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# A mathematical Model and Solving Method for Multi-Depot and Multi-level Vehicle Routing Problem with Fuzzy Time Windows

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**Abstract** – Multi-depot and multi-level vehicle routing problem with fuzzy time windows is considered as one of the most important and complicated decision problems in transportation field. Up to present, different heuristic, Meta-heuristic and accurate methods have been represented for solving vehicle routing problems. In this article, a mathematical model for multi-depot and multi-level vehicle routing problem with fuzzy time windows (MD-MLVRPFTW) and it's solving method has been offered. Never In transportation scheduling problems, the time window is achieved completely and precisely. And actually the deviation of the servicing time from the customer's time window is a parameter that can indicate the level of customer's satisfaction. In multi-depot vehicle routing problems, in order to determine the level of service related to time window, fuzzy functions has been used. In this article the multi-depot vehicle routing problem with fuzzy time windows (MDVRPFTW) has been solved as a multi-level model. This essay pursues its purposes to minimize the servicing time and distances passed by vehicles and also to maximize the level of service. In order to solve the multi-level model, a three-section algorithm has been developed. In the first section, for reducing the servicing time, customers are assigned to distribution centers. In the next section, a single-depot vehicle routing problem with time windows ( $\mathrm{VRPTW}-\alpha$ ) is solved in order to reduce the distribution costs. The third section concerns solving the improvement problem. Consideration of designed tests in this article illustrates a huge reduction in servicing time and distribution costs and actually elucidates a remarkable increase in customers' level of satisfaction.

**Keywords** – Multi-depot vehicle routing problem; time window; service level; minimizing the distribution costs.

## 1. Introduction

One of the most important matters in transportation problems is to economically manage the fleet of heterogeneous vehicles which are collecting or distributing specific number of products among a set of sellers and customers with specific supply and demand. The manager should make decision not only about the number of vehicles, but about which customer each vehicle gives service and also about the priority of customers in getting services. Products which must be distributed are uploaded in depot and also products which are collected from the customers, are returned to depot again. This problem begins with assigning a vehicle to a route. Each route starts from a depot and ends in it. For each route, limitations about the lead time and capacity of vehicles must be satisfied. Moreover, each customer should be serviced by one vehicle if demand dividing is not allowed. Till now, different kinds of vehicle routing problem (VRP) have been studied. Although each of these different kinds of problem alludes to an assorted practical condition, they are all about the efficient managing of vehicles for servicing a set of customers. In VRPs, the significance of routing is so clear and incontrovertible. Routing, usually forces a lot of money to the system. Therefore, controlling these routes is so important. Before the routing, actually it is so important to categorize the customers and determine the amount of products which must be distributed. We are seeking for the routes which regarding to the vehicles' capacity, can give products to customers before the lead time and also with the lowest prices. Solving the VRP includes anticipation of customers' demand, categorizing the customers, vehicle assigning, determining the amount of products which must be distributed and also the routing. On-time and accurate decisions in these sections culminate in a huge reduction in distribution costs and consequently play a key role in strengthening a distribution company among the others in various competitions. Thus, the multi-depot and multi-level vehicle routing problem with fuzzy time windows (MD-MLVRPFTW) has been considered in this article.

Vehicle routing problem and its importance has caught researcher's attention for many years. VRPTW was first introduced by Solomon (1987) and since then it has remained a test domain for various meta-heuristics, mainly because of its relevance to the real world, its difficult fitness landscape and its potential to deliver in economic terms.

Many efficient heuristic and meta-heuristic approaches have been proposed recently, including the works of Chiang and Russell (1997), Potvin and Rousseau (1993), Rochat and Taillard (1995), Taillard et al. (1997) and Thangiah et al. (1991). More recently, Schulze and Fahle (1999), Gehring and Homberger (2000) proposed new parallel tabu search heuristics that enable large-scale VRPTW instances to be solved .Several works have been carried out advocating the hybrid use of constraint programming and local search. For example, Pesant and Gendreau applied constraint programming to evaluate the local neighborhood to find the best local moves (1999). Shaw (1998) presented a method called large neighborhood search (LNS) for VRP in which a part of a given solution is extracted and then reinserted into the partial solution using a quasi-complete search process. If the reinsertion procedure generated a better solution, then the solution is kept. This process is repeated until certain stopping criterion is met. The result produced with this technique is competitive with other meta-heuristic approaches. In terms of exact algorithms, Desrochers et al. (1992) applied column generation that was able to solve some 100-customer problems optimally. Based on this, Kohl et al. (1999) developed a more efficient optimization algorithm by introducing a new valid inequality within a branch-and-cut algorithm, called k-path cuts, which solves 70 of the 87 Solomon benchmark problems to optimality. However, due to the exponential size of the solution space, it is unlikely that these optimization procedures can be used for larger-scale problems.

