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The Research and Analysis of Behavioral Finance on the Asset Price Bubbles

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Abstract –In the paper, we will use the Behavioral Finance (BF) to analyze the influence of rational investors and noise investors on the price dynamics of risky assets, and research the endogenous factors of the bubbles. The rational investors use utility maximization as their investment strategy. They will make allocation of their risky assets and risk-free assets, according to the continuously evolving expectations on the future return and risk of these assets. As for the noise investors, they will be more likely to invest in what most people choose, and put more emphasis on the past return and risk of assets. The implementation of rational strategy can stabilize price, while the noise one may increase the volatility of price. Through the stability analysis of the price system, we will find that the different relative influence between rational and noise investors will result in different kinds of price dynamics, even an explosive bubble.

Keywords: – Behavioral Finance; Dynamical System; Stability Analysis; Endogenous Bubbles

1. Introduction

1.1 Asset price bubbles

Financial asset is the value carrier of virtual economy, expanded by the real economy. Its price is also depended on market demand and supply. The interactions of supply and demand in virtual economy make larger and more frequent changes of price than ones in real economy. When the asset price continually rises and then rapidly falls in a short time, we call it as ‘the accumulation and burst of asset price bubbles’.

Asset price bubbles have destructive power. Bubbles will not only destroy the order and stability of the market, but also will bring millions loss of market value. Meanwhile, they can affect the market effectiveness and the allocation of assets in the macro-economy. When an explosive bubble occurs, it will even lead to a global crisis.

The earliest model of capital price bubbles is created by Blanchard (1979) [1]. He used run test and fat-tail test to prove the existence of bubbles, and named it ‘rational speculative bubbles’. On the basis of Blanchard model, Evans (1991) proposed ‘periodic explosive bubble’ model [2]. And then many kinds of bubble models emerged one after another. Finally Froot and Obstfeld (1991) created endogenous bubble model [3], analyzing the exponential relationship between price and variables of asset fundamentals.

1.2 Behavioral finance

Behavioral Finance (BF) is an innovation and challenge to the traditional finance. BF takes psychology and behavioral science into financial systems and research economic problems from a microcosmic and individual angle. Through distinguishing the different actions, we can roughly divide market participants into two categories: rational and noise traders, and analyze diversified business philosophy, investment strategies and making decision.

In traditional finance, the efficient market theory thinks rational traders can seize the every arbitrage opportunities created by noise traders, and drive noise ones out of the market. So there are only rational traders surviving.

But from real situation and historical data, we would find out that people will not be so rational normally. They may invest according to the judgment of themselves, may take account of price momentum, or make decisions according to the profit and loss of historic investments.

BF thinks such ‘diverging from reason’ can not be removed by statistical average. And noise trades play such an important role that we should take account of them when discussing economic problems.

The paper will research price dynamics, asset price bubbles, and the different investment strategies of different market participants. Based on such diversity, we can make a model to explore the asset price and the key factors of making bubbles.

2. The investment strategy of rational traders

Rational traders will continuously evolve their expectations on the future return and risk of risky assets. And based on the evolution, they decide their investment portfolios between risk-free assets and risky assets to maximize utility.

To describe the dynamics of rational investments in mathematical equations, we will use the 'one-period ahead optimization strategy' [4-5] proposed by Chiarella (2002). This means at present time t, the rational trader estimates the profit and risk of risky assets in the next time t+1. Then based on the estimation, trader adjusts the portfolio holding at present, to make sure the new one will result in the utility maximization in the next time t+1.

2.1 The parameter of rational traders

We will set the number of rational traders as a constant N_r, and also set the number of noise ones as a constant N_n. For simplify, we assume all the N_r traders have the same investment idea and action.

We can describe the adjusted portfolio at present time t as follows:

X_t The number of risky assets

P_t The price of risky assets

X_{ft} The number of risk-free assets

P_{ft} The price of risk-free assets

d_t The dividend paid by risky assets

We will get the value of the adjusted portfolio at present time t is: $Y_t = P_t X_t + P_{ft} X_{ft}$ (1)

Then in the next time t+1, the adjusted portfolio will lead to utility maximization. The adjusted portfolio depends on the strategy decided at present time t, new price of assets in the next time t+1 and the dividend of risky assets. So the value of new portfolio in the next time t+1 is:

$$Y_{t+1} = P_{t+1} X_t + P_{ft+1} X_{ft} + d_{t+1} X_t \tag{2}$$

Making a difference equation of equation (1) and (2) to get the value increment of portfolio in the one period:

$$\Delta Y = Y_{t+1} - Y_t = (P_{t+1} - P_t) X_t + (P_{ft+1} - P_{ft}) X_{ft} + d_{t+1} X_t \tag{3}$$

