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DIFFUSION OF HEAT IN INFINITE CYLINDER HAVING SOURCE OF HEAT WITHIN IT

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ABSTRACT: In this paper we have discussed a problem of diffusion of heat in an infinite cylinder of radius a assuming that the surface r = a is kept at zero temperature having boundary conditions and source of heat different form those taken by Sneddon . We assume that the rate of generation of heat is independent of the temperature. Since the cylinder is infinitely long so the variation of temperature with respect to axial coordinate z is neglected.

Keyword: Hankel transform, Laplace transform, Convolution, Diffusion, Heat.

AMS subject classification: 44A10, 44A20, 44A35, 76R50

1. INTRODUCTION

Conduction of heat in solids in the interior of which heat is generated or absorbed plays an important role in Engineering. Heat may be produced in solids by the induction heating, chemical reaction, absorbing radiation, radioactive decay, and the passage of an electric current etc. Conduction of heat in anisotropic solids such as crystals, wood, transformer cores and sedimentary rocks etc., is of great use in technical applications.

Cinelli [1] has discussed the problem of transient temperature in a finite hollow cylinder due to asymmetrical internal heat generation when radiation is taking place on all the four surfaces.

Marchi and Fasula [3] have studied the problem of conduction of heat in which sources are generated according to the linear function of the temperature in medium in the form of sectors of hollow cylinders of finite height and sectors of semi infinite and infinite hollow cylinders.

Sneddon [5] have discussed problem of diffusion of heat in a cylinder of radius a assuming that the surface r = a is kept at zero temperature.

In this paper we have discussed a problem of diffusion of heat in an infinite cylinder of radius a assuming that the surface r = a is kept at zero temperature having boundary conditions and source of heat different form those taken by Sneddon [5].

2. FORMULATION OF PROBLEM

Let us consider infinite cylinder of radius ‘a’. Let there are source of heat within it. The equation for diffusion of heat is given by

$$\frac{\partial u}{\partial t} = k\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] + \theta(r,t) \tag{2.1}$$

Where θ (r, t) is source of heat within it which lead to an axially symmetrical temperature distribution. We assume that the rate of generation of heat is independent of the temperature. Since the cylinder is infinitely long so the variation of temperature with respect to axial coordinate z is neglected.

Initial condition

$$u(r,t)\Big|_{t=0} = f_1(r) \tag{2.2}$$

Boundary Condition

Let the surface r = a is maintained at fixed temperature u_0 i.e.

$$u(r,t)\Big|_{r=a} = f_2(a,t) \tag{2.3}$$

The Finite Hankel transform [4] of zero order is given by the integral

$$\int_0^a rf(r)J_0(\xi_i, r)dr = \bar{f}(\xi_i) \tag{2.4}$$

where ζ_i is the root of the equation

$$J_0(a\xi_i) = 0 \tag{2.5}$$

The inversion theorem of above transform is

$$f(r) = \frac{2}{a^2}\sum_i \bar{f}(\xi_i)\frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2} \tag{2.6}$$

where the summation extends over all the positive roots of equation (2.5). The operational property of this transform is written as

$$\int_0^a r\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] J_0(r, \xi_i)dr$$

$$= a\xi_i u(a)J_1(a, \xi_i) - \xi_i^2 J_0(u) \tag{2.7}$$

The Laplace transform is defined as

$$\int_0^\infty e^{-pt}\bar{f}(t)dt = \bar{f}(p) \tag{2.8}$$

Inversion integral is

$$f(t) = \int_0^\infty e^{-pt}\bar{f}(p) \tag{2.9}$$

3. SOLUTION of THE Problem

Applying finite Hankel transform to the equation (2.4) then using operational property of the transform and boundary condition [3].

$$\frac{d\bar{u}}{dt}(\xi_i t) + k\xi_i^2\bar{u}(\xi_i, t)$$

$$= ka\xi_i f_2(a, t)J_1(a\xi_i) + \bar{\theta}(\xi_i, t) \tag{3.1}$$

$$\bar{u}(\xi_i, t) = \int_0^a ru(rt)J_0(\xi_i r)dr$$

Solving equation (3.1) with the help of Laplace transform and convolution theorem and using initial conditions (2.2), we get

$$\bar{u}(\xi_i, t) = e^{-k\xi_i^2 t}\bar{f}_1(\xi_i) + ka\xi_i J_1(a\xi_i)$$

$$\int_0^1 f_2(a, q)e^{-k\xi_i^2(t-q)}dq + \int_0^1 \bar{\theta}(\xi_i, t)e^{-k\xi_i^2(t-q)}dq \tag{3.2}$$

Applying Hankel inversion theorem given by equation (2.6)

$$u(r, t) = \frac{2}{a^2}\sum_i \frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2}$$

$$\left[e^{-k\xi_i^2 t}\bar{f}_1(\xi_i) + ka\xi_i J_1(a\xi_i)\int_0^1 f_2(a, q)e^{-k\xi_i^2(t-q)}dq\right]$$

$$+\int_0^1 \bar{\theta}(\xi_i, t)e^{-k\xi_i^2(t-q)}dq \tag{3.3}$$

If the rate of junction of heat is taken in the form

$$\text{then } q(r, t) = \frac{k}{k}f(r)g(t) \tag{3.4}$$

where k is the diffusivity and k the conductivity of the material. Therefore $\bar{\theta}(\xi_i, q) = \frac{k}{k}g(q)\bar{f}(\xi_i)$

