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A Feedback Linearization based Leader-follower Optimal Formation Control for Autonomous Underwater Vehicles

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Abstract –Leader-follower formation of autonomous underwater vehicles is investigated. Firstly, relative motion equations with disturbances are constructed, and the equations are converted into a linear system by feedback linearization and then feedforward and feedback optimal control (FFOC) law is designed for the linear system. Numerical simulations show the effectiveness of the control scheme.

Keywords –Autonomous Underwater Vehicle, Leader-follower Formation, Feedback Linearization

1. Introduction

Cooperative control of multiple autonomous underwater vehicles (AUV) plays an important role on marine scientific investigation and marine development. The formation of multi-AUV can significantly enhance applications on the marine sampling, imaging, surveillance and communications. Compared to multi-robot, formation control of multi-AUV is particularly difficult, especially on controlling attitude and direction of AUV; what is more, the communication way is acoustic. When communication distances increases, the communication qualities deteriorate quickly, which mainly make time-delay, signal attenuation and distortion. Although formation control of multiple AUVs obtains a wide range of attention in recent years, the fruits on formation control problem are less than ones on land multi-robot problems. For example, Fiorelli conducted a collaborative and adaptive sampling research of multi-AUV at the Monterey Bay [1]; Yu and Ura carried out the cable-based modular fast-moving and obstacle- avoidance experiments, and presented an interconnected multi-AUV system with three-dimension sensors. On the aspect of formation control framework [2-3], [4] proposed a four-layer cooperative control strategy based on hierarchical structure; [5] proposed a hierarchical control framework based on hybrid model. In addition, Yang converted nonholonomic system into a chain one and designed a controller and implemented the leader-follower formation for multiple AUVs in [6]. Kalantar et al. studied the distributed formation control [7]. References [8] adopted the centralized formation control based on virtual structure.

The principal idea of formation control based on leader-follower mode is that an AUV tracks one or more o AUVs in certain desired bearings and separations. This control mode is of the capacity of switching easily between different predefined formation modes and avoiding obstacles quickly. We investigate the formation control problem of autonomous underwater vehicles based on leader-follower. Firstly, we model the relative kinetics equations of two mobile AUVs containing one leader and one follower. The relative velocity vector is projected on two directions, along connection line between them and perpendicular to it. Taking into account the disturbance in underwater environment, we establish the system equations of relative movement, and then we convert the system equations into linear system equations with a disturbance based on feedback linearization. Moreover, the feedforward and feedback optimal control law of the linear system is designed to reject the effect of the disturbance. Finally, simulations demonstrate the effectiveness of the control scheme.

2. Problem Description

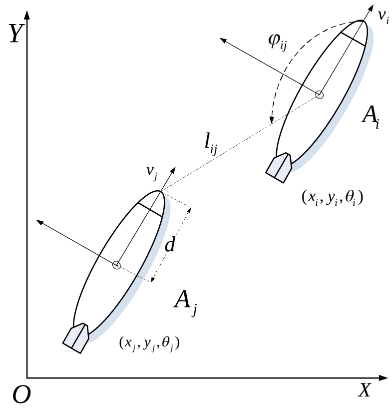

Figure 1. Leader-follower formation

Assume that AUV A_j tracks AUV A_i in a predefined separation l_{ij}^d and bearing φ_{ij}^d . So the formation control pair is shaped naturally. $p_i = \begin{bmatrix} x_i & y_i & \theta_i \end{bmatrix}^T$ and $p_j = \begin{bmatrix} x_j & y_j & \theta_j \end{bmatrix}^T$ denote their coordinates in the inertial coordinate frame, respectively. v_i and v_j denote the linear velocity of A_i and A_j , respectively. l_{ij} denotes the separation between them. φ_{ij} denotes the angle between the linear velocity direction of A_i and their connection line. d denotes the distance from the velocity reference point to the centre of gravity of A_j . f_1 and f_2 denote the underwater disturbances. So the relative velocity vector is projected on the two directions, illustrated in Fig. 1, we obtain the following equations:

$$\begin{aligned} \dot{l}_{ij} &= v_j \cos\gamma_{ij} + d\omega_j \sin\gamma_{ij} - v_i \cos\varphi_{ij} + d_{11} f_1 + d_{12} f_2 \\ l_{ij}\dot{\varphi}_{ij} &= -v_j \sin\gamma_{ij} + d\omega_j \cos\gamma_{ij} + v_i \sin\varphi_{ij} - l_{ij}\omega_i + \\ & d_{21} f + d_{22} f_2. \end{aligned} \tag{1}$$

We define

$z_1 = [l_{ij} \quad \varphi_{ij}]^T$, $u_i = [v_i \quad \omega_i]^T$, $f = [f_1 \quad f_2]^T$,

$u_j = [v_j \ \omega_j]^T$, and $\beta_{ij} = \theta_i - \theta_j$, then

$\dot{z}_1 = G_1 \cdot u_j + F_1 \cdot u_i + D_1 \cdot f$, $\dot{\beta}_{ij} = \omega_i - \omega_j$,

and $\gamma_{ij} = \theta_i - \theta_j + \varphi_{ij} - \pi = \beta_{ij} + \varphi_{ij} - \pi$,

where $G_1 = \begin{bmatrix} \cos\gamma_{ij} & d\sin\gamma_{ij} \\ -\sin\gamma_{ij}/l_{ij} & d\cos\gamma_{ij}/l_{ij} \end{bmatrix}$,

$F_1 = \begin{bmatrix} -\cos\varphi_{ij} & 0 \\ \sin\varphi_{ij}/l_{ij} & -1 \end{bmatrix}$, and $D_1 = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$.

