology DE and Section 4 comprises computational results along with the discussion of results and Section 5 concludes the paper.

## 2. Mathematical Formulation Of Multi-Pass Turning Operation

In the present study paper we have considered the machining model proposed by [9] for multi-pass turning operations which involve multiple rough cuts and a single finish cut. The optimization criteria considered here is the minimization of total unit production cost. The machining parameters to be determined are cutting speed, feed rate depth of cut and number of rough cuts under constrained machining environment. The model is described as below.

### 2.1. The Cost Function

The turning process is divided into multi-pass roughing and single-pass finishing. The components may be either a straight-cut type or of continuous forms. Here we have considered bar components. The unit production cost,  $UC$ , for multi-pass turning operations is divided in four basic cost elements and is given by:

$$UC = C_M + C_I + C_R + C_T$$

All of these components are described as below.

1. Cutting cost by actual time in cut,  $C_M$ , is given by:

$$C_M = k_0 t_m$$

Where,  $t_m$  is the actual cutting time expressed as the sum of the times for roughing and finishing cutting phases:

$$t_m = t_{mr} + t_{ms}$$

where,

$$t_{ms} = \frac{\pi DL}{1000 V_s f_s} \quad t_{mr} = \frac{\pi DL}{1000 V_r f_r} \times n$$

here number of passes ( $n$ ) is given by:

$$n = \left( \frac{d_t - d_s}{d_r} \right)$$

2. Machine idle cost due to loading and unloading operations and idling tool motion,  $C_I$ , is given by:

$$C_I = k_0 t_i$$

Where,  $t_i$  is machine idle time, and can be expressed as the sum of idle tool motion ( $t_v$ ) and time due to loading and unloading operations which is a constant term ( $t_c$ ).

$$t_i = t_c + (h_1 L + h_2) \times \left( \frac{d_t - d_s}{d_r} + 1 \right)$$

3. Tool replacement Cost,  $C_R$ , is given by:

$$C_R = k_0 t_e (t_m / T_p)$$

Where,  $t_e$  is the time required to exchange a tool and  $T_p$  is the tool life. Usually tool life ( $T$ ) is determined using Taylor's tool life equation;

$$T = \frac{C^{1/\alpha}}{V^{1/\beta} f^{1/\gamma} d^{1/\delta}} = \frac{C_0}{V^p f^q d^r}$$

But due to different machining conditions for wear rate changes for roughing and finishing phases. In such cases the tool life can be expressed as

$$T_p = T_r + T_s$$

$$\text{Where, } T_r = \frac{C_0}{V_r^p f_r^q d_r^r} \quad \text{and } T_s = \frac{C_0}{V_s^p f_s^q d_s^r}$$

4. The Tool Cost,  $C_T$  is given by:

$$C_T = k_t (t_m / T_p)$$

Where,  $k_t$  is cutting edge cost.

### 2.2. Constraints on Cutting Conditions

In the considered model, several constraints are imposed during roughing and finishing operations on machining environment. The detailed description of these constraints for both roughing and finishing operations are given below.

*Rough Machining:*

1. Parameter bounds: The available range of cutting speed, feed rate and depth of cut are expressed in terms of lower and upper bounds respectively as below:

$$V_{rL} \leq V \leq V_{rU}, \quad f_{rL} \leq f \leq f_{rU}, \quad d_{rL} \leq d \leq d_{rU}$$

Tool-life constraint: The constraint on tool life is expressed as

$$T_L \leq T_r \leq T_U$$

Operating constraints: The constraints on machine operating conditions are as:

(i) Cutting force constraint: Cutting force constraint is taken as

$$F_r = k_1 (f_r)^u (d_r)^v \leq F_u$$

(ii) Power constraint: The power required during the cutting operation should not exceed the available power of the machine tool. The power is given as

$$P_r = \frac{F_r V_r}{6120\eta} \leq P_U,$$

Where, efficiency  $\eta = 0.085$

(iii) Stable cutting region constraint: Stable cutting region is given by

$$f_r (V_r)^\lambda (d_r)^\nu \geq SC$$

(iv) Chip-tool interface temperature constraint: This constraint is expressed as

$$Q_r = k_2 (V_r)^\tau (f_r)^\phi (d_r)^\delta \leq Q_u$$

*Finish Machining:*

For finish machining operation, all the constraints other than the surface finish constraint are similar as for rough machining. Therefore all finishing constraints are as

