Advances in Intelligent Transportation Systems (AITS)
Vol. 1, No. 4, 2012, ISSN 2167-6399

www.worldsciencepublisher.org

Solving first-order ordinary differential equations by Modified Adomian decomposition method

Yahya Qaid Hasan
Department of Mathematics, Faculty of Applied Science, Thamar University
Thamar, 87246, Yemen
E-mail:yahya217@yahoo.com

Abstract- In this paper, we use modified Adomian decomposition method to solve singular and nonsingular initial value problems of the first order ordinary differential equation. Theoretical considerations have been discussed and the solutions are constructed in the form of a convergent series. Some examples are presented to show the ability of the method for linear and non-linear problems of the first- order ordinary differential.

Keywords- Adomian decomposition method; First-order ordinary differential equation; singular and nonsingular ordinary differential equations.

1 Introduction

The first order ordinary differential equation can be consider as:

$$y' + p(x)y + f(x,y) = g(x), \qquad (1)$$

with boundary condition $y(0) = A$.
Where A is constant, $p(x)$and $g(x)$ are given functions and f(x,y) is a real function. A large amount of literature developed concerning Adomian decomposition method [1-4,6], and the related modification [5,7,8,10,11] to investigate various scientific models. The Adomian decomposition method cannot find the solution of (1) directly at $x = 0$. For example, we can not find the solution of $y' + \frac{sec^2 x}{tan x} y = 2sec^2 x$, at $x = 0$ by Adomian decomposition method. The purpose of this paper to introduce a new reliable modification of Adomian decomposition method. For this reason, a new differential operator is proposed which can be used for singular and nonsingular ODEs. In addition, the proposed method is tested for some examples.

2 The method

We define a new differential operator L in terms of the one derivative contained in the problem. We rewrite(1) in the form

$$Ly = g(x) - f(x,y). \qquad (2)$$

Where the differential operator L is defined by

$$L(y) = e^{-\int p(x)dx} \frac{d}{dx}(e^{\int p(x)dx} y). \qquad (3)$$

The inverse operator L^{-1} is there for consider a one-fold integral operator, as below,

$$L^{-1}() = e^{-\int p(x)dx} \int_0^x e^{\int p(x)dx} ()dx. \qquad (4)$$

Applying L^{-1} of (4) to the first two terms $y' + p(x)y$ of Eq.(1)we find

$$L^{-1}(y' + p(x)y) = e^{-\int p(x)dx} \int_0^x e^{\int p(x)dx} (y' + p(x)y)dx$$

$$= y - y(0)\Phi(0)e^{-\int p(x)dx}dx,$$

where $\Phi(x) = e^{\int p(x)dx}$.
By operating L^{-1} on (2), we have

$$y(x) = y(0)\Phi(0)e^{-\int p(x)}dx + L^{-1}g(x) - L^{-1}f(x,y). \quad (5)$$

The Adomian decomposition method introduces the solution $y(x)$ by an infinite series of components

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \quad (6)$$

and the nonlinear function $f(x,y)$ by an infinite series of polynomials

$$f(x,y) = \sum_{n=0}^{\infty} A_n, \quad (7)$$

where the components $y_n(x)$ of the solution $y(x)$ will be determined recurrently and A_n are Adomian polynomial that can be constructed for various classes of nonlinearity according to specific algorithms set by Wazwaz[6,9]. for anonlinear $F(u)$, the first few polynomials are given by

$$A_0 = F(u_0),$$

$$A_1 = u_1 F^{'}(u_0),$$

$$A_2 = u_2 F^{'}(u_0) + \frac{u_1^2}{2!}F^{''}(u_0), \quad (8)$$

$$A_3 = u_3 F^{'}(u_0) + u_1 u_2 F^{''}(u_0) + \frac{u_1^3}{3!}F^{'''}(u_0),$$

$$\vdots$$

Substituting(6)and (7) into (5) gives

$$\sum_{n=0}^{\infty} y_n(x) = y(0)\Phi(0)e^{-\int p(x)}dx$$

$$+L^{-1}g(x) - L^{-1}\sum_{n=0}^{\infty} A_n. \quad (9)$$

To determine the components $y_n(x)$, we use Adomian decomposition method that suggests the use of the recursive relation

