

# Forecasting abrupt changes in the Chinese stock market via wavelet decomposition

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**Abstract** – As an abrupt change becomes imminent, the stock markets are usually preceded by an oscillatory behavior with amplitudes that tend to become smaller and smaller, while the frequencies tend to increase, which can be detected by the wavelet analysis because it provides simultaneous information on the frequency (scale) and localization in time. According to analysis of Shanghai Stock Exchange Composite Index before the crash of 2007 by the leading indicator constructed through wavelet decomposition coefficients, we find that the leading indicator presents good capacity of monitoring drawdowns and crashes in the Chinese stock market, which also provides a valuable forecasting method for the stock market.

**Keywords** – Abrupt changes in stock market; Wavelet analysis; Leading indicator

## 1. Introduction

Since the worldwide financial crises of 1929, substantial efforts have been directed to elucidate the underlying causes of this type of event. Abrupt changes in stock price may be either upward or downward. A successful characterization of the dynamics of the stock prices, particularly of sudden large drops, can have a profound impact on risk management.

A series of abroad studies firstly started from financial bubbles. A bubble is the accelerating ascent of the market price. The development of bubble theory has three stages, namely the rational bubble theory (Blanchard and Watson, 1982), the irrational bubble theory (De Long, Shleifer, Summers, et al. , 1990) and the nonlinear dynamics theory. Unlike the former two, nonlinear dynamics theory doesn't need to suppose investors to be rational or consider the complex relationship between behavioral finance and the modern theory of finance. The elaborate and successful depiction of bubbles operation mechanism has made it the most widely used method at present.

Based on the theories of nonlinear dynamics, the Log-Periodic Power Law Model (LPPLM) was firstly put forward by Sornette, Johansen and Bouchaud (1996) when they studied the bubble before the 1987 stock crash in the United States and identified the log-periodic signatures, which can be demonstrated by a physical model and the crash time can be estimated. Afterwards, Sornette and Johansen (1997) used the modified LPPL model by including the first non-linear correction to test on the 1929 and 1987 stock crashes, results show that modeling is desirable and it promotes the development of the stock crash theory. Through the analyses of 21 important bubbles, Johansen and Sornette (2001) find that LPPL model can also describe the speculative bubbles of emerging market. Johansen and Sornette (2006) also systematically studied the evolution of 49 stock prices or

indexes all over the world before crash and find that 25 of them can be depicted by the LPPL model.

As LPPL model fits need constant readjustment of the time windows, a few parameters to be estimated, so it is a little complex. Moreover, analysis of abrupt changes via LPPL model premises on log-periodic oscillations of the stock price, so for smaller drops, due to limited time ranges, could not be analyzed by LPPL model. The wavelet may be proper since it provides simultaneous information on the frequency (scale) and localization in time.

Generally speaking, the abrupt changes of stock price, whether upward or downward, when it closes to the transition of trend, because of the more cautious behavior of buy-sides and sell-sides, it usually precedes by an oscillatory behavior with amplitudes that tend to become smaller and smaller, while the frequencies tend to increase, which resembles the waveforms associated with imminence of earthquakes (Johansen, 2003) and can be reflected by the distribution of wavelet coefficients.

Distribution of wavelet coefficients, namely scale-time chart, with lighter grey represents greater magnitude coefficients while vice versa. The lighter grey also stands for the predominant frequencies and by protracting the predominant frequencies and the threshold line, so the cross point is the critical instant we want to make sure. According to the analysis of the scale-time chart of Brazil Sao Paulo Stock Exchange IBOVSPA and stock prices of major companies, Caetano and Yoneyama (2007) find a feasible way to ascertain the crash time.

However, the drawback of the proposed method is that it's affected by the subjective factors in the interpretation of graphical data and not suitable for quantitative analysis. Through further study of wavelet coefficients, Caetano and Yoneyama (2009) introduced an index  $\zeta$ , which is computed as a percentage of the number of coefficients with values greater than a convenient threshold in a vertical strip over the total number of coefficients in the strip. When  $\zeta$  is close to 1, the probability of drawdown

is high and if  $\zeta$  is close to 0, the market maintains its trend. When applied to analyze the Dow Jones Average index, Hang Seng Index and Brazil IBOVESPA, the new index presents good capability of monitoring crashes and drawdowns.