Figliozzi (2010) proposed an iterative route construction and improvement heuristics for the vehicle routing problems with soft time windows, which penalties are imposed to the total cost in case the customer delivery time window cannot be met. Also, Duan et al. (2010) proposed a mixed integer programming model for the route construction algorithm for solving time dependent vehicle routing problem. In this category of vehicle routing problems, the travel time between customer locations is not a constant, but dependent to the timing of the travel between the locations.

To deal with the issues pertaining to the violation of time windows, researchers have proposed the concept of “soft time windows”. In the vehicle routing problem with soft time windows (VRPSTW), a penalty cost is added once a time window is violated, and the penalty cost is often assumed to be linear with the degree of violation. The VRPSTW successfully models many scenarios where time window constraints do not hold strictly. Sexton and Choi (1986) were among the first to consider soft time windows in the context of a single-vehicle problem involving pickups and deliveries. They used a Bender’s decomposition approach to solve the problem. Ferland and Fortin (1989) applied a heuristic approach for solving a related sliding time window problem in which the time windows of pairs of customers are adjusted in order to achieve a lower cost solution. Balakrishnan (1993) used simple route construction heuristics to develop fast solutions to VRPSTW problems. Chiang and Russell (2004) developed a tabu search method for VRPSTW. Calvete et al. (2007) modeled and solved the VRPSTW using goal programming techniques. However, “soft time windows” have one disadvantage. In some cases, violation of time windows does not directly incur any penalty cost, although the satisfaction levels of customers (the service level of suppliers) may drop and lead to profit loss in the long term. As is known, the supplier’s service level greatly influences customer satisfaction; modeling these service level issues is therefore a natural idea. Ever since Zadeh first proposed fuzzy theory (1965), researchers from different areas have found fuzzy theory to be a useful tool to describe subjective opinions.

Usually, scheduling in transportation includes multi-depot vehicle routing problem with specific time window for the customer, which is related to both customer’s satisfaction and minimization of distribution costs. Multi-depot vehicle routing problem with time windows (MDVRPTW) and its various types has been widely studied by researchers and different algorithms have been represented to solve it. For the common MDVRPTW, a solution is feasible if the customer servicing time is in the time window. This is also called crisp time window. But in the real world, usually, this crisp time window is not satisfied and therefore, the fuzzy time window has been offered. In MDVRPFTW, a special cost will be added as a penalty once the time window is not satisfied.

Up to present, no study has mentioned the fuzzy concepts for modeling the service level in multi-depot vehicle routing problems (MDVRP). This essay tries to do it by making case for fuzzy time windows in MDVRP. Structure of this essay is as follows: in second section, definitions, parameters and decision variables of the MD-MLVRPFTW have been introduced and also mathematical models of MD-MLVRPFTW and $\text{MDVRPTW}-\alpha$ have been delineated completely. Allocation of customers to the distribution centers has been done in third section. The relationship between customer’s satisfaction and time window has been introduced in fourth section. In fifth section improvement of service level and customer’s satisfaction has been explained and model of improvement of customer’s service level (CSLIP) has been described. Finally in the last section, the conclusion has been represented.

## 2. Vehicle routing problem with fuzzy time window

In vehicle routing problems with fuzzy time windows, each customer has the [eia , Lia] delivery time window as congenial servicing time window and the [ESTia , LSTia] as acceptable servicing time window which both exclusively belong to that customer. For the supplier, it is imperative to avoid servicing some customers so early and some customers so late because it will ruin the relationship between customer and supplier in long time. A practical method for avoiding this problem is to determine the lowest service level $\alpha_{ia}(I,1,2,...,n)$ for each customer in each area. In fact, $\alpha_{ia}$ is the lowest level of service that satisfies the customer i. In order to minimize the operational costs and satisfy customers, supplier must choose a routing that makes vehicles service their customers regarding to time window and the lowest service level.