Next we will bring some classical rates into our model:

$W_t = \dfrac{P_t X_t}{Y_t}$ The risky assets ratio

$R_{t+1} = \dfrac{P_{t+1}}{P_t} - 1$ The price growth rate of risky assets

$R_f = \dfrac{P_{ft+1}}{P_{ft}} - 1$ The price growth rate of risk-free assets

Thus the equation (3) turns into:

$$Y_{t+1} = Y_t + [R_f + W_t (R_{t+1} - R_f + \frac{d_{t+1}}{P_t})] Y_t \tag{4}$$

Making further calculations, we will get its expectation and variance:

$$\begin{aligned} E_t[Y_{t+1}] &= Y_t + R_f Y_t + W_t (E_t[R_{t+1}] - R_f + \frac{E_t[d_{t+1}]}{P_t}) Y_t \\ Var_t[Y_{t+1}] &= Y_t^2 W_t^2 (Var[R_{t+1}] + \frac{Var[d_{t+1}]}{P_t^2}) \end{aligned} \tag{5}$$

2.2 Utility maximization

For satisfying the condition, utility maximization in the next time, we will adjust our portfolio. The first thing is to get two key adjusting factors X_t and X_{ft}.

We can use the mean-variance optimization model to deal with the problem of utility maximization. And we assume that the rational trader in the model is constant-absolute risk aversion (CARA) with risk aversion factor γ.

Thus, the expected utility function is:

$$EU(Y_{t+1}) = E_t[Y_{t+1}] - \frac{\gamma}{2} Var_t[Y_{t+1}] \tag{6}$$

Combine equation (5) with equation (6), the whole expected utility function will be obtained. Applying the first-order condition to the whole function with respect to $W_t = \dfrac{P_t X_t}{Y_t}$ leads to:

$$P_t X_t = \frac{E_t[R_{t+1}] - R_f + \frac{E_t[d_{t+1}]}{P_t}}{\gamma(Var[R_{t+1}] + \frac{Var[d_{t+1}]}{P_t^2})} \tag{7}$$

So if trader adjusts his portfolio according to the first-order condition, he will maximize his utility in the next time t+1.

2.3 The excess demand of rational traders

During $(t-1, t)$, the adjustment of risky assets in the portfolio is called excess demand of risky assets.

$$\Delta D_{rt} = N_r \left(P_t X_t - P_t X_{t-1} \right) = N_r \left(P_t X_t - \frac{P_t}{P_{t-1}} P_{t-1} X_{t-1} \right) \tag{8}$$

Otherwise, the expression $E_t[R_{t+1}] - R_f + \dfrac{E_t[d_{t+1}]}{P_t}$ means the total excess expected rate of return of the risky assets above the risk-free ones from t to t+1. To indicate the reassessment of risky assets made by rational traders is varying with time, we define the excess expected rate of return is also varying with time continuously in the following way.

$$E_t[R_{t+1}] - R_f + \frac{E_t[d_{t+1}]}{P_t} := \bar{\mu}(1 + \alpha_t) \tag{9}$$

Where $\bar{\mu}$ is mean of excess expected rate of return, and α_t is a random variable with zero mean.

Combining the first-order condition (7) and the excess demand (8) with the definition (9) leads to:

$$\Delta D_{rt} = \frac{\bar{\mu} \cdot N_r}{\gamma \sigma^2} [1 + \alpha_t - \frac{P_t}{P_{t-1}} (1 + \alpha_{t-1})] \tag{10}$$

Where $\sigma^2 = Var[R_{t+1}] + \dfrac{Var[d_{t+1}]}{P_t^2}$.

2.4 Stabilizing force on price

If σ^2 is a constant, we will get:

$$E_t[\Delta D_{rt}] = -\frac{\bar{\mu} \cdot N_r}{\gamma\sigma^2} \frac{P_t - P_{t-1}}{P_{t-1}} \quad (11)$$

This equation describes the mean of rational traders' excess demand of optimal risky assets. We will assume $\bar{\mu} > 0$, because investing in risky assets is profitable.

From the equation (11), we can know the excess demand is mean-reversing.

When $P_t > P_{t-1}$, then $E_t[\Delta D_{rt}] = \frac{\bar{\mu} \cdot N_r}{\gamma\sigma^2}(1 - \frac{P_t}{P_{t-1}}) < 0$. So trader will sell risky assets, and price of risky assets will fall and stop the rising trend during from t-1 to t.