Compared to the studies of bubbles, domestic studies on abrupt changes still have a long way to go. Early researches are centralized on the detection of singular points, a singular point is the magnitude away from the overall distribution. Ping Wang, Yan-yan Jin and Jie-ming Yang (2005) introduced the method of using Lipschitz index to describe the local singularity of signals after wavelet transform and put forward that time corresponding to the maximum modulus is just the critical instant. Based on bi-orthogonal spline wavelet, Zhong-fa Zheng (2006) analyzed the singularity of the Shanghai stock exchange composite index and found ten singular points which all have their important political, economic, social or policy background. To promote the study, Xue-shen Sui and Zhong-hai Yang (2007) systematically studied the singularity of the Shanghai index monthly returns sequence, empirical results show that the location and number of the singular point are correct and exact. Yu-hong Kang and Zhao-yu Xu (2007) explored a model on stock prices on the basis of catastrophe theory. However, since factors influence both directly and indirectly the stock price, it's not easy to control all these variables.

What we should notice is that, though singularity detection by wavelet transform may give accurate indications of large crashes of stock price oscillations, for periodic smaller fluctuations, it may not be applicable. So in this paper, inspired by Caetano and Yoneyama, we use the leading indicator in the Chinese stock market to explore the applicability and differences and it is also a test in emerging markets.

## 2. The wavelet analysis

Wavelet, just as its name implies, is small wave, "small" means it has limited duration and "wave" means its volatility with positive and negative alternate amplitude. Under some assumptions (Daubechies, 1992), the functions in a family  $\psi$  of wavelets can be obtained by scaling and translating a single function  $\varphi$ , called mother wavelet. Hence, if the function  $\varphi$  is a mother wavelet, then the family  $\psi$  is constructed by using scaling  $a \neq 0$  and translations  $b$ :

$$\psi = \{ \varphi^{a,b}(\cdot) \in L^2(R); \varphi^{a,b}(t) = |a|^{-1/2} \varphi(\frac{t-b}{a}), a \in R, b \in R \} \quad (1)$$

Where  $L^2(R)$  denotes the set of square integrable functions, i.e.,  $\{ f: R \rightarrow R \quad st. \quad \int_{-\infty}^{\infty} |f(t)|^2 dt < \infty \}$

When  $b = 0$ , then the parameter  $a$  simply stretches or shrinks the waveform in the direction of the t-axis. On the other hand, when  $a = 1$ ,  $b$  simply moves the waveform along the t-axis.

As  $a, b$  is continuously changing value, hence the family function  $\psi$  is called continuous wavelet basis function. When the function  $f(t)$  in  $L^2(R)$  is expanded under the continuous wavelet basis function, the continuous wavelet transform is obtained, with its expression as:

$$W_f(a, b) = \int_{-\infty}^{\infty} f(t) \varphi^{a,b}(t) dt \quad (2)$$

We define  $W_f(a, b)$  as the wavelet coefficients.

For continuous wavelet transform, if adopted  $\varphi$  is subjected to the admissible condition, then its inverter exists, in other words, given  $W_f(a, b)$ ,  $f(t)$  can be recovered by the formula

$$f(t) = \frac{1}{C} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(a, b) \varphi^{a,b}(t) db \frac{da}{a^2} \quad (3)$$

where the constant C depends on the mother wavelet:

$$C = \int_{-\infty}^{\infty} \frac{|\hat{\varphi}(\omega)|^2}{|\omega|} d\omega \quad (4)$$

when  $C \in (0, +\infty)$ , it is the admissible condition.

$\hat{\varphi}(\omega)$  is the Fourier Transform of  $\varphi$ .

Among many possibilities for the choice of the mother wavelet  $\varphi(\cdot)$ , the Meyer function is selected in this paper. As Meyer is both smooth and orthogonal (Hui-qin Wang, 2011), it is adequate to present wavelet coefficients in the graph. Still, it is easier to present the Meyer wavelet in the frequency  $\phi(\omega)$  domain

$$\varphi(\omega) = \begin{cases} \frac{e^{i\omega/2}}{\sqrt{2\pi}} \sin(\frac{\pi}{2} v(\frac{3}{2\pi} |\omega| - 1)) & \frac{2\pi}{3} \leq \omega \leq \frac{4\pi}{3}, \\ \frac{e^{i\omega/2}}{\sqrt{2\pi}} \cos(\frac{\pi}{2} v(\frac{3}{2\pi} |\omega| - 1)) & \frac{4\pi}{3} \leq \omega \leq \frac{8\pi}{3}, \\ 0 & |\omega| \notin [\frac{2\pi}{3}, \frac{8\pi}{3}]. \end{cases} \quad (5)$$

The function  $v: R \rightarrow R$  used in this work is the standard one and is given by

$$v(\xi) = \xi^4 (35 - 84\xi + 70\xi^2 - 20\xi^3)$$

## 3. Empirical analysis

### 3.1. Data and pre-processing

In this paper, we'll apply the leading indicator to the Chinese stock market. Compared to the prediction of upward abrupt changes, it's easier to forecast the drawdowns and crashes (Sornette, 2003). So the data sample in our paper consists of the daily close index of Shanghai Stock Exchange Composite Index (SSEC) from Nov. 16, 2006 to Nov. 28, 2007, before the 2008 global financial crisis broke out.