(1) Bounds on finishing parameters are

$$V_{sL} \leq V \leq V_{sU} ; f_{sL} \leq f \leq f_{sU} ; d_{sL} \leq d \leq d_{sU}$$

$$(2) T_L \leq T_s \leq T_U$$

$$(3) F_s = k_1 (f_s)^\mu (d_s)^\nu \leq F_u$$

$$(4) P_s = \frac{F_s V_s}{6120\eta} \leq P_U$$

$$(5) f_s (V_s)^\lambda (d_s)^\nu \geq SC$$

$$(6) Q_s = k_2 (V_s)^\tau (f_s)^\phi (d_s)^\delta \leq Q_u$$

In addition with these constraints imposed on finish machining, the constraint related to surface finish is given by

$$(7) \frac{f_s^2}{8R} \leq (SR)_U$$

The functional relationships between roughing and finishing parameters are as

$$V_s \geq k_3 V_r, f_r \geq k_4 f_s, d_r \geq k_5 d_s$$

Finally the bound on number of finish cuts  $n = \left( \frac{d_t - d_s}{d_r} \right)$ , which are considered similarly as rough cuts and is given by

$$\left( \frac{d_t - d_{sU}}{d_{rU}} \right) \leq \left( \frac{d_t - d_s}{d_r} \right) \leq \left( \frac{d_t - d_{sL}}{d_{rL}} \right)$$

All the notations used in modelling objective function and constraints are given below.

### 3. Optimization Methodology: Differential Evolution

Differential Evolution (DE) belongs to the category evolutionary algorithms and developed by Storn and Price in 1995 [13]. The algorithm shares many similarities with other Evolutionary Algorithms (EA) on the basis of genetic operators for generating and refining solutions at every iteration. However, the order of operators in DE differs from other evolutionary algorithms quite significantly on the working of these operators, particularly the mutation operator. These operators are defined in the following subsections.

#### 3.1. Mutation

The mutation operator in DE produces a trial vector corresponding to each individual of the current population by mutating a target vector with a weighted differential. This trial vector is then used by crossover operator to produce offspring. The trial vector  $u_i(t)$ , corresponding to the target vector  $X_i(t)$ , is generated, as follows;

Select a target vector  $X_{i_1}$ , from the population, such that  $i \neq i_1$ . Then, randomly select two individuals,  $X_{i_2}$  and  $X_{i_3}$  from the population such that  $i \neq i_1 \neq i_2 \neq i_3$  and  $i_2, i_3 \sim U(1, n_s)$ . Using these individuals, the trial vector is calculated by perturbing the target vector as follows:

$$u_i(t) = X_{i_1}(t) + \beta (X_{i_2}(t) - X_{i_3}(t))$$

Where  $\beta \in (0, 1+)$  is the scale factor which controls the amplification of the differential variation,  $(X_{i_2}(t) - X_{i_3}(t))$ . The smaller value of  $\beta$  leads to smaller step sizes that increases the computational time of algorithm, on the other hand the larger value of  $\beta$  provides faster convergence but may result in premature convergence. Therefore an appropriate value of  $\beta$  should be chosen to maintain exploration-exploitation trade off.

#### 3.2. Crossover

The Crossover operator, combines the trial vector  $u_i(t)$  and the parent vector  $X_i(t)$ , to produce offspring, using the following rule

$$X'_{ji}(t) = \begin{cases} u_{ji}(t) & \text{if } randb(j) \leq CR \text{ or } j = rnbr(i) \\ X_{ji}(t) & \text{if } randb(j) > CR \text{ or } j \neq rnbr(i) \end{cases}$$

Where,  $randb(j) \in [0,1]$  is the  $j$ th evaluation of random number generator,  $rnbr(i)$  is a randomly chosen index  $\in \{1, 2, \dots, d\}$ , which ensures that offspring,  $X'_{ji}(t)$  has at least one component from

trial vector  $u_i(t)$ .  $CR$  is the crossover constant to be determined by the user.