When $P_t < P_{t-1}$, then $E_t[\Delta D_{rt}] = \frac{\bar{\mu} \cdot N_r}{\gamma\sigma^2}(1 - \frac{P_t}{P_{t-1}}) > 0$. So trader will buy risky assets, and price of risky assets will rise and stop the falling trend. We will find the investment strategy of rational traders play a stabilizing role in the price of assets.

3. The investment strategy of noise traders

Noise traders are defined as people who cannot obtain the insider information, irrationally invest with noise information. Their investment strategy depends on herd behavior [6] and momentum effect [7].

Herd behavior means traders are affected by social force. They will be more likely to follow the major direction of investment. Such group psychology leads to over-reaction and the lack of variety.

Momentum effect means the rate of asset return trends to follow that of history. Based on the point, traders will buy those have high historic return and sell those with low historic return to get profit through technical analysis and chart analysis.

3.1 The parameter of noise traders

We assume noise traders have only two choices: buy risky assets or buy risk-free ones. Unlikely rational traders, they will partly purchase risky assets and partly purchase risk-free ones. Such assumption is called all-or-nothing strategy.

At present time t, the number of noise traders who choose to invest in risky assets is $N_+(t)$. In contrast, the number of noise traders who choose to invest in risk-free assets is $N_-(t)$.

So $N_+(t) + N_-(t) = N_n$ (12)

Introduce a new variable S_t. It describes the main social direction of investment, namely describes the herd effect.

$$S_t = \frac{N_+(t) - N_-(t)}{N_n} \quad (13)$$

When $S_t > 0$, then $N_+(t) > N_-(t)$, which means the majority of noise traders takes a bullish view on the risky assets.

When $S_t < 0$, then $N_+(t) < N_-(t)$,which means the majority of noise ones takes a bearish view.

Through simple algebraic manipulation, combined equation (12) and (13), we can obtain:

$$N_+(t) = \frac{N_n}{2}(1 + S_t), \quad N_-(t) = \frac{N_n}{2}(1 - S_t) \quad (14)$$

3.2 Herd behavior

Firstly, during time from t-1 to t, $p_\pm(t-1)$ is defined as the probability of sell (or buy) risky assets decided by the $N_\pm(t-1)$ noise traders previously holding risky assets (or risk-free assets).

Secondly, $\eta_k(p_\pm)$ is a random variable which obeys the (0-1) distribution. It means the investment decision made by a noise trader k holding risky assets (or risk-free assets). When $\eta_k(p_\pm) = 1$, the trader k will sell (or buy) risky assets with the probability $p_\pm$. When $\eta_k(p_\pm) = 0$, trader k will continuously hold risky assets (or risk-free assets) with the probability $1 - p_\pm$.

Aggregating all these decisions made by all noise traders, we can get:

$$\begin{aligned}\Delta S_t &= S_t - S_{t-1} \\ &= \frac{2}{N_n}[N_+(t) - N_+(t-1)] \quad (15) \\ &= \frac{2}{N_n}\left\{\sum_{j=1}^{N_-(t-1)} \eta_j[p_-(t-1)] - \sum_{k=1}^{N_+(t-1)} \eta_k[p_+(t-1)]\right\}\end{aligned}$$

$\eta_k(p_+)$ and $\eta_j(p_-)$ are random variables following the (0-1) distribution. So $\{\eta_k(p_+)\}$ and $\{\eta_j(p_-)\}$ are i.i.d. According to the property of independent identical distribution, we get the expectation and variance of ΔS_t:

$$E[\Delta S_t] = p_-(t-1) \cdot (1 - S_{t-1}) - p_+(t-1) \cdot (1 + S_{t-1})$$

$$Var[\Delta S_t] = \frac{2}{N_n}\begin{bmatrix} p_-(t-1)(1 - p_-(t-1))(1 - S_{t-1}) + \\ p_+(t-1)(1 - p_+(t-1))(1 + S_{t-1}) \end{bmatrix}$$

(16)

3.3 Momentum effect

We try to use the exponential moving averaging to measure the momentum effect of price:

$$H_t = \theta H_{t-1} + (1 - \theta)\left(\frac{P_t}{P_{t-1}} - 1\right) \quad (17)$$

Where H_t describes the price momentum. And θ is the weighting coefficient between 0 and 1, describing the memory length of historic price.

$p_\pm$ is the probability of changing invested assets. Like what we have said, the investment of noise traders depends on two features: herd behavior and momentum effect, so we can also use such two features to define $p_\pm$:

$$p_\pm(t-1) = p_\pm(S_{t-1}, H_{t-1})$$

We assume this function obeys symmetrical relationship. Namely one unit of bullish view (or one unit of rising price momentum) has the same absolute influence on the investment decisions of noise traders, with one unit of bearish view (or one unit of falling price momentum). $p_-(S,H)=p_+(-S,-H)$.