$$y_0(x) = y(0)\Phi(0)e^{-\int p(x)}dx + L^{-1}g(x), \quad (10)$$

$$y_{k+1}(x) = -L^{-1}(A_k), \quad k \geq 0, \quad (11)$$

which gives

$$y_0(x) = y(0)\Phi(0)e^{-\int p(x)}dx + L^{-1}g(x),$$

$$\begin{aligned} y_1(x) &= -L^{-1}(A_0), \\ y_2(x) &= -L^{-1}(A_1), \\ y_3(x) &= -L^{-1}(A_2, \\ &\vdots \end{aligned} \quad (12)$$

from(8)and (12), we can determine the components $y_n(x)$, and hence the series solution of $y(x)$ in(6)can be immediately obtained. For numerical purposes, the n-term approximant

$$\Psi_n = \sum_{n=0}^{n-1} y_k,$$

can be used to approximate the exact solution.

3 Numerical illustrations

Example 1. We consider the linear singular initial value problem :

$$y^{'} + \frac{\sec^2 x}{\tan x}y = 2\sec^2 x, \quad (13)$$

$$y(0) = 0.$$

We put $L(.) = \frac{1}{\tan x}\frac{d}{dx}\tan x(.)$.
So

$$L^{-1}(.) = \frac{1}{\tan x}\int_0^x \tan x(.)dx.$$

In an operator form Eq.(13) becomes

$$Ly = 2\sec^2 x. \quad (14)$$

Now by applying L^{-1} to both sides of (14) we have

$$L^{-1}Ly = \frac{1}{\tan x}\int_0^x 2\sec^2 x(\tan x)dx.$$

$$y(x) = \tan x.$$

Example 2. Consider the nonlinear initial value problem:

$$y' + 3x^2 y = e^x + 3y(\ln y)^2, \qquad (15)$$

$$y(0) = 1.$$

We put $L = e^{-x^3}\frac{d}{dx}e^{x^3}$.
So

$$L^{-1}(.) = e^{-x^3}\int_0^x e^{x^3}(.)dx.$$

In an operator form, Eq.(15) becomes

$$Ly = e^x + 3y(\ln y)^2. \qquad (16)$$

Applying the inverse operator L^{-1} to both sides of Eq.(16), we have

$$y(x) = e^{-x^3} + L^{-1}(e^x) + 3L^{-1}y(\ln y)^2,$$

$$y_0 = e^{-x^3} + e^{-x^3}\int_0^x e^{x^3+x}dx.$$

By using Taylor series of e^{-x^3} and e^{x^3+x}, with order 8 and Adomain polynomials mentioned we obtain

$$y_0 = 1 + x + \frac{x^2}{2} - \frac{5x^3}{6}$$

$$-\frac{17x^4}{24} - \frac{7x^5}{24} + \frac{301x^6}{6!} + \frac{1531x^7}{7!} + \frac{4411x^8}{8!} + ...$$

$$y_1 = x^3 + \frac{3x^4}{4} - \frac{9x^5}{10} - \frac{5x^6}{3} - \frac{127x^7}{280} + \frac{353x^8}{320} + ...$$

$$y_2 = \frac{6x^5}{5} + \frac{5x^6}{4} - \frac{183x^7}{140} - \frac{253x^8}{80} + ...$$

$$y_3 = \frac{51x^7}{35} + \frac{39x^8}{20} - \frac{1027x^9}{560} - \frac{15531x^{10}}{2800} + ...$$

.

.

.

This means that the solution in a series form is given by

$$y(x) = y_0 + y_1 + y_2 + y_3 + ...$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + ...,$$

and in the closed form

$$y(x) = e^x.$$

Example 3. Consider the nonlinear initial value problem:

$$y' + 2xy = 1 + x^2 + y^2, \qquad (17)$$

$$y(0) = 1.$$

We put $L = e^{-x^2}\frac{d}{dx}e^{x^2}$.
So

$$L^{-1}(.) = e^{-x^2}\int_0^x e^{x^2}(.)dx.$$

In an operator form, Eq.(17) becomes

$$Ly = 1 + x^2 + y^2. \qquad (18)$$

Applying the inverse operator $L^{-1}$to be the sides of Eq.(18)