3. Controller design for formation control

We transform the equations (1) to linear forms using input-output feedback linearization.

Choose

$$u_j = G_1^{-1} \cdot (p_1 - F_1 \cdot u_i) \tag{2}$$

where $p_1 = \left[k_1(l_{ij}^d - l_{ij}) - v \quad k_2(\varphi_{ij}^d - \varphi_{ij}) - w \right]^T$ is an auxiliary control input.

Further, we choose appropriate gains $k_1, k_2 > 0$. So the close-loop linear equations are

$$\begin{aligned} \dot{l}_{ij} &= k_1(l_{ij}^d - l_{ij}) - v + d_{11} f_1 + d_{12} f_2 \\ \dot{\varphi}_{ij} &= k_2(\varphi_{ij}^d - \varphi_{ij}) - w + d_{21} f_1 + d_{22} f_2 \end{aligned}$$
(3)

Let $x_1 = l_{ij}^d - l_{ij}$ and $x_2 = \varphi_{ij}^d - \varphi_{ij}$, so

$$\begin{aligned} \dot{x}_1 &= -k_1 x_1 + v - d_{11} f_1 - d_{12} f_2 \\ \dot{x}_2 &= -k_2 x_2 + w - d_{21} f_1 - d_{22} f_2 \end{aligned} \tag{4}$$

We define $x = [x_1 \quad x_2]^T$ and $u = [v \quad w]^T$, so (4) is transformed to

$$\dot{x} = Ax + Bu + Df \tag{5}$$

where $A = \begin{bmatrix} -k_1 & 0 \\ 0 & -k_2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -d_{11} & -d_{12} \\ -d_{21} & -d_{22} \end{bmatrix}$, and f denotes the underwater disturbance vector of the following characteristics

$$\dot{f} = Gf \tag{6}$$

where $G \in R^{2\times 2}$ is a constant matrix.

The dynamics of underwater disturbance is of the following characteristic:

a) All eigenvalues of G satisfy

$\mathrm{Re}[\lambda_i(G)] < 0, \quad i = 1,2$

b) The initial conditions of exosystem (6) are unknown.

To balance the tracking performance and energy consumption, we choose the following infinite-time quadratic performance index:

$$J = \int_0^{\infty} (x^{\mathrm{T}} Qx + u^{\mathrm{T}} Ru) dt \tag{7}$$

Next, we consider the optimal controller design of the system (5) with (6).

If and only if (A, B) is completely controllable, (A, C) is completely observable, C is a full rank matrix and satisfies the equation $Q = C^T C$, and the dynamics of the disturbance satisfies **Error! Reference source not found.** and **Error! Reference source not found.**,the only exiting feedforward and feedback optimal control law is as follows

$$u^*(t) = -R^{-1} B^T [Px(t) + \overline{P} w(t)]$$

where P is the unique positive definite solution of $Riccati$ matrix equation

$$A^T P + PA - PSP + Q = 0$$

, and $\overline{P}$ the unique solution of matrix equation

$$(A - SP)^T \overline{P} + \overline{P} G = -PD$$

with $S = BR^{-1}B^T$.

Substituting the optimal control law $u^* = \begin{bmatrix} v^* & w^* \end{bmatrix}^T$ into **Error! Reference source not found.**, we obtain the control law of A_j:

$$v_j = \cos(\gamma_{ij})/(\cos(\gamma_{ij})^2 + \sin(\gamma_{ij})^2)(k_1(l_{ij}^d - l_{ij}) - v^* + \cos(\varphi_{ij})v_i)$$
$$-l_{ij}\sin(\gamma_{ij})(k_2(\varphi_{ij}^d - \varphi_{ij}) - w^* - v_i\sin(\varphi_{ij})/l_{ij} + w_i)$$
$$\omega_j = \sin(\gamma_{ij})(k_1(l_{ij}^d - l_{ij}) - v^* + v_i\cos(\varphi_{ij}))/d$$
$$+l_{ij}\cos(\gamma_{ij})(k_2(\varphi_{ij}^d - \varphi_{ij}) - w^* - v_i\sin(\varphi_{ij})/l_{ij} + w_i)/d$$

Furthermore, the formation control strategy is as follows:

AUV A_i obtains its linear velocity v_i, angular velocity ω_i, steering angle θ_i and φ_{ij} using its velocity sensors and gyroscopes. Then A_i transmits the data packet containing the information of $v_i, \omega_i, \varphi_{ij}$ and θ_j to A_j. So AUV A_j obtains its control law with l_{ij} (through sensor) and γ_{ij} (through computing). In this way, the leader-follower formation is constructed where A_j tracks A_i in the desired separation l_{ij}^d and bearing φ_{ij}^d.