The simplest equations to satisfy the symmetrical relationship are linear ones:

$$\begin{aligned} p_-(S,H) &= \tfrac{1}{2}\left[p+k\cdot(S+H)\right] \\ p_+(S,H) &= \tfrac{1}{2}\left[p-k\cdot(S+H)\right] \end{aligned} \tag{18}$$

P is the average time of holding position without the influence of outside force. K measures the strength of herd behavior and momentum effect on the $p_\pm$, k>0.

3.4 The excess demand of noise traders

Likely rational traders, we use ΔD_{nt} to describe the excess demand of risky assets of noise traders during from time t-1 to t:

$$\Delta D_{nt} = A[N_+(t)-N_+(t-1)] \tag{19}$$

A is the amount of investment of one noise trader. We assume not only A is a constant, but also every noise trader has the same amount of investment.

Combine equation (14) with excess demand (19), we will get:

$$\Delta D_{nt} = \frac{1}{2}AN_n\cdot(S_t-S_{t-1}) = \frac{1}{2}AN_n\cdot\Delta S_t \tag{20}$$

Further, combine equation (18) and (16), the expectation will be obtained as following:

$$E[\Delta S_t] = -(p-k)S_{t-1}+kH_{t-1} \tag{21}$$

4. The dynamical system of asset price

4.1 Market clearing condition

Market clearing condition [8] indicates if the price of the asset market is fully flexible, then it will adjust itself rapidly according to the market supply and demand. Such flexibility leads non-existence of excess supply and demand. So

$$\Delta D_{rt}+\Delta D_{nt}+Supply_t=0$$

$Supply_t$ is the excess supply of risky assets. Of course $Supply_t$ is a variable, but we would treat it as a constant $Supply$ for simplification.

Combine equation (10) and (20) with the market clearing condition, we will get:

$$\frac{\bar{\mu}\cdot N_r}{\gamma\sigma^2}\left[1+\alpha_t-\frac{P_t}{P_{t-1}}(1+\alpha_{t-1})\right]+\frac{1}{2}A\cdot N_n\cdot\Delta S_t+Supply=0$$

$$\frac{P_t}{P_{t-1}}(1+\alpha_{t-1})-(1+\alpha_t)=\frac{AN_n\cdot\Delta S_t\cdot\gamma\sigma^2}{2\bar{\mu}\cdot N_r}+\frac{Supply\cdot\gamma\sigma^2}{\bar{\mu}\cdot N_r}$$

Assume $\tau:=\frac{A\gamma\sigma^2}{2\bar{\mu}}\cdot\frac{N_n}{N_r}$ and $\delta=\frac{\gamma\sigma^2}{\bar{\mu}N_r}\cdot Supply$, then the equation will be simplified as:

$$\begin{aligned} &\frac{P_t}{P_{t-1}}(1+\alpha_{t-1})-(1+\alpha_t)=\tau\cdot\Delta S_t+\delta \\ &P_t=\frac{1+\alpha_t+\tau\cdot\Delta S_t+\delta}{1+\alpha_{t-1}}\cdot P_{t-1} \end{aligned} \tag{22}$$

4.2 The whole dynamical system

Through arrangements, we will get the whole dynamical system of asset price.

$$\begin{cases} P_t=\dfrac{1+\alpha_t+\tau\cdot\Delta S_t+\delta}{1+\alpha_{t-1}}\cdot P_{t-1} \\ H_t=\theta H_{t-1}+(1-\theta)\left(\dfrac{1+\alpha_t+\tau\cdot\Delta S_t+\delta}{1+\alpha_{t-1}}-1\right) \\ \Delta S_t=\dfrac{2}{N_n}\left\{\sum_{j=1}^{\frac{N_n}{2}(1-S_{t-1})}\eta_j[p_-(t-1)]-\sum_{k=1}^{\frac{N_n}{2}(1+S_{t-1})}\eta_k[p_+(t-1)]\right\} \\ E[\Delta S_t]=-(p-k)S_{t-1}+kH_{t-1} \end{cases} \tag{23}$$

In the system, the first dynamical equation describes the interaction of rational strategy and noise strategy. The price dynamics obey the modified geometric Brownian process with the multiplicative factor $\frac{1+\alpha_t+\tau\cdot\Delta S_t+\delta}{1+\alpha_{t-1}}$.

The second one is the equation of risky-asset price momentum. It is a key factor of noise traders to make investment decisions.