### 3.3. Selection

Selection operator decides which individual should be forwarded to next generation, the offspring  $X'_i(t)$  is compared to the target vector  $X_i(t)$  using the greedy criterion. If the vector  $X'_i(t)$  has better fitness value than target vector  $X_i(t)$ , it will replace the target vector in next generation, otherwise the target vector retains its place for at least one more generation. By comparing each offspring with its target vector from which it inherits parameters, DE strongly integrates recombination and selection in comparison to other EAs:

$$X_i(t+1) = \begin{cases} X'_i(t) & \text{if } f(X'_i(t)) \leq f(X_i(t)) \\ X_i(t) & \text{otherwise} \end{cases}$$

Once the new population is installed, the process of mutation, recombination and selection is repeated until the optimum is located, or a pre specified termination criterion is satisfied.

### 4. Experimental Results and Discussion

Here in this study, the machining data for bar components which was provided by [9] is adopted for experimental evaluation of turning model. The data is shown in Table 4. The parameters used in DE are as: the initial population size is fixed to 20; number of simulations is taken as 50 with maximum number of generations set to 500 during each simulation. Table 1 show the optimal unit production cost with non dimensional constraint violation using DE and other algorithms as quoted in previous study [5]. The parameters that are to be determined for optimizing the machining conditions are feed rate, depth of cut, cutting speed and number of passes for roughing and finishing. Table 2 depicts the optimal value of these parameters using DE and other algorithms as recorded in literature.

**Table 1.** Optimal total unit cost and constraints at the optimum point

Method	Optimum unit cost(\$)	Non-dimensional constraint violation
DE	1.962581	0
PSO	2.2721	0
ACO	1.8450	0.5396
GA	1.7842	0.5148
SA/PS	2.3135	0.0667

**Table 2.** Optimal cutting parameters using different methods

Method	DE	PSO	ACO	GA
$V_r$	122.81120	106.69	103.05	114.22
$f_r$	0.576182	0.897	0.9	0.7
$d_r$	2.95612	2	3	2.9745
$V_s$	169.24555	155.89	162.02	164.369
$f_s$	0.23047	0.28	0.24	0.2978
$d_s$	2.956125	2	3	2.963

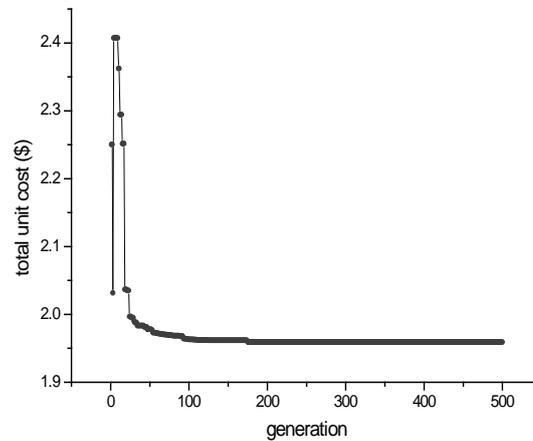
**Table 3.** Number of rough passes and optimal unit cost for different values of total depth of cut( $d_t$ ) for  $d_{sL} = d_{rL} = 1$  mm

Total depth of cut( $d_t$ )	Maximum depth of cut (mm) for $d_{sU} = d_{rU}$	Bounds on n	$d_{opt}$		Unit cost in \$	
			PSO	DE	PSO	DE
6	3	$1 \leq n \leq 5$	2	2.99	2.272	1.9626
8	4	$2 \leq n \leq 7$	3	2.66	3.306	2.438
10	5	$2 \leq n \leq 9$	3	3.32	3.853	2.754
12	3	$3 \leq n \leq 11$	5	2.99	3.846	3.237