### 2.1. Mathematical model of MD-MLVRPFTW

In this part the mathematical model of MD-MLVRPFTW has been represented. In this model, a main distribution center exists and products requested by customers, are served by one of these centers. Neither product deficiency nor delay in arrival is allowed and demand of each period for each customer in each area, is definite. Actually all needs of a customer, are carried by one vehicle. Savelsbergh has shown that finding a solution for vehicle routing problem with time window (VRPTW) is a NP-COMPLETE.

Parameters and decision variables for mathematical model of MD-MLVRPFTW are defined as follows:

I= 1,2,…,n starting point
J= 1,2,…,m endpoint
k= 1,2,…,k set of k vehicles
a= 1,2,…,A set of A areas
d= 1,2,…,D set of D depots

$C^k_{ija}$ : the cost for passing the distance between point I and point J in the area a and by the vehicle k. $(C^k_{jai} = C^k_{jia})$

$t^k_{ija}$ : the time for passing the distance between point I and point J in the area a and by the vehicle k. $(t^k_{jai} = t^k_{iaj})$

$S_{ia} = [e_{ia}, L_{ia}]$ : time window of customer i in area a.

$e_{ia}$ : the earliest time that servicing the customer i in area a is started.

$L_{ia}$ : the latest time that servicing the customer i in area a is started.

$d_{ia}$ : amount of demand of customer i in area a.

$n_a$ : number of customers in area a.

$m_a$ : number of homogeneous vehicles with capacity c in area a.

$Z^k_{ija}$ : if vehicle k passes the distance between i and J; $Z^k_{ija} = 1$

Else; $Z^k_{ija} = 0$.

$T^k_{ia}$ : the time when vehicle k starts servicing customer i in area a.

$W^k_{ia}$ : the time when vehicle k arrives to customer i in area a.

Objective function:

$$MIN \quad \sum_{a=1}^{A}\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{k} C^k_{ija} \ . \ Z^k_{ija} \tag{1}$$

$$MAX \quad \sum_{a=1}^{A}\sum_{i=1}^{n} S_i(t) \tag{2}$$

S.t.

$$\sum_{i=1}^{n}\sum_{k=1}^{k} Z^k_{ija} = 1 \qquad j = 1,2,...,m, a = 1,2,...,A \tag{3}$$

$$S_i(t) = \sum_{k=1}^{k}\sum_{j=1}^{m} Z^k_{ija} T^k_{ia}, \ a = 1,2,...,A \quad ,i = 1,2,...,n \tag{4}$$

$$\sum_{i=1}^{n} Z^k_{ija} - \sum_{j=1}^{m} Z^k_{ija} = 0 \quad i \neq j \ , \quad K = 1,2,...,K \ ,a = 1,2,...,A \tag{5}$$

$$\sum_{i=0}^{n}\sum_{i=0}^{m} Z^k_{ija} .d_{ia} \leq C \quad k = 1,2,...,K \quad a = 1,2,...,A \tag{6}$$

$$Z^k_{ija} \ (T^k_{ia} + t_{ijk} + W^k_{ia} - T^k_{ja}) \leq 0 \quad i = 1,2,...,n \ \ k = 1,2,...,K \quad j = 1,2,...,m \ \ a = 1,2,...,A \tag{7}$$

$$\sum_{i=1}^{n}\sum_{R=1}^{k} \frac{1}{2} S_i(t) \geq \alpha_{ia} \quad a = 1,2,...,A \quad ,i = 1,2,...,n \tag{8}$$

$$Z^k_{ija} = (0,1) \quad i = 1,2,...,n \ , \quad j = 1,2,...,m \quad k = 1,2,...,K \ , \quad a = 1,2,...,A \tag{9}$$