The third one is another key factor of noise traders to make investment decisions, the difference of herd effect. And the forth one is its expectation.

4.3 The stability of system

To make sense of stability of system, we should get rid of the random variables from the system firstly, meanwhile reduce the system into deterministic components. But we will take account of the random variables into numerical simulation stated on the next chapter 5.

Thus, let α_t identically equal to zero. Then $E[\Delta S_t]$ will be substituted for ΔS_t to eliminate the interference of random variables. Then the system is simplified as:

$$\begin{cases} P_t=\left\{1+\tau\cdot\left[kH_{t-1}-(p-k)S_{t-1}\right]+\delta\right\}\cdot P_{t-1} \\ H_t=\left[\theta+(1-\theta)\tau k\right]H_{t-1}-(p-k)S_{t-1}(1-\theta)\tau+\delta(1-\theta) \\ S_t=kH_{t-1}+(1-p+k)S_{t-1} \end{cases} \tag{24}$$

Carefully observe the three key equations, we will find they are correlative. The first one describes the present price can be obtained by the previous price and a multiplicative factor, a linear structure made by variables H_{t-1} and S_{t-1}.

The second and third equations are used to explain the first key equation. Otherwise they are an autonomous system, because they all depend on H_{t-1} and S_{t-1}. Consequently, we will start with the second and third equations to analyze the stability of system.

Choose a balance point to make a stable price dynamics. Let $H^*=\delta$, $S^*=\frac{k\delta}{p-k}$ as chosen balance point. Then how can we know it is the balance one? If we put them into system (24), we will find:

$$P_t = P_{t-1}(1+\delta) \approx P_0(1+\delta)^t \approx P_0 e^{\delta t}$$

Introduce random variables $\{\alpha_t\}$ and $\{\Delta S_t\}$ again, the price dynamics will correspond to a geometric Brownian motion.

The problem of stability is essentially equal to the stability of system on the balance point. This means we will research whether the system will keep its scheduled moving condition steadily after occasional or continued interferences. On the other words, see whether H_t and S_t will still converge to the balance point H^* and S^*. Equally, make sense of whether $H_t - H^*$ and $S_t - S^*$ converge to zero. Thus, we define auxiliary variables:

$$\tilde{H}_t := H_t - H^*,\ \tilde{S}_t := S_t - S^* \qquad (25)$$

Then the two equations of system (24) will be turned into:

$$\begin{cases} \tilde{H}_t = \left[\theta + (1-\theta)\tau k\right]\tilde{H}_{t-1} + (1-\theta)\tau(k-p)\tilde{S}_{t-1} \\ \tilde{S}_t = k\tilde{H}_{t-1} + (1+k-p)\tilde{S}_{t-1} \end{cases} \qquad (26)$$

Analyze the equation set by the eigenvalue λ, we get:

$$\det(A-\lambda E) = \begin{vmatrix} \theta + (1-\theta)\tau k - \lambda & (1-\theta)\tau(k-p) \\ k & 1+k-p-\lambda \end{vmatrix}$$

So the characteristic equation for λ reads:

$$(1-\lambda+k-p)\left[\theta-\lambda+(1-\theta)\tau k\right] - (1-\theta)\tau(k-p)k = 0$$

The solution of the characteristic equation is:

$$\lambda_\pm(\tau) = 1 + \sqrt{(1-\theta)(p-k)}\,\omega_\pm$$

Where $\omega_\pm(x) = x \pm \sqrt{x^2-1}$,

$$x = x(\tau) = \tau k\rho - \rho - \frac{1}{4\rho}$$

$$\rho = \frac{1}{2}\sqrt{\frac{1-\theta}{p-k}}$$

5. The bifurcations and numerical simulation

There are some theorems [9] of Lyapunov function about the linear discrete-time dynamical system.

Definition 5.1:

Set A as a $n \times n$ matrix. And λ_i ($i = 1,2,\ldots,n$) is its eigenvalue. Thus we can call $\rho(A) = Max\{|\lambda_i|, i = 1,2,\ldots,n\}$ as the spectral radius of A.

Theorem 5.2:

$x(n+1) = Px(n)$ is the linear discrete-time dynamical system. Where $x \in R^m, n \in I, P \in R^{m\times m}$ (constant matrix), and $\rho(P)$ is the spectral radius of P. Thus the necessary and sufficient condition of a asymptotic stable system is $\rho(P) < 1$.

Apply the theorems into the solution of eigenvalue above. We will find the solution is a function of variable τ. So the stability of system will vary with τ. Now what needs our discussion is which τ can make the system stable, and which τ will leads to a divergence system and a financial bubble.