**Table 4.** Numerical data for cutting model

Parameter	Value	Parameter	Value
$V_{rU}$	500 m/min	$d_t$	6 mm
$V_{rL}$	50 m/min	p	5
$V_{sU}$	500 m/min	q	1.75
$T_L$	25 mm	$k_2$	132
$V_{sL}$	50 m/min	r	0.75

$T_U$	45 mm	$k_3$	1
$f_{rL}$	0.1 mm/rev	$u$	0.75
$t_c$	0.75 min/piece	$k_4$	2.5
$f_{rU}$	0.9 mm/rev	$v$	0.95
$t_e$	1.5 min/edge	$k_5$	1
$f_{rL}$	0.1 mm/rev	$\eta$	0.85
$P_U$	5 kW	$k_t$	2.5 \$/edge
$f_{rU}$	0.9 mm/rev	$\lambda$	2
$F_U$	200 kgf	$D$	50 mm
$d_{rL}$	1 mm	$v$	-1
$d_{rU}$	3 mm	$\tau$	0.4
$d_{sL}$	1 mm	$\phi$	0.2
$Q_U$	1,000 <sup>0</sup> C	$L$	300 mm
$SR_U$	10 $\mu$ m	$R$	1.2 mm
$h_2$	0.3	$k_1$	108
$h_1$	$7 \times 10^{-4}$	$k_0$	0.5 \$/min
$SC$	140	$d_{sU}$	3 mm
$\delta$	0.105	$C_0$	$6 \times 10^{11}$



**Figure1.** Generation wise convergence graph of optimal unit cost using DE for  $d_t = 6.0$  mm

Table 3 presents the optimal value of unit production cost using DE and along with cutting parameters for different values of total depth of cut. The average computational time taken by DE over 50 runs is 0.2665 *Sec*. Table 1 shows that optimal unit production using DE is much

better than PSO and Simulated annealing (SA/PS) and also encouraging over other algorithms on feasibility basis. It is observed from Table 3 that the corresponding values of unit production cost obtained using DE are improved significantly over PSO for all considered values of depth of cut. The convergence of total unit cost over max number of generations using DE is shown in Fig.1, which illustrates the algorithm, converges up to 200 generations. Thus, the proposed method successfully obtained the desired optimal conditions within fewer computations.

## 5. Conclusion

In this study, the problem of parameter optimization in constrained machining environment is addressed and solved using Differential Evolution, a potential candidate

of non-traditional global optimizers. The mathematical simulation turns out as a complex and highly constrained machining model where the aim is to minimize the total production cost. The machining parameters as feed rate, cutting speed and depth of cut during roughing and finishing passes are the main process parameters whose optimal values affect the machining process to greater extent. The results obtained using DE are compared with PSO, Binary GA, ACO and SA as reported in literature. The observation of optimal results show that the proposed methods (DE) provides promising results on quality and feasibility basis as compared to other existing algorithms. The proposed algorithm finds significantly better optimal solution with less computational efforts. Thus, DE can be recommended as a reliable and efficient method for solving such complex machining problems and those with higher degree of complexity.

## Acknowledgements

The author's would like to thank the editor's of this special issue: Tarun Kumar Sharma and the unknown reviewers/ referees for giving their valuable suggestions, which helped us in improving the shape of the paper.