$$T^k_i \geq 0 \quad i = 1,2,...,n \ ,k = 1,2,...,K \tag{10}$$

In this model, equations (1) and (2) are the two objective functions of the routing problem. Limitations (3), (5) and (6) are the prevalent and conventional limitations of the MDVRP that illustrate meeting each customer, limitation for capacity and maintaining the passing route. Limitation (4) demonstrates the amount of Si(t) in objective function. Inequality (7) introduces the relationship of servicing time between two consecutive customers and Inequality (8) addresses the lowest service level for customer i in area a. Model of MD-MLVRPFTW is a multi-purpose integer scheduling problem and also it proves to be NP-Hard. In model of MD-MLVRPFTW if $\alpha_{ia} = 1$ then this model changes to model of MDVRPTW. In this case a specific time window must be satisfied for all customers. In order to solve the problem, firstly, we break the model of MD-MLVRPTW to MDVRPTW - $\alpha$ for area a. Then, the multi-depot model will be altered to single-depot model by assigning customers to depots. Thereafter, a vehicle routing problem for satisfaction level of $\alpha$ will be solved by one of the common methods of VRP and finally the service level will be improved. Regarding to membership functions $S_i(t)$, the earliest and latest acceptable servicing time for customers equals $(\hat{e}_i, \hat{L}_i)$ and they can be calculated by using figure 1 and equations (11) and (12).

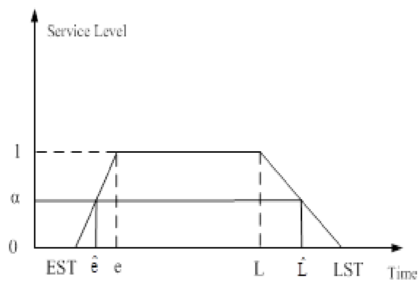


Fig. 1. Altering the model of VRPFTW to model of VRPTW- α

$$\hat{e}_i = e_i^{-1}(\alpha_i) \qquad i = 1,2,...,n \tag{11}$$

$$\hat{L}_i = L_i^{-1}(\alpha_i) \qquad i = 1,2,\ldots,n \tag{12}$$

$\hat{e}_i$, $\hat{L}_i$ Have been calculated by using the $\alpha-cut$ of fuzzy membership function.

Therefore, VRPFTW model will be changed to VRPTW- $\alpha$ through the procedure mentioned below:

Modifying the time window $(e_i,\ l_i)$ to time window $(\hat{e}_i,\ \hat{L}_i)$

Eliminating the service level maximization function

Eliminating the limitation of service level

VRPTW- $\alpha$ can be solved by existing algorithms for VRPTW. After solving VRPTW- $\alpha$, the improvement of customers' service level (CSLIP) has been considered.

### 2.2 Model of MDVRPTW- $\alpha$ for area a

$$MIN \quad \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{k} C_{ija}^{k} \ . \ Z_{ija}^{k} \tag{13}$$

S.t.

$$\sum_{i=1}^{n}\sum_{k=1}^{k} Z_{ija}^{k} = 1 \qquad j = 1,2,\ldots,m \tag{14}$$

$$\sum_{i=1}^{n} Z_{ija}^{k} - \sum_{j=1}^{m} Z_{ija}^{k} = 0 \quad i \neq j \ , \ k = 1,2,\ldots,K \tag{15}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{m} Z_{ija}^{k} \ .d_{ia} \leq C \quad k = 1,2,\ldots,K \tag{16}$$

$$Z_{ija}^{k}\ (T_{ia}^{k} + t_{ijk} + W_{ia}^{k} - T_{ja}^{k}) \leq 0$$
$$i = 1,2,\ldots,n \ \ k = 1,2,\ldots,K \tag{17}$$
$$j = 1,2,\ldots,m$$

$$\hat{e}_{ia} \leq T_{ia}^{k} \leq \hat{L}_{ia}$$
$$i = 1,2,\ldots,n \quad ,k = 1,2,\ldots,K \tag{18}$$

$$Z_{ija}^{k} = (0,1) \quad i = 1,2,\ldots,n \ ,$$
$$j = 1,2,\ldots,m \quad , \ k = 1,2,\ldots,K \tag{19}$$

$$T_i^k \geq 0 \quad i = 1,2,\ldots,n \ \ ,k = 1,2,\ldots,K \tag{20}$$

This model must be considered and analyzed for each area and regarding to set of areas (a=1,2,…A) , number of problems which must be solved is A.