The trend must be subtracted from the actual data in order to make the wavelet expansion possible because in equation (2)  $f(t)$  is required to be  $L^2(R)$ . Suppose the actual data series is  $y(t)$ , if we estimate the trend,

denoted by  $y_L(t)$ , using the least squares method, then the data without trend is  $y_R(t) = y(t) - y_L(t)$ . Cyclic terms with low frequencies are not related to the signal to be studied, though it can be tackled by the wavelet transform, they add extra coefficients that tend to clutter the scale-time chart. Hence, the cyclic component, denoted by  $y_C(t)$  is estimated by using the least squares method again. The wavelet decomposition method can now be applied to the series obtained by subtracting from  $y_R(t)$ , namely  $f(t) = y_R(t) - y_C(t)$ .

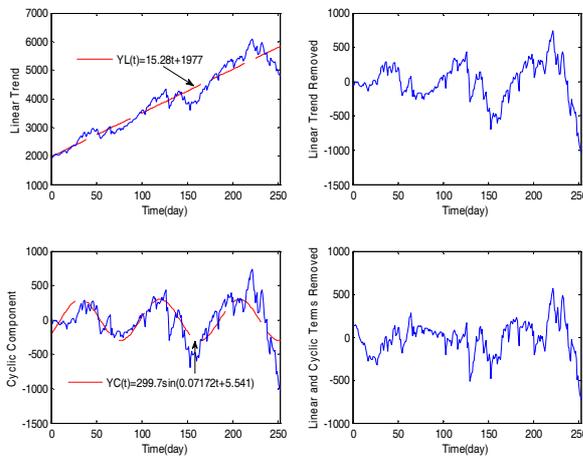
In order to describe the pre-processing phase, consider the case of SSEC and referred to as series  $y(t)$ . Firstly, the trend component is estimated by using the least squares method. Since the goodness of fit is better for linear trend estimate than exponential trend, the trend is found to be given by

$$y_L(t) = 15.28t + 1977 \quad (6)$$

After removal of the linear trend  $y_L(t)$ , the series  $y_R(t) = y(t) - y_L(t)$  presents a cyclic component. Again using least squares method, an estimate of the cyclic component is found to be

$$y_C(t) = 299.7 \times \sin(0.07172t + 5.541) \quad (7)$$

The pre-processing phase can be seen in Fig. 1. After the removal of linear trend and cyclic component, wavelet transform can be applied to the final high frequencies series  $f(t) = y_R(t) - y_C(t)$ .

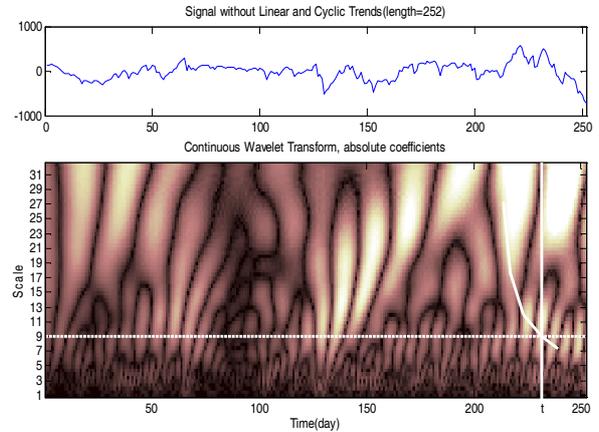


**Fig. 1.** Pre-processing phase for SSEC. Upper left: Original SSEC data and estimated linear trend; Upper right: Linear trend removed; Lower left: Estimated cyclic component; Lower right: pre-processed data.

### 3.2. Wavelet transform analysis

The wavelet analysis, as mentioned before, provides a simultaneous view of the frequency components (scales) and their localization in time. Wavelet decomposition can be used on the pre-processed data and in our paper we choose the Meyer wavelet function, since Meyer wavelet decomposition has a more favorable ability to distinguish the wavelet coefficients. In the scale-time chart (Fig. 2), the vertical axis represents the scale and the horizontal axis the time. As the scale is inversely related to the frequency, so the wavelet coefficients can be well

analyzed by time-frequency analysis through wavelet decomposition scale-time chart. Regions in white color represent existence of coefficients with greater magnitude while regions in dark color represent smaller magnitude. According to the white and dark grey stripes characteristics staggered by time, one can be alerted of the possibility of an abrupt change in the stock prices.



