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Study of Risk Value Based on GARCH model

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Abstract – the paper introduces VaR calculation method by GARCH model, concluding the biggest expected losses in the stock market portfolio during a holding period which is significant for manage asset risk. In addition, this paper analyzes the advantages and disadvantages of the VaR method, indicating the direction for further research.

Keywords: (Value at risk)VaR, GARCH, Portfolio management, Risk aversion.

1. Introduction

J.P.Morgan developed Risk Metrics model In 1998 which powerfully promoting the popularity of the Value-at-Risk (VaR) which is a statistical measure of downside risk of financial portfolio among the most important measures of market risk. It has been widely used for financial risk management by financial institutions, regulators and portfolio managers. The greatest advantage of VaR is that it can summarize risks in a single number. It is surveyed by Wharton/CIBC Wood Gundy that, there was 29 percent of America's non-financial Group using VaR to assess the risk of derivative. Additionally, the of Institute Investor investigated that 32 percent of surveyed groups evaluate risk by VaR, meanwhile, the investigation of New York University indicated 60 percent of some pension fund management companies control the risks by the tool of VaR.

Financial institutions use the methodology to measure the risk of their trading portfolios. The topic has been discussed in the literatures [1]-[4] and studied several topic aspects in the papers [5]-[11]. In addition, J.P. Morgan has established a market standard through RiskMetrics system [12]-[14].

In this article, we evaluate the VaR of four stocks, i.e, IBM, Dell, Apple, and Microsoft based on GARCH model. Furthermore, we provide the lower bound of the stock prices which is important for controlling risk is also very clear and intuitive. Finally, we give the VaR of the portfolio of these four stocks.

2. The theory of VaR

2.1. Introduction of VaR

Compared with the standard deviation risk measure in the traditional Markowitz framework, VaR provides a more flexible and comprehensive risk measurement framework. Thus it is one of the most widely accepted risk measure for quantifying downside risk.

Assuming that the return of asset is normally distributed, i.e,

$$R_t \mid \mathrm{I}_{t\text{-}1} \sim N(0,\sigma_t^2) \tag{1}$$

Where R_t denotes stock return at time t and $\mathrm{I}_{t\text{-}1}$ denotes the available information at time $t-1$. Then we get

$$\mathrm{P}(\mathrm{R_t} > \text{-}\mathrm{VaR_t}) = 1\text{-}\alpha \tag{2}$$

where α denotes significance level and VaR_t denotes the value-at-risk at time t. This equation means the probability that the maximum of asset's loss would not surpass VaR_t is $1-\alpha$. Then we can get the following equation

$$(\mu_t + \mathrm{VaR_t}) / \sigma_t = u_{1-\alpha} \tag{3}$$

Where $u_{1\text{-a}}$ denotes $1-\alpha$ quantile of conditional normal distribution and σ_t the volatility of asset return at time t, μ_t is often set to be zero. So after transformation, (2) could be rewritten as

$$\mathrm{VaR_t} = u_{1-\alpha}\sigma_t \tag{4}$$

That is to say, when the return of asset obey conditional normal distribution, VaR_t is a linear function of σ_t. In recent years, both theoretical and empirical research show that the non-normality of financial time series have the root of heteroscedasticity, therefore, it is appropriate to deal with heteroscedasticity with GARCH model to describe the volatility of stock market returns, which can obtain more accurate value-at-risk.

2.2. Introduction of GARCH model

Since the articles by Engle [15] on ARCH processes and Bollerslev [16] on GARCH processes, a large variety of papers has been devoted to the statistical inference of these models. It is the aim of the present paper to estimate the asset volatility with GARCH model in order to calculate VaR of the assets. Recall that the time series (R_t in this article) is called a GARCH process of order $(\mathrm{p},\ \mathrm{q})$ for some integers $\mathrm{p},\mathrm{q} > 0$ if it satisfies the recurrence equations

$$\begin{aligned} &R_t = \sigma_t Z_t,\ Z_t \sim N(0,\ 1) \\ &\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{5}$$