## References

- [1] A. Yazdekhasti, I. Sadeghkhan, Optimal Tuning of TCSC Controller Using Particle Swarm Optimization, *Advances in Electrical Engineering Systems*, 2012, 1:1, 24-29.
- [2] D. S. Ermer, Optimization of constrained machining economics problem by geometric programming, *ASME Journal of Engineering for Industry*, 1971, 93, 1067-1072.
- [3] G. C. Onwubolu, T. Kumalo, Multi-pass turning operations optimization based on genetic algorithms, *Proceedings of the Institution of Mechanical Engineers*, 2001, 215, 117-124
- [4] J. S., Agapiou, The optimization of machining operations based on a combined criterion. Part I: the use of combined objectives in single-pass operations, *ASME Journal of Engineering for Industry*, 1992, 114, 500-507.
- [5] J. Srinivas, R. Giri, S.-H. Yang, Optimization of multi pass turning using particle swarm intelligence, *International journal of advanced manufacturing technology*, 2009, 40, 56-66.
- [6] K. Vijayakumar, G. Prabhakaran, P. Asokan, R. Saravanan, Optimization of multi-pass turning operations using ant colony system, *International journal of Machine Tools & Manufacture*, 2003, 43, 1633-1639.
- [7] M. R. Rad, M. R. Rad, S. Akbari, S. A. Taher, Using ANFIS, PSO, FCN in Cooperation with Fuzzy Controller for MPPT of Photovoltaic Arrays, *Advances in Digital Multimedia*, 2012, 1:1, 37-45.
- [8] M. Ruy, E. Krasteva, S. Doytchinov, Computer-aided selection of optimum machining parameters in multi pass turning, *The International Journal of Advanced Manufacturing Technology*, 1995, 10, 19-26.
- [9] M.-C. Chen, D.-M. Tsai, A simulated annealing approach for optimization of multi-pass turning operations, *International Journal of Production Research*, 1996, 34, 2803-2825.
- [10] M.-C., Chen, Optimizing machining economics models of turning operations using the scatter search approach, *International Journal of Production Research*, 2004, 42, 2611-2625.
- [11] N. Yusup, A. M. Zain, S. Z. M. Hashim, Evolutionary techniques in optimizing machining parameters: Review and recent applications (2007–2011), *Expert Systems with Applications*, 2012, 39: 10, 9909-9927.
- [12] N. Yusup, A. M. Zain, S. Z. M. Hashim, Overview of PSO for Optimizing Process Parameters of Machining, *Procedia Engineering*, 2012, 29, 914-923.
- [13] R. Storn, K. Price, Differential Evolution-A Simple and Efficient Adaptive Scheme for Global Optimization over Continuous Spaces, *Technical Report TR-95-012, Berkeley, CA*, 199.
- [14] S. A. Ghoreishi<sup>1</sup> and A. Ahmadvand, State Feedback Design Aircraft Landing System with Using Differential Evolution Algorithm, *Advances in Computer Science and its Applications*, Vol. 1, No. 1, March 2012, pp. 16-20.
- [15] T. K. Sharma, M. Pant, V. P. Singh, Adaptive Bee Colony in an Artificial Bee Colony for Solving Engineering Design Problems, *Advances in Computational Mathematics and its Applications*, 2012, 1:4, 213-221.
- [16] T. K. Sharma, M. Pant, V. P. Singh, Improved Local Search in Artificial Bee Colony using Golden Section Search, *Journal of Engineering*, 2012, 1:1, 14-19.
- [17] Y. C. Shin, Y. S. Joo, Optimization of machining conditions with practical constraints, *International Journal of Production Research*, 1992, 30, 2907-2919.

## Vitae



**Yograj Singh:** Yograj Singh is currently working as an Assistant Professor in Mathematics at University of Delhi. He was born on August 13, 1983 in Greater Noida, Uttar Pradesh, India.

He obtained his Bachelor degree in 2004 and Masters degree in Mathematics in 2006 from CCS University, Meerut with first division. He has also received a Masters degree in Computer Science from Indian Statistical Institute, Kolkata, India with first grade and also qualified many national level exams as JAM, CSIR-JRF, GATE (Mathematics and Computer Science). His research interests include Cryptology, Evolutionary Computing and their Applications.



**Pinkey Chauhan:** Pinkey Chauhan is currently pursuing her Ph.D. degree in Mathematics at Indian Institute of Technology Roorkee, India. She was born on May 3, 1985 in Noida, Uttar Pradesh, India and finished her schooling with distinction marks. She received her Bachelor and Masters degree in Mathematics from C.C.S University, Merrut, India with first division. Her research interest includes development of Nature Inspired Algorithms and their applications for solving optimization problems arising in various industries.