In this model, equation (13) defines the objective function. Equations (14), (15) and (16) indicate meeting the customers, capacity limitation and maintaining the passing route. Inequality (17) addresses the relationship of servicing time between consecutive customers and inequality (18) illustrates the time window for customer i. In this essay, model of VRPTW- $\alpha$ has been solved by demrocher algorithm.

## 3. Assigning the customers to distribution centers; using the coordinal distances

In this part, each of the customers is allocated to one of the distribution centers. The criterion for this allocation is the coordinal distance between the customer and the center. For instance, if we have depots $d_A$ and $d_B$, then the customer $C_i$ must be exactly assigned to one of them and this is how the depot will be chosen:

If Dis ($C_i$, $d_A$) < Dis ($C_i$ , $d_B$) ; then $C_i$ will be assigned to dA

If Dis ($C_i$ , $d_A$) > Dis ($C_i$ , $d_B$) ; then $C_i$ will be assigned to dB

If Dis ($C_i$ , $d_A$) = Dis ($C_i$ , $d_B$) ; then $C_i$ will be randomly assigned to one of the two depots.

The distance between customer I and distribution center k is calculated like equation 21.

$$Dis(C_i, d_k) = \sqrt{(X_{ci} - X_{dk})^2 + (Y_{ci} - Y_{dk})^2} \tag{21}$$

Assigning the customers to the closer center can result in minimization of distribution costs, distributing time and lead time. It causes an increase in customers' satisfaction as well. In addition, the multi-depot problem will be denominated into some single-depot problems.

## 4. Customer's satisfaction and fuzzy time window

In vehicle routing problem with time windows, a solution is possible when it satisfies all time windows. Supplying service level is fine when customer gets service in time windows otherwise; service level is bad. Hence, the customer's satisfaction can be explained by a binary variable. Level of customer's satisfaction equals 1, if servicing time is in time window and it equals 0 if servicing time is out of time window. Service level and satisfaction function for customer i and area a is as the following figure:

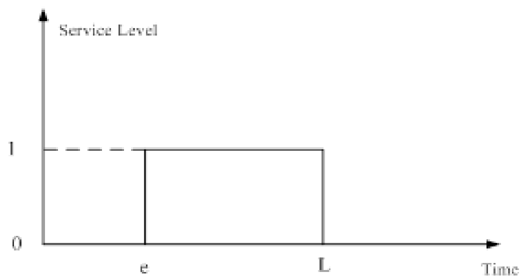


Fig. 2. Service level function with crisp time window

Crisp time windows may be not satisfied for economic and operational reasons but there are always limitations for acceptable amounts of earliest start time (EST) and latest start time (LST). For EST, the acceptable amount is the earliest servicing time that customer can accept it and for LST, the acceptable amount is the latest servicing time that customer can accept it. Servicing out of the time window [eia ,Lia] has some amounts of delays or early-starts. Delays and early-starts have a direct relationship with quality of servicing. In fuzzy time windows, response of customer's satisfaction to a specific servicing time can be one of the options of "good", "average" or "bad". Theory of fuzzy sets is a strong and efficient tool for modeling the problems which are related to human's emotions. In order to explain the earliest acceptable servicing time for customer (EST) and the latest one

(LST), the supplying service level for each customer can be defined as a fuzzy membership function:

$$S_i(t) = \begin{cases} 0 & t < EST \\ e_i(t) & EST \le t < e \\ 1 & e \le t < L \\ L_i(t) & L \le t < LST \\ 0 & LST \le t \end{cases} \qquad (22)$$

Regarding to Si(t) function which indicates different levels of servicing time, time window is called “fuzzy time window”. In the simplest condition for fuzzy time window, Li(t) , ei(t)functions are linear as it has been shown below:

$$e_i(t) = \frac{t - EST}{e_i - EST} \qquad (23)$$

$$L_i(t) = \frac{t - LST}{e_i - LST} \qquad (24)$$

The service level function for fuzzy time window has been shown in figure 3.