4. Simulation

The initial state of A_i (the leader) in the inertial coordinate frame is

$p_i(0) = \begin{bmatrix} x_i(0) & y_i(0) & \theta_i(0) \end{bmatrix}^T = \begin{bmatrix} 2 & 1 & \pi/6 \end{bmatrix}^T$, and its linear velocity and angular velocity is $v = 3.5$ and $\omega = 0$, respectively, and they remain constant during the simulation period.The initial state of A_j (the follower) in the inertial coordinate frame is $p_j(0) = \begin{bmatrix} x_j(0) & y_j(0) & \theta_j(0) \end{bmatrix}^T = [1.8 \quad -1.2 \quad 0]^T$. The desired separation is $l_{ij}^d = 7.2$ and the desired bearing is $\varphi_{ij}^d = \pi/4$. So we know that the initial state is

$x_1(0) = l_{ij}^d - l_{ij}(0) = 0.64$ and $x_2(0) = \varphi_{ij}^d - \varphi_{ij}(0) = -0.7$.

The related underwater disturbance matrices are

$$D = \begin{bmatrix} -0.5 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } G = \begin{bmatrix} -1 & 10 \\ -10 & -2 \end{bmatrix}.$$

The related weight matrices are

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } R = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}.$$

The time step is $\tau = 0.05$ and simulation period is $\mathrm{T} = 200$; the related gains are $k_1 = 1, k_2 = 1$, and $d = 0.2$.The performance of the proposed algorithm is simulated and compared with classic feedback optimal control (FOC) algorithm. The simulation results demonstrate that the feedforward and feedback optimal control law has better suppression effect on the underwater disturbance, shown in Fig.2 through Fig. 5.

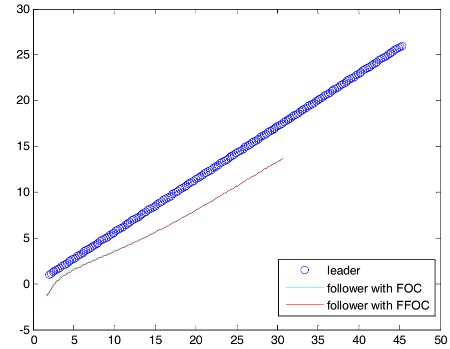

Figure 2. Leader-follower

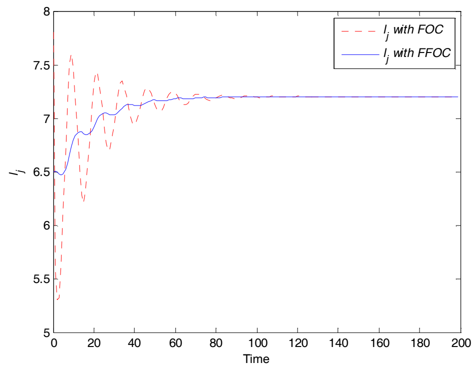

Figure 3. l_{ij} with time

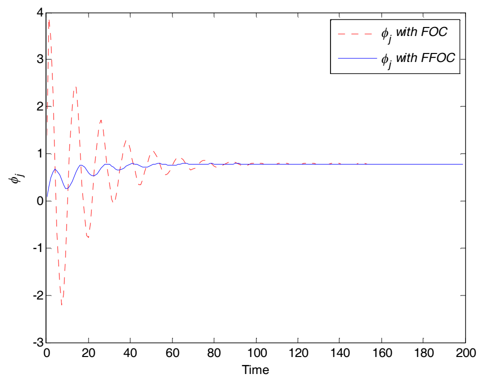

Figure 1. φ_{ij} with time

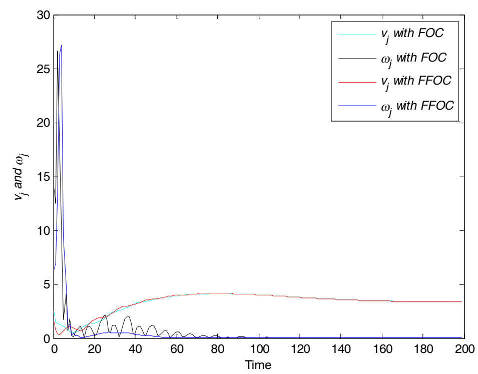

Figure 2. A_j with time

5. Conclusions

Leader-follower formation control of autonomous underwater vehicles is investigated. The performance of the proposed algorithm is simulated and compared with FOC. The simulation results demonstrate that FFOC has better suppression effect on the underwater disturbance compared to FOC.

6. References

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