**Fig. 2.** Wavelet transform coefficients for SSEC data. The upper graph shows the actual data with linear and cyclic trends removed. The lower graph shows the scale-time chart of Meyer wavelet transform coefficients.

As the small-scale corresponds to the high-frequency and the large-scale corresponds to the low-frequency. We can also see from Fig. 2, small-scale wavelet coefficients' stripes are narrow, indicating that the volatility of coefficients is more intense, while the large-scale wavelet coefficients' stripes are wide, indicating that the volatility is slow, which also shows a positive correlation between frequency and volatility of wavelet coefficients. In general, more short-term tradings will appear in the stock market near the transition, with the price spreads becoming smaller, and prices begin to appear high-frequency fluctuations. Crashes lead to a larger shock, so the magnitude of the wavelet coefficients is relatively large, especially at small-scales which mean high frequency parts. In scale-time, the white stripes accumulate to the horizontal axis, and then go forward with time around a certain scale when white stripes reach a certain level. If we indicate that scale with dotted line and solid line representing the dominant frequencies in Fig. 2, then the point of intersection of the solid and dotted line is just the critical time when the abrupt change may be imminent. Fig. 2 shows that a large abrupt change of SSEC occurs on around  $t=230$  day.

Distribution of the wavelet coefficients can warn abrupt changes in stock markets, but it is highly subjective to determine the critical time of transitions through the scale-time chart, because the shade of grey may differ according to the chosen mother wavelet, and it's not a trivial work to determine the location of the solid and dotted line. In order to make the judgment of the critical time more objective, we introduce the leading indicator  $\zeta$  which is proposed by Caetano and Yoneyama (2009) and defined by

$$\zeta(t) = \frac{n(t)}{N} \quad (8)$$

where  $N$  is the total number of coefficients in the vertical strip with the base spanning a small time range and  $n(t)$  is the number of coefficients greater than the adopted threshold. The variation of  $\zeta$  lies in the interval  $[0, 1]$ .

The larger the time range is chosen, the smoother of the leading indicator, so the prediction will be less accurate. Here, we base the time range on day, and adopt the overall means of wavelet coefficients magnitude as threshold (quantile of the coefficients magnitude can also be considered). So we can get the relationship between the indicator and the time. To facilitate the analysis, we superimpose the SSEC with the graph of the proposed wavelet index ( $\zeta$ ) as shown in Fig. 3.

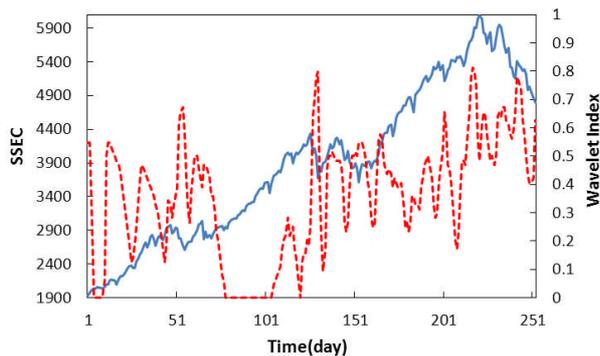


Fig. 3. SSEC prior to 2007 crash (left axis) and the leading indicator (Wavelet Index in the right axis)

### 3.3. Test of the leading indicator

Moreover, in order to test the validity of the indicator, we use some statistical methods to analyze signals given by the leading indicator. According to the local maximum points of wavelet index, combined with the actual stock price index movements, we extract the following abrupt changes, as shown in Table 1.

In Table 1, Crash starts from the last trading day before the drawdown, corresponding to Date. The variable Days counts the number of days that the price continues to fall and Percentage corresponds to the losses with respect to the initial maximum peak. There are three ways to define a persistent decrease in the price (Johansen, 2004). As the strict continuous decrease ceases when there is a rebound, even it's a very small one, and if  $\varepsilon$ -drawdown is used, we should estimate the index history volatility, which proves a hardwork for a new index. Hence, we compute using the closing price, ignoring price rebounds less than 3%.

From Table 1, for the 21 abrupt change signals the indicator gives for SSEC, there is only one false indication, which happened on Feb. 14, 2007. One this day, though the stock closing index rose 2.59%, market outlook didn't expect a reversal but continued to climb up high.

