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Study on Distribution Center Location Model based on Economics and Timeliness

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Abstract –Location of distribution center is the key step for logistics system optimization. Reasonable location can not only save cost, but also improve the service level of logistics system. This paper studies the distribution center location problem based on economics and timeliness. Firstly, a model with time constraint is proposed to optimize location under target of minimizing total cost. Secondly, the sequential quadratic programming, suitable for solving this model, is discussed. Finally, a distribution center location case study is presented. The results indicate that the location model and SQP can optimize the distribution center location with time constraint effectively.

Keywords –Distribution Center; Time Constraint; Location Model; Sequential Quadratic Programming

1. Introduction

Distribution center is the hub of logistics network, and is also one of the important facilities in the supply chain management. It is the circulating logistics node with the main function of organizing to sell, supply ,distribute goods and perform physical delivery, is also modern distribution facility engaged in providing goods and organizing client's delivery in order to achieve a high service level.The location of distribution center is a planning process to select a reasonably address in an economic region with a number of supply points and demand points and set up a distribution center. Rationality of distribution center location, as the integration point of economy, information and value, is of great significance to realize its function as well as improve the efficiency of logistics system. It also has important impact on urban planning, traffic condition, environment, and so on.

There are many factors influencing distribution center location, mainly including: traffic condition, supporting infrastructure, market efficiency, policy and legislation. Taking these factors into account, academia studies the problem of distribution center location and put forward different models and algorithms. Research methods mainly consist of the center-of -gravity method, capacitated facility location problem, cell automaton method, 0-1 integer programming, brain-storming, expert investigation method. In addition, some scholars use artificial intelligence algorithms such as simulate anneal arithmetic, ant colony optimization, genetic algorithm to solve the problem of distribution center location. Previous studies mainly consider economic goal, which is to minimize the total consumption cost including transportation cost, inventory cost and other costs of by selecting the location of distribution center reasonably, but don't consider clients' requirements of delivery time (time constraint) in the distribution process fully. In fact, the delivery time is one of the criterions to measure the service level of logistics, and an important factor needed to be considered in the distribution center location. This article studies the problem of distribution center location based on time constraint.

Firstly, the article focuses on the mathematical modeling on the basis of comprehensive consideration of the economics and timeliness; Secondly, it investigates traditional iterative algorithm suitable for the distribution center location without time constraint and sequential quadratic programming (SQP) algorithm with time constraint; Thirdly, it verify the feasibility of the model and algorithm combining practical case; Finally, it concludes the research work and points out the development direction in the future.

2. Location Model

Assumptions: ① The target area of location is continuous, any point is candidate in the region; ②The straight-line distance between two points is used instead of the transportation distance approximately; ③ Time constraint is described as the maximum allowable delivery distance.

Problem definition: We intend to set up a distribution center for the client of number in the region; goods from the factory are transported to the client via the distribution center. As shown in Figure 1, it is known that factory's coordinate is, coordinate of client ( ) is, is the maximum allowable delivery distance. Try to determine the distribution center's address coordinate, to make sure that the total cost is the least under the request of time constraint.

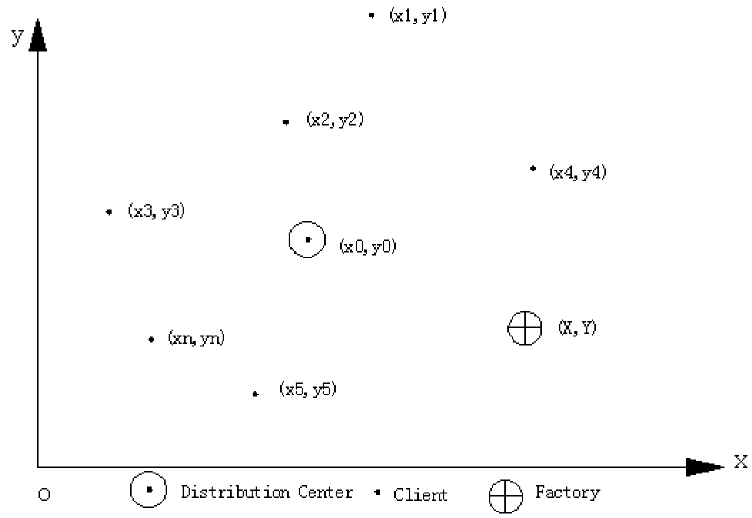

Figure1 Distribution center、Client、Factory

Based on the above assumptions, the model of distribution center location with time constraint can be proposed as:

$$\min_{(x_0, y_0)} \quad f(x_0, y_0) \tag{1}$$

$$s.t. \quad \sqrt{(x_0 - x_j)^2 + (y_0 - y_j)^2} - D \le 0 \tag{2}$$

The objective function $f(x_0, y_0)$ can be written as the following forms:

$$\begin{aligned} f(x_0, y_0) &= T + G + C \\ &= eW + aWd + \sum_{j=1}^{n} a_j w_j d_j \\ &= eW + aW\sqrt{(x_0 - X)^2 + (y_0 - Y)^2} \\ &+ \sum_{j=1}^{n} a_j w_j \sqrt{(x_0 - x_j)^2 + (y_0 - y_j)^2} \end{aligned} \tag{3}$$