Because of the complexity of the solution of eigenvalue, our research will be separated. Starting from the equation $x = x(\tau) = \tau k\rho - \rho - \frac{1}{4\rho}$, we will get some key definitions.

When $x(\tau) = -1$,

$$x(\tau) = \tau k\rho - \rho - \frac{1}{4\rho} = -1$$
$$\Leftrightarrow \quad \tau = \left(\rho + \frac{1}{4\rho} - 1\right)\frac{1}{k\rho} \quad := \tau_{-1} \qquad (27)$$

When $x(\tau) = 0$,

$$x(\tau) = \tau k\rho - \rho - \frac{1}{4\rho} = 0$$
$$\Leftrightarrow \quad \tau = \left(\rho + \frac{1}{4\rho}\right)\frac{1}{k\rho} \quad := \tau_0 \qquad (28)$$

When $x(\tau) = 1$,

$$x(\tau) = \tau k\rho - \rho - \frac{1}{4\rho} = 1$$
$$\Leftrightarrow \quad \tau = \left(\rho + \frac{1}{4\rho} + 1\right)\frac{1}{k\rho} \quad := \tau_1 \qquad (29)$$

5.1 The first kind of price dynamics, monotonously exponential convergence [10]

According to the historic data, we assume $0 < k < p$.

When $0 \le \tau < \tau_{-1}$, it means $x(\tau) < -1$. And because of $0 < \sqrt{(1-\theta)(p-k)} < 1$, so:

$$x(\tau) < -1 \Leftrightarrow \omega_\pm(x) = x \pm \sqrt{x^2-1} < 0$$
$$\Leftrightarrow \lambda_\pm(\tau) = 1 + \sqrt{(1-\theta)(p-k)}\,\omega_\pm < 1$$

The above expression indicates the system is stable on the balance point (H^*, S^*). And H_t and S_t will monotonously exponentially converge to the balanced point. In this case, price dynamics shows:

$$P_t = P_{t-1}(1+\delta) \approx P_0(1+\delta)^t \approx P_0 e^{\delta t}$$

After introducing the random variables $\{\alpha_t\}$ and $\{\Delta S_t\}$ again, the price dynamics will show the feature of a standard geometric Brownian motion.

Now we will try to do numerical simulation to confirm our conclusion.

We will choose one day as our time interval between time t-1 and t. In the calculation of momentum effect of noise traders, the length of memory of historic price is regarded as 20 days. Then $\frac{1}{1-\theta} = 20$, so the weight is $\theta = 0.95$.

P is the twice probability of selling or buying risky assets of noise traders without outside force. We will assume $p=0.2$. This means without interference, the normal trading frequency of noise investors is about one week. As for k, a coefficient measuring the outside interference on noise traders, we will set it close to but smaller than p. $k=0.95p=0.19$

According above data, and through the definition (27), (28) and (29), we get $\rho \approx 1.12$, $\tau_{-1} \approx 1.88$, $\tau_0 \approx 6.58$ and $\tau_1 \approx 11.3$.

Additionally, for rational traders, we will assume the mean of excess expected rate of return of risky assets $\overline{\mu} \approx 0.2\%$ and $A\gamma \approx 1$. The variance of risky asset price is normally deemed to be $\sigma^2 \approx 0.001$.

According to the statement made by Coyne and Witter in 2002, price is pushed by a small part of investors. So we make $N_n \approx 10^3$, and we will get:

$$\tau \approx \frac{0.001}{0.2\%} \cdot \frac{N_n}{2N_r}$$

Because $\tau=\tau_0$ is the cut-off point (this will be stated in the next section), if $\tau_0 \approx 6.58$, then we will get $N_n \approx 25N_r$.

Finally using above data, we can get the charts from the experiments of MATLAB as following:

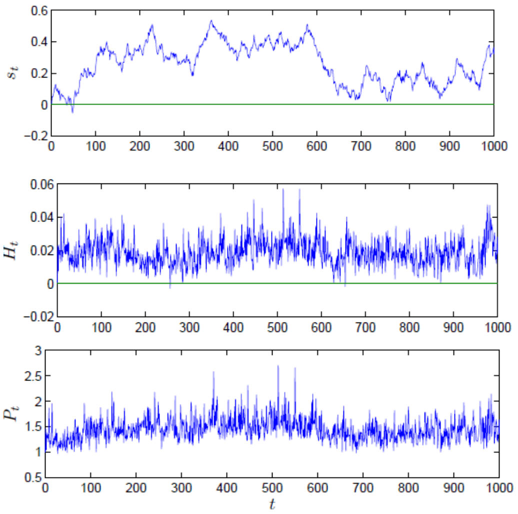


Figure 1. When $\tau=1, \delta=10^{-4}$, the simulated diagram of S_t, H_t, P_t

From the above diagram, when $\tau=1<\tau_{-1} \approx 1.88$, S_t has large changes. This means noise traders are changing their investments frequently. But the price is changing slowly around its fundamental value. This is because in the case, rational traders have a larger strength of controlling the price than one of noise traders, and rational strategy fully plays a stable role in the price dynamics.

