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Monte Carlo Simulation for Branching Process with Delay at Reproduction Periods

Mojtaba Moradi [a] **, Behrouz Fathi Vajargah** [b]

Department of Statistics, Faculty of Mathematical Sciences, University of Guilan, Rasht, Iran

a. Email: mmorad @guilan.ac.ir
b. Email: fathi@guilan.ac.ir

Abstract: The main goal of this paper is introduced a new efficient Monte Carlo algorithm for simulating number of particles in *n*th generation of a Branching Process with Delay at Reproduction Periods.

Keywords: Monte Carlo Methods; Branching Process; Bellman–Harris Process

Introduction

The theory of branching processes is an area of mathematics that describes situations in which an entity exists for a time and then may be replaced by one, two, or more entities of a similar or different type. It is a well-developed and active area of research with theoretical interests and practical applications.

The theory of branching processes has made important contributions to biology and medicine since Francis Galton considered the extinction of names among the British peerage in the nineteenth century. More recently, branching processes have been successfully used to illuminate problems in the areas of molecular biology, cell biology, developmental biology, immunology, evolution, ecology, medicine, and others. For the experimentalist and clinician, branching processes have helped in the understanding of observations that seem counterintuitive, have helped develop new experiments and clinical protocols, and have provided predictions which have been tested in real-life situations. For the mathematician, the challenge of understanding new biological and clinical observations has motivated the development of new mathematics in the field of branching processes.

The branching process is a system of particles (individuals, cells, molecules, etc.) which live for a random time and, at some point during lifetime or at the moment of death, produce a random number of progeny. The Bellman–Harris branching process is described as follows [6].

A single ancestor particle is born at $t = 0$. It lives for time τ, which is a random variable with cumulative distribution function $G(\tau)$. At the moment of death, the particle produces a random number of progeny according to a probability distribution with pgf $f(s)$. Each of the first-generation progeny behaves, independently of each other and the ancestor, as the ancestor particle did [i.e., it lives for a random time distributed according to $G(\tau)$ and produces a random number of progeny according to $f(s)$]. If we denote the particle count at time t by $Z(t)$, we obtain a stochastic process $\{Z(t), t \geq 0\}$. This so-called age-dependent process is generally non-Markovian, but two of its special cases are Markovian: the Galton–Watson process and the age-dependent branching process with exponential lifetimes.

On the lines of this research, we suppose that any particle in the branching process lives for one or more unit of time, and at the moment of death it produces a random number of progeny according to a prescribed probability distribution. In other word, lifetime of particles is discrete random variable. In fact, this new type of branching process has delay at reproduction in the moment of death. For this property, we call it as

Branching Process with Delay at Reproduction Period (BPDRP). Studying overcome gene is the application of this class of branching process. Figure 1 illustrates this process, where the second particle from second period, has 2 delaying periods at reproduction.

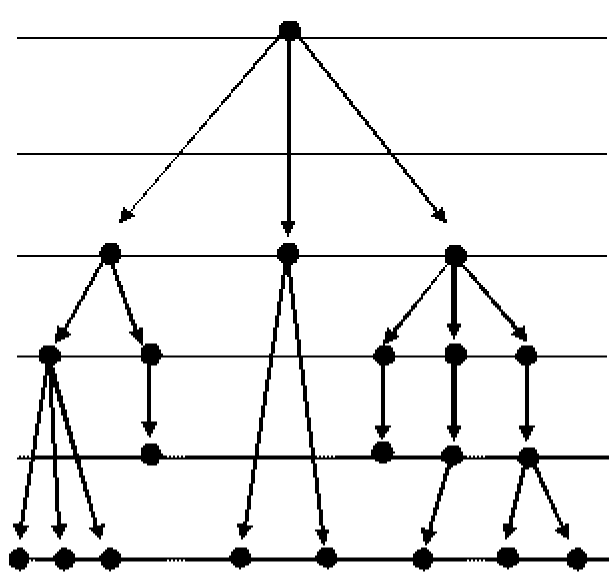

Figure 1: An example of Branching Process with Delay at Reproduction Period (BPDRP).

Modeling of BPDRP

Suppose that the discrete random variable X denotes the number of progeny of a particle and its lifetime be a nonnegative discrete random variable and we denote it by M . So, the probability mass function for BPDRP is given by

$$f_{X,M}(x,m) = P(X = x, M = m), \; x = 0,\ 1,\ 2,\ ...,\ m = 1,\ 2,\ .... \tag{1}$$

Example 1: If particle in a BPDRP has the following probability function

$$f_{X,M}(x,m) = \begin{cases} 1/8, & m = 1,2; \quad x = 0,1,2,3 \\ 0 & \text{Otherwise} \end{cases}$$

then any particle can reproduce at most 3 progeny independently from other particle, and it may have one delay on reproduction. In this example, random variables X and M are considered independent.