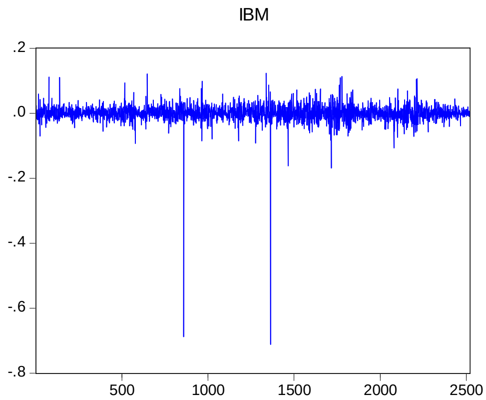
Where ω, α_i, β_j are non-negative parameters ensuring non-negativity of the squared volatility process $\{\sigma_t^2\}$ and Z_t is a sequence of iid symmetric random variables with $\mathrm{Var}(Z_t) = 1$. For simplicity the value of σ_t^2 based on past available information usually could be simplified to order of (1, 1), that is GARCH(1, 1)

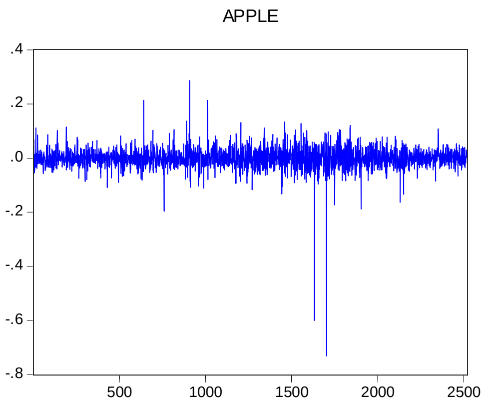
$$\sigma_t^2 = \omega + aR_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (6)$$

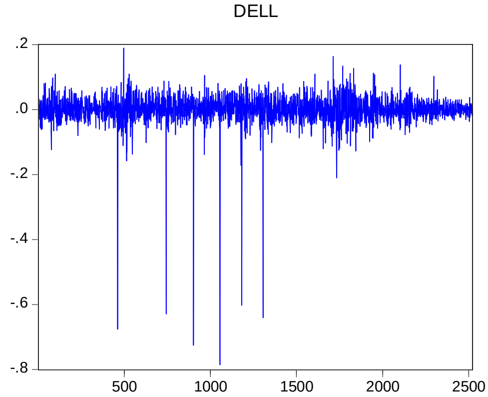
3. Empirical analysis

3.1. Data and hypothesis test

All data are retrieved from site: http://finance.yahoo.com. The research subject of this paper are four companies in the United States named IBM, Dell, Apple, and Microsoft. We use the stock price during the period of 1998 to 2008 to estimate GARCH model and then outside predict the value-at- risk of the four companies from Jan 2008 to May 2010, furthermore, we can provide the lower bound of the stock price. Then compose daily total stock returns $R_t = \ln(p_t / p_{t-1})$, where R_t denotes the continuously compounded return at time t, and p_t denotes the stock price at time t. Figure 1 shows each stock's return which strongly indicate that the return series are stationary. Table 1 presents some descriptive statistics for the return series. As showed in Table 1, all the return series is negatively skewed and their Kurtosis all excess 3, that indicate all the stock return series are leptokurtosis and fat-tail. Furthermore, Table 2 shows the Ljung–Box tests which clearly suggest the presence of GARCH effects.







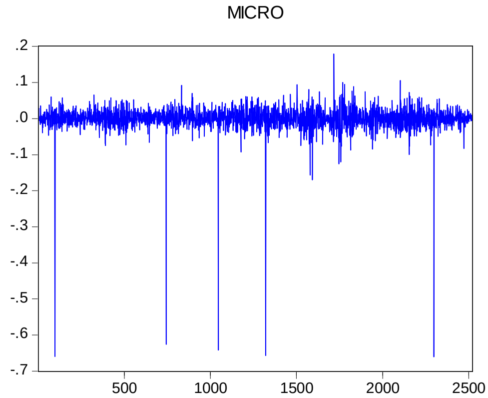


Figure 1. The graphs for stock return series

Table 1. Data descriptive statistics

	IBM	DELL	APPLE	MICRO
Mean	0.00019	0.000132	-0.000133	-0.000427
Maximum	0.123636	0.188798	0.286796	0.178773
Minimum	-0.7113	-0.785674	-0.731247	-0.660157
Std. Dev.	0.029783	0.047735	0.039124	0.037615
Skewness	-10.25082	-6.961821	-3.731906	-10.18071
Kurtosis	245.0572	102.8354	75.03741	177.1183

Note: All the return series negatively skewed and their Kurtosis all excess 3, that indicate all the countries stock return series are leptokurtosis and fat-tail.