Where T is the inventory cost of distribution center, $T = eW$; G is the transportation cost from factory to distribution center, $G = aWd$; C is the sum of the transportation cost from distribution center to all clients, $C = \sum_{j=1}^{n} a_j w_j d_j$; e is the inventory cost of per unit amount in the distribution center; W is the total inventory amount (the total transportation amount), $W = \sum_{j=1}^{n} w_j$; a is the transportation cost of per unit amount and per unit distance from factory to distribution center; d is the straight-line distance between factory and distribution center, $d = \sqrt{(x_0 - X)^2 + (y_0 - Y)^2}$; a_j is the transportation cost of per unit amount and per unit distance from distribution center to client j ; w_j is the demand of client j ; d_j is the straight distance between distribution center and client j , $d_j = \sqrt{(x_0 - x_j)^2 + (y_0 - y_j)^2}$.

3. Optimization algorithm

3.1 Traditional iterative algorithm

If we don't consider the distribution distance constraint of formula (2), the distribution center location model is the center-of-gravity model. We can take advantage of the traditional iterative algorithm to solve the problem. If (x_0^*, y_0^*) can meet the constraint $\frac{\partial f(x_0, y_0)}{\partial x_0} = 0$, $\frac{\partial f(x_0, y_0)}{\partial y_0} = 0$, then (x_0^*, y_0^*) is the optimal location of the distribution center we are looking for. We can conclude from formula (3):

$$\frac{\partial f(x_0, y_0)}{\partial x_0} = aW(x_0 - X)/d + \sum_{j=1}^{n} a_j w_j (x_0 - x_j)/d_j = 0 \tag{4}$$

$$\frac{\partial f(x_0, y_0)}{\partial y_0} = aW(y_0 - Y)/d + \sum_{j=1}^{n} a_j w_j (y_0 - y_j)/d_j = 0 \tag{5}$$

Further, x_0^* and y_0^* can be obtained by the formula (4) and formula (5).

$$x_0^* = \frac{aWX/d + \sum_{j=1}^{n} a_j w_j x_j / d_j}{aW/d + \sum_{j=1}^{n} a_j w_j / d_j}$$
$$y_0^* = \frac{aWY/d + \sum_{j=1}^{n} a_j w_j y_j / d_j}{aW/d + \sum_{j=1}^{n} a_j w_j / d_j} \quad (6)$$

From the mathematical expressions of x_0^*, y_0^*, d, d_j, we know that can't be obtained immediately by formula (6) . In fact, if we use the traditional iterative algorithm, the location coordinate of distribution center and the minimum total cost are obtained through repeated iterative calculation based on the initial solution.

Theory and practice show that the traditional iterative algorithm can ensure the economic goal of the site program, but does not satisfy the time constraint generally. Therefore, the traditional iterative algorithm cannot solve the problem of location of distribution center with time constraint effectively.

3.2 Sequential quadratic programming

Model of distribution center location with time constraint (formula (1) – (2)) belongs to constrained nonlinear optimization problem. We can take advantage of the sequential quadratic programming (SQP) to solve this kind of problem. The basic idea of SQP is to transform nonlinear programming problem into a series of quadratic programming sub-problems. Every sub-problem determines a downward direction. Finally, we obtain the optimal solution of the problem.

Consider nonlinear constrained optimization problem as follows:

$$\min \quad f(x) \quad (7)$$
$$s.t. \quad g_j(x) \le 0,\ j \in I = \{1,2,...,m\} \quad (8)$$

Where f and $g_j(j \in I)$: $R^n \to R$ is continuously differentiable.

The corresponding Lagrangian function $L(x,u)$ and feasible set X of formula (7)-(8) are described as:

$$L(x,u) = f(x) + \sum_{j=1}^{m} u_j g_j(x) \quad (9)$$
$$X = \{x \in R^n \mid g_j(x) \le 0,\ j \in I\} \quad (10)$$

Where u is the Lagrangian multiplier.

If $x^* \in X$ and $u^* = (u_j^*, j \in I) \in R^m$ can meet the request of the following formula (11), then x^* is called the Karush-Kuhn-Tucker point (KKT point for short) of formula (7) - (8). (x^*, u^*) is the KKT point pair of formula (9).

$$\nabla f(x^*) + \sum_{j=1}^{m} u_j^* \nabla g_j(x^*) = 0$$
$$g_j(x^*) \le 0, u_j^* g_j(x^*) = 0, u_j^* \ge 0,\ j \in I \quad (11)$$

Now consider the quadratic programming problem as follows:

$$\min \quad \nabla f(x)^T d + \frac{1}{2} d^T H d$$
$$s.t. \quad g_j(x) + \nabla g_j(x)^T d \le 0,\ j \in I \quad (12)$$

Where $H \in R^{n \times n}$ is a positive definite matrix. If (d, λ) is the KKT point pair of the formula (12), then

$$\nabla f(x) + Hd + \sum_{j=1}^{m} \lambda_j \nabla g_j(x) = 0$$
$$g_j(x) + \nabla g_j(x)^T d \le 0, (g_j(x) + \nabla g_j(x)^T d)\lambda_j = 0, \quad (13)$$
$$\lambda_j \ge 0,\ j \in I$$

Where d (the solution of the formula (12)) is usually used as downward search direction. If $d = 0$, then formula (13) is equivalent to KKT conditions of formula (7) - (8).