The threshold adopted to calculate the leading indicator in equation (8) for SSEC is the overall mean of the magnitude of wavelet decomposition coefficients, so it can be regarded as a measure of the average degree of the shade of grey of the scale-time chart. In this way, the higher wavelet coefficients the average level, the greater the energy of the wavelet coefficients characterized is, which shows that high-frequency oscillation for the moment is more intense and this is exact the precursory patterns before an abrupt change. Leading indicators of local maxima in Fig. 3 are corresponding to these moments. While the value is greater, the probability of a large drawdown is higher.

Carefully examining the actual SSEC, we find that there are still some abrupt changes that not predicted by the leading indicator. Not all of these drawdowns correspond to the case of  $\zeta = 0$ , the unpredicted abrupt changes of SSEC are shown in Table 2.

Table 1. Drawdowns and crashes of SSEC

Date	$\zeta$	Crash	Days	Percentage
2006/12/5	0.5469	2173.28-2093.62	3	-3.67
2007/1/24	0.3438	2975.12-2857.36	1	-3.96
2007/1/29	0.6719	2945.26-2612.53	5	-11.30
2007/2/14	0.5000	false indication		
2007/2/26	0.4688	3040.59-2785.3	5	-8.40
2007/5/10	0.2813	4049.7-4021.65	1	-0.69
2007/5/14	0.2500	4046.37-3899.15	1	-3.64
2007/5/29	0.7969	4334.91-3670.38	4	-15.33
2007/6/19	0.5156	4296.52-3041.08	4	-7.69
2007/7/3	0.5156	3899.72-3615.87	2	-7.28
2007/7/9	0.5469	3883.21-3853.02	1	-0.78
2007/7/23	0.5781	4213.35-4210.31	1	-0.07
2007/7/31	0.4531	4471.02-4300.56	1	-3.81
2007/8/14	0.3594	4872.77-4656.56	3	-4.44
2007/8/28	0.5000	5194.68-5109.41	1	-1.64
2007/9/10	0.6563	5355.27-5113.96	1	-4.51
2007/10/11	0.8125	5913.23-5903.25	1	-0.17
2007/10/16	0.6563	6092.04-5562.39	7	-8.69
2007/10/24	0.5000	5843.1-5562.39	1	-4.80
2007/10/31	0.6719	5954.75-5158.1	9	-13.38
2007/11/14	0.7813	5412.68-4803.39	10	-11.26

Table 2. Leading indicator failed to predict the drawdowns and crashes of SSEC

Date	$\zeta$	Crash	Days	Percentage
2007/1/10	0.3750	2825.57-2668.11	2	-5.57
2007/4/18	0.0000	3612.37-3449.01	1	-4.52
2007/6/27	0.2656	4078.59-3820.7	2	-6.32

In this way, including the 3 unpredicted drawdowns, for SSEC, the indicator succeed to forecast 20 abrupt changes of all the 24, so that

$$P(\text{drawdown} = \text{true}) = \frac{20}{24} \approx 83\%$$

where drawdown=true signify successful predictions of drawdowns or crashes.

Therefore, the leading indicator presents good capability of monitoring drawdowns or crashes for the Chinese stock market.

#### 4. Conclusions

As it closes to an abrupt change, the stock price will often present high frequency oscillations, with price spreads become smaller and smaller, while trading volume and energy become larger and larger, which can be detected by wavelet since it provides simultaneous information on the frequency (scale) and localization in time and can concentrate on every detail of the signal.

After pre-processing of the data of SSEC for the Chinese stock market before the 2007 crash, the wavelet decomposition can be used. Wavelet coefficients are the inner product of the analyzed signal and the wavelet function and can be presented in a scale-time chart. Through the characterization of shade of grey of wavelet decomposition coefficients, one can be very alert to the crash of stock market. Case of SSEC indicates that this method is feasible.

The leading indicator of abrupt changes in the stock market is a deeper level of understanding of the wavelet coefficients distribution, and this kind of quantitative analysis can avoid the effects of different wavelet base functions as well as subjective factors, also it can be explained qualitatively from the angle of energy. The indicator also presents good capacity of monitoring drawdowns and crashes in the Chinese stock market, there is only one false indication of the indicator for all the 21 signals of the index and the accuracy rate of the indicator can reach as high as 83 percent!

However, different from studies of Caetano of U.S. Dow Jones Average and Brazil IBOVESPA to predict the crash in 1929 and 2008 respectively, the leading indicator here doesn't give values as large enough in predicting the crash of 2007. As the leading indicator based on some assumptions, such as information symmetry. If the Chinese stock market can't effectively reflect the expectations of the future market outlook, then it's likely that the proposed indicator value could be small. On the other hand, the indicator may also give a local maximum at some instant, but market afterwards doesn't expect a drawdown or a crash, this can be seen from Table 1 and further study is required to elucidate the underlying causes.

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