5.2 The second kind of price dynamics, exponential damped oscillation

When $\tau_{-1}<\tau<\tau_0$, equally $-1<x(\tau)<0$, the characteristic equation has not real solution but an imaginary solution:

$$\lambda_{\pm}(\tau)=1+\sqrt{(1-\theta)(p-k)}x(\tau) \pm \sqrt{(1-\theta)(p-k)} \cdot \sqrt{1-x^2}\, i \tag{30}$$

We will find $0<\mathrm{Re}\left[\lambda_{\pm}(\tau)\right]<1$, so the real part of complex number satisfies the stability condition. But the existence of imaginary part will bring some interference. So the system is still stable on the balance point (H^*, S^*), but H_t and S_t will converge to the balance point with damped oscillations, the amplitude gradually getting smaller.

Still use the data from section 5.1.

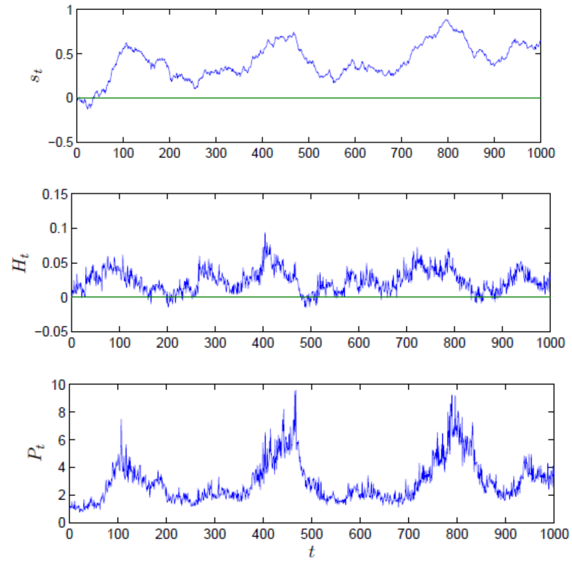
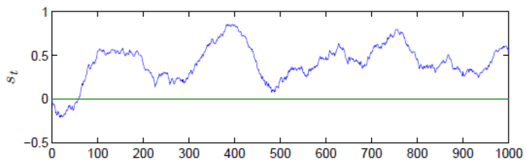


Figure 2. When $\tau=5, \delta=1.85 \cdot 10^{-3}$, the simulated diagram of S_t, H_t, P_t

When $\tau=5$, $1.88 \approx \tau_{-1}<\tau<\tau_0 \approx 6.58$ partly satisfies the condition of stability. In figure 2, H_t and S_t have lager waves than that of figure 1. And the price dynamics become not very smooth but with some medium oscillations. Price diverges from its fundamental value in some times.

5.3 The third kind of price dynamics, exponential divergence with a swing

When $\tau_0<\tau<\tau_1$, equally $0<x(\tau)<1$, the characteristic equation also has an imaginary solution. But at this time the real part is larger than one, $1<\mathrm{Re}\left[\lambda_{\pm}(\tau)\right]$. So the system is unstable on the balance point (H^*, S^*), meanwhile H_t and S_t will exponentially diverge with a swing.



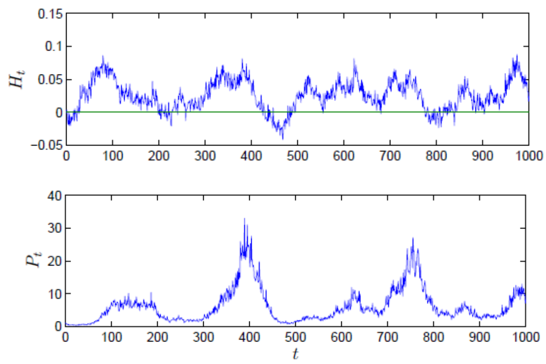


Figure 3. When $\tau = 8, \delta = 4.7 \cdot 10^{-3}$, the simulated diagram of S_t, H_t, P_t