In the view of the above facts, we can take advantage of the SQP algorithm for solving the problem of distribution center location with time constraint. The specific steps of the algorithm are described as follows:

Step 1: $k = 0$, Give initial point of distribution center $x^0 = (x_0, y_0)$,Initialize positive definite matrix H^0.

Step 2: When $x = x^k$, Solve the quadratic programming

$$\min \quad \nabla f(x^k)^T d + \frac{1}{2} d^T H^k d$$
$$s.t. \quad g_j(x^k) + \nabla g_j(x^k)^T d \le 0,\ j \in I$$

Where d^k and λ^k are interpreted as corresponding multipliers.

Step 3:

$x^{k+1} = x^k + \alpha^k d^k$

Where step α^k is determined by some line search. If x^{k+1} satisfy the termination condition, then $x^* = x^{k+1}$, the algorithm stops; Or turn to Step 4.

Step 4: Correct H^k to keep H^{k+1} positive.

Step 5: $k = k + 1$, Return to Step 2.

4. Case study

There is a fresh vegetable production base to supply vegetables to seven major supermarkets in the city. The position coordinate of the production base, the position coordinate of the every supermarket and the supermarket's demand for vegetables are shown in Table 1, the distance between the production base and every supermarket is shown in Table 2. We plan to set up a vegetable distribution center near the production base or in the city to reduce total logistics cost. Inventory cost of per ton goods in the distribution center, transportation cost of per ton goods from production base to the distribution center , transportation cost of per ton goods from distribution center to the supermarket . While we transport goods from production base to the supermarket directly, cost of per ton goods. To avoid vegetable's quality problem due to long transportation, distance from distribution center to supermarket distribution shall be shorter than 6.5km. Try to determine address coordinate of the distribution center.

Tab.1 Location and requirement of production base and supermarket

Client	Abscissa /km	Ordinate /km	Demand/t
R1	4	14	10
R2	3	9	5
R3	7	11	15
R4	13	8	5
R5	7	6	20
R6	3	4	15
R7	11	4	10
P	21	12	80(Output)

Tab.2 Distance between production base and supermarket

Client	Distance/km
R1	17.1
R2	18.2
R3	14.0
R4	8.9
R5	15.2
R6	19.7
R7	12.8

This paper uses the math software Matlab 7.0 to program the algorithm for solving the problem of distribution center location, and chose the center of geographic gravity as the initial feasible coordinate to run the optimization algorithm.

(1) If we transport goods from production base to the supermarket directly, then, transportation cost is the total cost:

$$C = \sum_{j=1}^{n} r_j w_j d_j$$

=5,000×10×17.1+5,000×5×18.2+5,000×15×14.0+5,000×5×8.9+5,000×20×15.2+5,000×15×19.7+0.5×10×12.8
=6,220,000$

(2) If we ignore the time constraint, traditional iterative algorithm is used for solving the problem. Address coordinate of distribution center is [8.39 km, 7.86 km], the total cost is 5,570,000$.

Tab.3 Transport distance and cost without time constraint

Client	1	2	3	4	5	6	7	P
Transprotation distance/km	7.55	5.51	3.43	4.61	2.32	6.63	4.66	13.27
Transportation cost/thousand $	377.5	137.8	257.3	115.3	232.0	497.3	233.0	2,139.2

(3) Take the time constraint into account and use SQP algorithm to solve the problem. Address coordinate of distribution center is [7.60km, 8.59km], the total cost is 5,610,000$.

Tab.4 Transport distance and cost with time constraint

Client	1	2	3	4	5	6	7	P
Transprotation distance/km	6.50	4.62	2.48	5.43	2.66	6.50	5.71	13.83
Transportation cost/thousand $	325.0	115.5	186.0	135.8	266.0	487.5	285.5	2,212.8

The following conclusions can be drawn by analyzing the case:

(1) Reasonable construction of distribution center is more economic compared with transporting goods directly.

(2) Program of distribution center location with time constraint may be a slight increase in total cost (in this example, less than 1%), but the economic cost rise in the permitted extent. It meets all the clients' requestments of delivery time and improves the level of service significantly. It is of great importance to improve the operational efficiency of the logistics system.

5 Conclusion

The location of distribution center is complex system engineering, and many other factors need to be considered. This article not only takes the transportation cost, inventory cost, and other economic factors into account, but also considers the requirements of the client for delivery time. It sets up the distribution center location model based on economics and timeliness, to make up for the traditional model's negligence of service level. The case study shows that the model is a good guide to the location of distribution center. However, the model just admits timeliness requirement into constraint, but don't admit time penalty cost into the objective function. Therefore, the model needs to be further improved in order to adapt to broader situation.

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