Table 2. Test for heteroscedasticity

	LB(10)	LB(20)	Test result
IBM	76.20 (0.000)	06.39 (0.000)	H1
DELL	82.86 (0.000)	96.93 (0.000)	H1
APPLE	69.92 (0.000)	71.00 (0.000)	H1
MICRO	42.22 (0.000)	51.81 (0.000)	H1

Note: Ljung－Box (LB) at 10 lag lengths and 20 lag lengths statistics are computed for returns and squared returns. And the P-value is given in the parenthesis. Test results all are H1 indicate this hypothesis reject the null hypothesis of no heteroscedasticity.

3.2. Estimation results

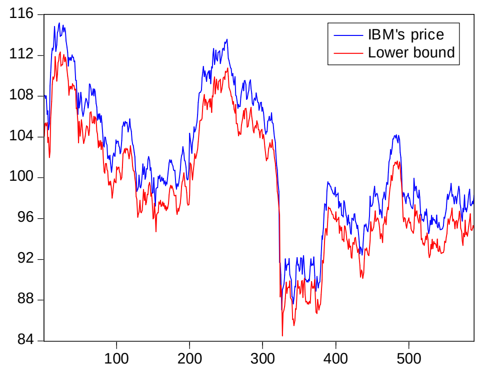
The parameters of (6) and its corresponding p-value for the these four companies are presented in table 3.

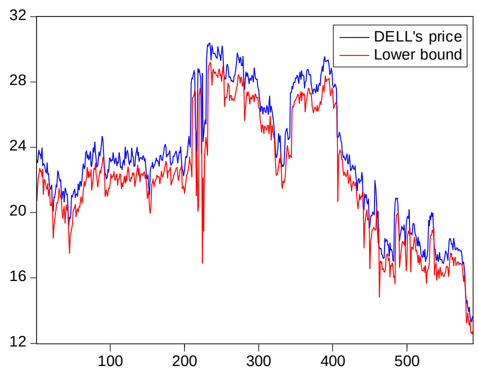
Table 3. Estimation result

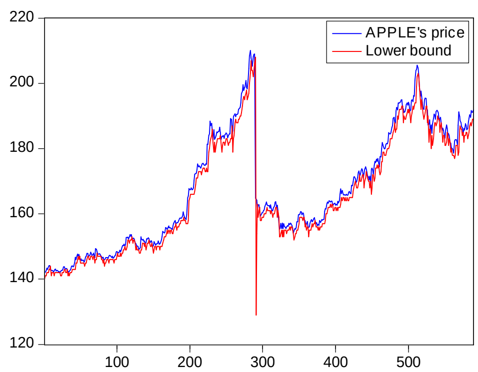
	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$
IBM	3.0943e-004 (8.4132e-006)	0.0543 (0.0190)	0.9457 (0.0122)
DELL	2.8927e-004 (1.4518e-005)	0.6677 (0.0068)	0.3323 (0.0067)
APPLE	3.1446e-004 (2.487e-005)	0.5170 (0.0112)	0.3561 (0.0203)
MICRO	0.0014 (0.0193)	0.0167 (0.0032)	0 -

Note: The P-value given in the parenthesis indicate the significance of the parameters.

We can see in table 3, the persistence parameters $\hat{\alpha}+\hat{\beta}$ are all close to unity except that of MICRO, which means shocks of the current conditional variance would last a long time, while diversified portfolio could eliminate in some extent. Based on estimated (6) we can outside predict VaR of these four stocks from January 2, 2008 to May 5, 2010(590 trading days altogether) and then calculate the lower bound of its stock price. Finally compare with the actual data we can test the capacity of this method.







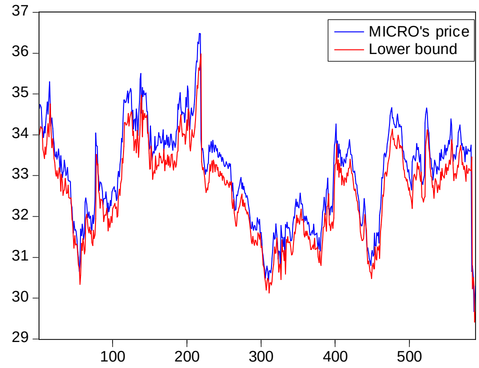


Figure 2. Graphs for stock price and its lower bound

As shown in figure 2, the blue line is very close to the red line which indicates that the predicted value fit the realization very well. There are 2, 15, 33, 2 days respectively for each company when the stock price lower bound surpass the actual price, which all are small respect to the whole sample period(590 trading days).