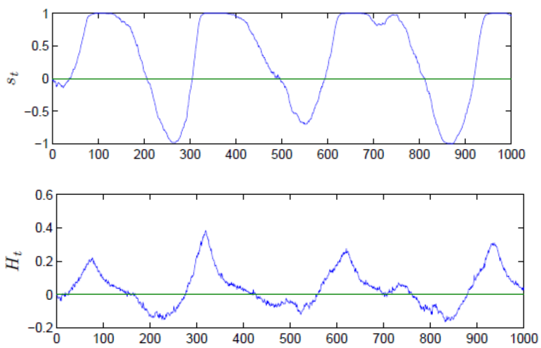
On $\tau = 8$, it will leads to $6.58 \approx \tau_0 < \tau < \tau_1 \approx 11.3$. We will find when the majority of noise traders take a bullish view on the risky assets, S_t will be larger than zero and increasing continuously. The same conclusion of H_t can be obtained. At the same time, from analyzing the diagram above, we find price ignore its fundamental value, rising and rising with a peak.

After the maximum, H_t and S_t begin rapid fall, because noise traders think it is time to sell risky assets through fundamental analyses. At the same time, this reflects on the diagram of P_t is a collapse.

And then the three diagrams show the process again and again which are called 'recurrent bubbles' [11]. We will find that the strength of controlling price of noise traders is larger than that of rational ones. This leads to unstable price dynamics.

5.4 The forth kind of price dynamics, exponential divergence

As $\tau_1 < \tau$, which means $1 < x(\tau)$, the eigenvalue is a real number and larger than 1, $\lambda_{\pm}(\tau) > 1$. At that time, the system is obviously unstable on the balance point (H^*, S^*). H_t and S_t will exponentially diffuse until their extreme. In this period, a giant explosive bubble will occur.



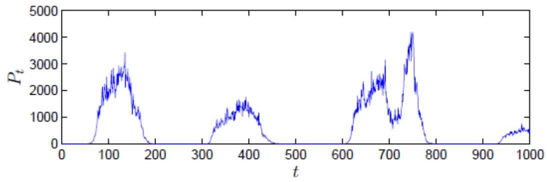


Figure 4. When $\tau = 15, \delta = 2.4 \cdot 10^{-2}$, the simulated diagram of S_t, H_t, P_t

As $\tau = 15$, then $\tau > \tau_1 \approx 11.3$. In the case, system is extremely unstable. Noise traders have overwhelming superiority in controlling the price. Reflecting in the figure is the huge swings of H_t and S_t. Also the price ignores its fundamental value with large swings. An explosive bubble [12] emerges.

5.5 Conclusions

From these discussions and analyses above, we will find an important conclusion.

$$\tau_0 = \left(\rho + \frac{1}{4\rho}\right)\frac{1}{k\rho}$$

The variable is a key cut-off point. When $\tau < \tau_0$, the system is basically stable and price will vary around its fundamental value. When $\tau > \tau_0$, the system is no longer stable, the divergence of price occurs, and bubbles emerge.

The essence of the conclusion is the game between noise traders and rational traders. Observe the following variable τ. It consists of two parts. One part describes the influence of noise traders on the price dynamics. Another part measures that of rational traders.

$$\tau = (A \cdot N_n) \Big/ \left(\frac{2\bar{\mu} \cdot N_r}{\gamma\sigma^2}\right)$$

In the numerator, A refers to the amount of investment of every noise trader, and N_n is the total number of noise traders. The increment of A and N_n indicates the investment of noise traders in risky assets is also getting large. At this time, τ rises too. Until τ is getting larger and larger to be superior to τ_0, noise traders have larger strength of controlling price than that of rational traders.

The strategy of noise traders doesn't care the expectation of future return, but is based on the majority of social investments and historic momentum. So it leads to the phenomenon of buying when the price is up and selling shares when the price goes down. So the price changes largely ignoring its fundamental value. And market becomes unstable with asset price bubbles.

Otherwise in the denominator, $\bar{\mu}$ is the mean of total expected rate of return over the risk-free rate of the risky assets, σ^2 is the variance of the price growth rate of risky assets, γ is the risk aversion factor of rational traders, and N_r is the total number of rational traders. The increment of $\bar{\mu}$ and N_r, or the decrement of σ^2 and γ, indicates the influence of rational traders on the risky assets is getting larger. At this time, τ also decreases.

Until τ is getting smaller and smaller to be inferior to τ_0, rational traders establish overwhelming control over the price and market.

The strategy of rational traders has the ability to stabilize the price dynamics, which we call it as 'mean reversing' stated above. So the price will vary around its fundamental value ignoring the tiny irrational trades made by noise traders. This moment, market is stable without asset price bubbles.

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