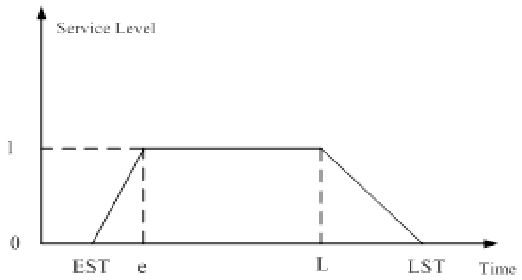


Fig. 3. Service level function with fuzzy time windows

## 5. Improvement of service level and customers’ satisfaction

VRPTW- $\alpha$ can be solved by existing algorithms for VRPTW. After solving VRPTW- $\alpha$ , the improvement of customers’ service level (CSLIP) has been considered. In order to increase the whole service level, scheduling for servicing time of each customer will be considered by CSLIP model. Mathematical formulating of CSLIP model has been shown below:

$$MAX \quad \sum_{i=1}^{A} \sum_{k=1}^{k} \frac{1}{n} .S_i(t) \qquad (25)$$

$$S_i(t) = \sum_{k=1}^{k} \sum_{j=1}^{m} Z_{ij}^k T_i^k , \quad i = 1,2,...,n \qquad (26)$$

$$Z_{ij}^k \ (T_i^k + t_{ij} + W_i^k - T_j^k) \le 0 \qquad i = 1,2,...,n \quad j = 1,2,...,m \quad k = 1,2,...,K \qquad (27)$$

$$\sum_{i=1}^{n} \sum_{k=1}^{k} Si(t) \ge \alpha_i \qquad (28)$$

$$k = 1,2,...,K \quad ,i = 1,2,...,n$$

$$T_i^k \ge 0 \quad i = 1,2,...,n \ ,k = 1,2,...,k \qquad (29)$$

In this model the amounts of $Z_{ij}^k$ are constant. These amounts are results of solving the VRPTW- $\alpha$ model. CSLIP model is a multi-condition linear scheduling problem. This model has been altered to integer linear scheduling model by the means of binary variables and then it has been solved. In CSLIP model, equation (25) shows the objective function and equation (26) completes it. Equation (27) indicates the sequence in servicing two consecutive customers and equation (28) demonstrates the acceptable service level for customers. Equation (30) has been used for altering the multi-condition linear scheduling model to linear scheduling model.

$$S_i(t) = Z_1 .e_i(t) + Z_2 + Z_3 .L_i(t) \qquad (30)$$

In equation (30), variable Zi equals 0 or 1 and it’s amounts can be calculated by equations (31), (32) and (33).

$$Z_1 = \begin{cases} 1 & t \ge e \\ 0 & EST \le t \le e \end{cases} \qquad (31)$$

$$Z_2 = \begin{cases} 1 & t \ge L \\ 0 & e \le t \le L \end{cases} \qquad (32)$$

$$Z_3 = \begin{cases} 1 & t \ge LST \\ 0 & L \le t \le LST \end{cases} \qquad (33)$$

Actually, in order to complete the model, this limitation must be added to it because service level of each customer is just in one of the interval of equations (31), (32) and (33)

$$Z_1 + Z_2 + Z_3 = 1 \qquad (34)$$

Equations (23) and (24) give amounts of ei(t) and Li(t) which are needed in equation (30). After solving this model, regarding to accessed minimum distributing cost, transporting operation will be managed so that the average service level of customers achieves it’s maximum amount.

## 6. Conclusion

In this article, multi-depot and multi-level vehicle routing problem with fuzzy time window (MD-MLVRPFTW) was considered. Fuzzy time window was used rather than crisp time window and based on it; a multi-level linear scheduling model was represented for multi-depot vehicle routing problem. This model was altered to some single-depot vehicle routing problems using the coordinal distances between customers and distribution centers. The purpose of minimization of lead time was considered in denomination of the problem and minimizing the distribution costs was reflected in solving VRPTW- $\alpha$ . Besides, maximizing the service level of customers was considered in CSLIP problem. Designed experiments convey that MD-MLVRPFTW model, in comparison with MDVRPTW, provides routings with lower costs and higher service level. Based on this research using larger time windows, vehicles with more capacity and bigger amounts for $\alpha$ , cause solutions with lower costs and higher service level as well.

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