3.3. *Backtesting*

Backtesing is a VaR calculation methodology involves looking at how often exceptions (loss>VaR) occur. Suppose that the theoretical probability of an exception is p. The probability of m or more exceptions in n days is

$$p_{backtesting} = \sum_{k=m}^{n} \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \tag{7}$$

where $p = m/n$. m is the number of days that exception occurs, n is the total number of trading days. If the probability $p_{backtesting}$ = 27.90% is more than 5% then the model should be considered acceptable. After calculation we obtain

IBM: $p_{backtesting}$ = 27.90% =27.90%>5%

Dell: $p_{backtesting}$ = 27.90% = 99.9%>5%

Apple: $p_{backtesting}$ = 27.90% =100%>5%

Microsoft: $p_{backtesting}$ = 27.90% =100%>5%

Therefore, we can accept this model. Considered the value at risk of portfolio, so that we presume that one investor hold the four shares, amounting to 30000 shares, IBM: 10000 shares, Dell: 3000 shares, Apple: 2000 shares, Microsoft: 150000 shares. At time t, VaR of this portfolio could be calculated as:

$$\mathrm{VaR}_t = u_{1-\alpha}\sigma_t = u_{1-\alpha}\sqrt{w'\Sigma w} \tag{8}$$

Where $u_{1-\alpha}$ is left quantile at the confidence level, w is the weight vector of the asset portfolio; Σ is asset portfolio of variance- covariance matrix, whose basic element $\Sigma(i, j)$ = $\rho_{ij} \times \sigma_i \times \sigma_j$. $w'\Sigma w$ is the asset portfolio standard deviation. $u_{1-\alpha}$ could be used to adjust the portfolio of standard deviation matrix.

We take the confidence at 95%, i.e, $u_{1-\alpha}$ =1.65, weight vector w = [1/3 1/10 2/30 1/2]. We randomly select a day to calculate the stock price in order to calculate the VaR of the portfolio . Choose the data on Jan, 5, 2008 for example, this day all the prices of the stock are: [93.05, 35.22, 22.17, 28.14] .So we are able to compute the value of the assets for the day. Covariance matrix. We take data from 1998 to 2008 with SPSS statistical software to calculate the correlation coefficient matrix about the four stocks, see table 4:

Table 4. Correlation matrix

	IBM	Dell	Apple	Micro
IBM	1	0.3870	0.1484	0.5200
Dell	-	1	0.0079	0.7246
Apple	-	-	1	0.1436
Micro	-	-	-	1

As shown in table 4, these four stocks are highly correlated, which indicates the diversified portfolio is necessary for eliminating risk. Then by combining the volatility, we can calculate its covariance. Based on correlation coefficient and volatility We can use Matlab to calculate covariance matrix on January, 5, 2008:

Table 4. Co-variance matrix

	IBM	Dell	Apple	Microsoft
IBM	0.0003	0.0002	0.0001	0.0002
Dell	-	0.0007	0.0000	0.0005
Apple	-	-	0.0017	0.0001
Microsoft	-	-	-	0.0006

Now we calculate the volatility of portfolio, $\sigma_t = \sqrt{w'\sum w}$ =0.018482, take it into (8), $\mathrm{VaR_t} = u_{1-\alpha}\sigma_t = u_{1-\alpha}\sqrt{w'\sum w} = 1.65 \times 0.018482$, we multiply $\mathrm{VaR_t}$ by the portfolio price P on Jan 4, in 2008 $\mathrm{VaR_t} \times P$ 1502600=48522.24 \$, This means the largest possible loss is 48522.24 dollars invested in these four stocks on Jan 5, in 2008.

4. Conclusion

This article discusses how to use GARCH model to calculate value-at-risk and outside predict the state of risky assets. This approach has the characteristic of dynamics and accuracy. It is open and shut for investors to know the asset risks and then manage them. However, there are several issues need to be extended: This article is only measure VaR based on the basic market factors of return, but actually we encountered some multiple portfolio potential losses, so I suggest extending GARCH (1, 1) model to measure the volatility of multiple market factors, seeking more suitable formula to the asset pricing, and calculating the specific risks of specific assets.

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