$$y(x) = e^{-x^2} + L^{-1}(1 + x^2) + L^{-1}(y^2)$$

$$y_0 = e^{-x^2} + e^{x^2}\int_0^x e^{x^2}(1 + x^2)dx.$$

By using Taylor series of e^{-x^2} and e^{x^2}, with order 6 and Adomain polynomials mentioned we obtain

$$y_0 = 1+x-x^2-\frac{x^3}{3}+\frac{x^4}{2}+\frac{2x^5}{15}-\frac{x^6}{6}-\frac{4x^7}{105}-\frac{143x^9}{3780}+...,$$

$$y_1 = x+x^2-x^3-\frac{7x^4}{6}+\frac{2x^5}{3}+\frac{32x^6}{45}-\frac{103x^7}{315}-\frac{383x^8}{1260}+...,$$

$$y_2 = x^2+\frac{4x^3}{3}-x^4-\frac{29x^5}{15}+\frac{5x^6}{9}+\frac{14x^7}{9}-\frac{619x^8}{2520}+...,$$

$$y_3 = x^3 + \frac{5x^4}{3} - \frac{13x^5}{15} - \frac{253x^6}{90} + \frac{7x^7}{45} + \frac{79x^8}{30} + ...,$$

$$y_4 = x^4 + 2x^5 - \frac{28x^6}{45} - \frac{236x^7}{63} - \frac{28x^8}{45} + ...,$$

$$y_5 = x^5 + \frac{7x^6}{3} - \frac{4x^7}{15} - \frac{2963x^8}{630} + ...,$$

.

.

.

This means that the solution in a series form is given by

$$y(x) = y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + ...$$

$$= 1 + 2x + x^2 + x^4 + x^5 + ...$$

and in the closed form

$$y(x) = x + \frac{1}{1-x}.$$

4 Conclusion

In the above discussion it was shown that, with the proper use of modified Adomian decomposition method, it is possible to obtain an analytic solution to first order differential equation, singular or nonsingular. The difficulty in using Adomian decomposition method directly to this type of equations, due to the existence of singular point ar $x = 0$, is over come here .
Here we use this method's for solving singular and nonsingular initial value problem of order one. It is demonstrated that this method has the ability of both linear and nonlinear ordinary differential equation.

References

[1] G. Adomian, A review of the decomposition method and some recent results for nonlinear equation, Math. Comput. Modelling 13(7)(1992) 17.

[2] G. Adomian, Solving forontier problems of physics: The Decomposition Method, Kluwer, Boston, MA, 1994.

[3] G. Adomian, R. Rach, Noise terms in decomposition series solution, Comput. Math. Appl. 24(11)(1992)61.

[4] G. Adomain, Differential coefficients with singulr coefficients, Apple. Math. Comput. 47 (1992) 179.

[5] M.M Hosseini, Adomian decomposition method with Chebyshev polynomials, Appl. Math. Comput. 175 (2006)1685-1693.

[6] A. M. Wazaz, A First Course in Integral Equation, World Scientific, Singapore, 1997.

[7] A.M. Wazwaz, A reliable modifications of Adomian's decomposition method, apple. Math.Comput. 102(1999)77.

[8] A.M .Wazwaz, Analytical approximaations and pade' approximations for Volterra's population model, Appl, Math. Comput 100 (1999) 13.

[9] A.M .Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators , Appl. Math. Comput. 111 (1) (2000) 33.

[10] Yahya Qaid Hasan,Modified Adomian decomposition method for second order singular initial value problems, Advances in Computational Mathematics and its Applications. 1(2012) 94-99.

[11] Yahya Qaid Hasan,The numerical solution of third-order boundary value problems by the modified decomposition method, Advances in Intelligent Transportation Systems 1(3)(2012) 71-74.

Vitae

Yahya Qaid Hasan, male, was born in the city of Taiz, Taiz Province, Yemen . He got bachelor degrees on Mathematics from Sana'a university in 1993, he got Master degree on Applied Mathematic from Anhui Normal University(China)in 2005 and he got PhD degree from Harbin Institute of Technology (China)in 2009. He works in Thamar University.His research interest includes Differential Equations and its applications, numerical solution of Differential Equations .