So that,

$$u(r, t) = \frac{2}{a^2}\sum_i \frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2}$$

$$\left[e^{-k\xi_i^2 t}\bar{f}_1(\xi_i) + ka\xi_i J_1(a\xi_i)\right]$$

$$\int_0^1 f_2(a, q)e^{-k\xi_i^2(t-q)}dq + \bar{f}_1(\xi_i)\int_0^1 g(q)e^{-k\xi_i^2(t-q)}dq \tag{3.5}$$

Let $f(r) = H_{P\,Q}^{M\,N}\left[r^\lambda \;\middle|\; \begin{matrix} {}_1(a_j, \alpha_j)_P \\ {}_1(b_j, \beta_j)_Q \end{matrix}\right]$. This $H_{P\,Q}^{M\,N}\left[r \;\middle|\; \begin{matrix} {}_1(a_j, \alpha_j)_P \\ {}_1(b_j, \beta_j)_Q \end{matrix}\right]$ is the H-function of one variable introduced by Fox [2]. The finite Hankel transform of

f (r) is given by

$$\bar{f}(\xi_i) = \int_0^b r \quad H_{P\ Q}^{M\ N}\left[r^{\lambda} \ \Big|_{\ _1(b_j,\beta_j)_Q}^{\ _1(a_j,\alpha_j)_P} \right] \quad J(r\xi_i)dr$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\xi_i\right)^{2m} b^{2m+2}}{m! \ \Gamma(m+1)}$$

$$H_{P\ Q}^{M\ N}\left[b^{\lambda} \ \Big|_{\ _1(b_j,\beta_j)_Q \ (-2m-2,\lambda)}^{(-2m-1,\lambda) \ _1(a_j,\alpha_j)_P} \right] \tag{3.6}$$

Putting the value of f (ζ_i) in equation

$$u(r,t) = \frac{2}{a^2}\sum_i \frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2} e^{-k\xi_i^2 t} \bar{f}_1(\xi_i)$$

$$+ ka\xi_i J_1(a\xi_i)\int_0^1 f_2(a,q)e^{-k\xi_i^2(t-q)}dq$$

$$+ \frac{k}{k}\sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\xi_i\right)^{2m} b^{2m+2}}{m! \ \Gamma(m+1)} (\xi_i t)$$

$$H_{P+Q+1}^{M\ N+1}\left[b^{\lambda} \ \Big|_{\ _1(b_j,\beta_j)_Q \ (-2m-2,\lambda)}^{(-2m-1,\lambda) \ _1(a_j,\alpha_j)_P} \right] \ h \tag{3.7}$$

where the sum is taken over all positive roots of the equation

$$J_0(a\xi_i) = 0$$

and

$$h(\xi_i t) = \int_0^1 g(q)e^{-k\xi_i^2(t-q)}dq \tag{3.8}$$

4. VERIFICATION OF THE SOLUTIONS

Using [6] in (3.7)

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{2}{a^2}\sum_i \frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2} e^{-k\xi_i^2 t} \bar{f}_1(\xi_i)$$

$$+ ka\xi_i J_1(a\xi_i)\int_0^1 f_2(a,I)e^{-k\xi_i^2(t-I)}dI$$

$$+ \frac{k}{k}\sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\xi_i\right)^{2m} b^{2m+2}}{m! \ \Gamma(m+1)}$$

$$H_{P+Q+1}^{M\ N+1}\left[b^{\lambda} \ \Big|_{\ _1(b_j,\beta_j)_Q \ (-2m-2,\lambda)}^{(-2m-1,\lambda) \ _1(a_j,\alpha_j)_P} \right]$$

$$\int_0^1 g(I)e^{-k\xi_i^2(t-I)}dI \tag{4.1}$$

From (3.4) and (3.6) we get

$$\theta(r,t) = \frac{k}{k}\sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\xi_i\right)^{2m} b^{2m+2}}{m! \ \Gamma(m+1)}$$

$$H_{P+Q+1}^{M\ N+1}\left[b^{\lambda} \ \Big|_{\ _1(b_j,\beta_j)_Q \ (-2m-2,\lambda)}^{(-2m-1,\lambda) \ _1(a_j,\alpha_j)_P} \right]$$

$$\frac{2}{a^2}\sum_i \frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2} g(t) \tag{4.2}$$

and from (3.7) we have

$$\frac{\partial u}{\partial t} = \frac{2}{a^2}\sum_i \frac{J_0(r\xi_i)}{[J_1(a\xi_i)]^2}$$

$$\left[e^{-k\xi_i^2 t}\bar{f}_1(\xi_i) + ka\xi_i J_1(a\xi_i)\int_0^1 f_2(a,q)e^{-k\xi_i^2(t-q)}dq \right]$$

$$+ \frac{k}{k}\sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{1}{2}\xi_i\right)^{2m} b^{2m+2}}{m! \ \Gamma(m+1)}$$

$$H_{P+Q+1}^{M\ N+1}\left[b^{\lambda} \ \Big|_{\ _1(b_j,\beta_j)_Q \ (-2m-2,\lambda)}^{(-2m-1,\lambda) \ _1(a_j,\alpha_j)_P} \right]$$

$$\left[-g(t)\int_0^1 g(q)e^{-k\xi_i^2(t-q)}dq \right] \tag{4.3}$$

Setting these values in (2.1), we observe that the equation is satisfied.

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