Example 2: Suppose that reproduction in a BPDRP has the following probability mass function.

M \ X	0	1	2
1	0.1	0.15	0.2
2	0.05	0.15	0.1
3	0	0.05	0.2

In this example, random variables X and M are dependent. The correlation coefficient between two random variables X and M is –0.47.

Since to generate *n*th period, ($n-1$) th, ($n-2$) th, ..., 1th and zero periods have to participate, so we suppose that the nonnegative integer-valued random variables $X_{ij}^{(n)},\quad i, j=\{1, 2, ...\}$ have the probability mass function (1) and show the number of generated offspring by *j*th particle, where parent particle is in ($n-i$) th period and generates its offspring in *n*th period. In fact, the parent particle delays its reproduction for *i* period.

If we set Z_n as the number of particles in *n*th generation, then we have

$$Z_n=\sum_{i=1}^{m}\sum_{j=1}^{X_{n-i}}X_{ij}^{(n)} \tag{2}$$

For example, in figure 1, we have

$$Z_4=X_{11}^{(4)}+X_{12}^{(4)}+X_{13}^{(4)}+X_{14}^{(4)}+(X_{21}^{(4)}+X_{22}^{(4)}+X_{23}^{(4)}+X_{24}^{(4)}+X_{25}^{(4)})+(X_{31}^{(4)}+X_{32}^{(4)}+X_{33}^{(4)})+X_{41}^{(4)}+X_{51}^{(4)}$$
$$=(0+0+1+2)+(3+0+0+0+0)+(0+2+0)+0+0=8$$

4. Properties of BPDRP

We denote $E(X_{ij}^{(n)})$ by μ_i, i.e., mean of particles in *i*th generation.

Theorem 1: *If* $\{Z_n\}_{n\geq 0}$ *be a BPDRP and* $E(X_{ij})=\mu_i$ *, then*

$$E(Z_n)=\sum_{i=1}^{n}\mu_i E(Z_{n-i}) \tag{3}$$

Proof: First, we calculate

$$E(Z_n\,|\,Z_{n-1}=a_{n-1},Z_{n-2}=a_{n-2},...,Z_1=a_1,Z_0=a_0)$$
$$=E(X_{11}^{(n)}+X_{12}^{(n)}+\cdots+X_{1a_{n-1}}^{(n)})$$
$$+E(X_{21}^{(n)}+X_{22}^{(n)}+\cdots+X_{2a_{n-2}}^{(n)})$$
$$\vdots$$
$$+E(X_{n1}^{(n)}+X_{n2}^{(n)}+\cdots+X_{na_0}^{(n)})$$
$$=a_{n-1}E(X_{11}^{(n)})+a_{n-2}E(X_{21}^{(n)})+\cdots+a_0E(X_{n1}^{(n)})=a_{n-1}\mu_1+a_{n-2}\mu_2+\cdots+a_0\mu_n$$

Using conditional expectation, we have

$$E(Z_n)=E(E(Z_n\,|\,Z_{n-1},...,Z_0))=E(\mu_1 Z_{n-1}+\cdots+\mu_n Z_0)=\mu_1 E(Z_{n-1})+\cdots+\mu_n E(Z_0)$$

For obtain more properties of BPDRP, see [2].

Monte Carlo Simulation

In this section, we introduce a Monte Carlo simulation for estimation of Z_n.

Monte Carlo Algorithm

1. Get n, for estimation of Z_n.
2. Get the $f_{X,M}(x,m) = P(X = x, M = m)$.
3. Set $i = 0$ and $Z_0 = 1$.
4. Set $j = 1$.
5. Generate the x and m according to $f_{X,M}(x,m)$.
6. Set $Z_{i+m} \leftarrow Z_{i+m} + x$.
7. Set $j \leftarrow j + 1$. If $j \le Z_i$ go to step 5.
8. Set $i \leftarrow i + 1$. If $i \le n$ go to step 4.
9. As the end of algorithm, show the value of Z_n.

Example 3: We apply the above Monte Carlo algorithm for probability mass function illustrated in Example 2. The simulation for $Z_0 = 1$ and $n = 10$ is

n	0	1	2	3	4	5	6	7	8	9	10
Z_n	1	2	1	2	4	1	2	4	6	6	5

Example 4: Suppose that probability mass function in a BPDRP be equal to

$$f_{X,M}(x,m) = \frac{1}{36} m\, x, \qquad m = 1,2,3; \; x = 1,2,3.$$

So, the Monte Carlo simulation for $Z_0 = 1$ and $n = 10$ is

n	0	1	2	3	4	5	6	7	8	9	10
Z_n	1	0	3	0	5	6	4	16	16	23	29

Conclusion

The presented Monte Carlo algorithm can simulate a path of BPDRP, efficiently.

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