

Fatigue Life Prognosis of Concrete Using Extended Grey Markov Model and PSOA

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ABSTRACT - The fatigue life prognosis of concrete is becoming more important with the development for demanding higher quality and safety in industrial. However, effective methods for this prognosis are still in need now, due to the feature of concrete. This paper proposes the extended grey Markov model (i.e. EGMM) for fatigue life prognosis of concrete. Firstly, the GM (1, 1, λ_1, λ_2) (i.e. EGM) is proposed by integrating the particle swarm optimization algorithm (PSOA) with GM (1, 1) (i.e. GM). Then the Markov model is integrated with EGM and a novel prognosis method of the extended grey Markov model is proposed. In the proposed method, the PSOA is used to optimize the parameters of EGM so that EGM can obtain better health states of concrete. The extended grey Markov model is used to combine the health states and transition probability, and obtain better and more accurate fatigue life of concrete. And a real case study is used to demonstrate the implementation and potential applications of the proposed fatigue life prognosis approach on concrete.

Keywords- Materials; Concrete; Intelligence; Fatigue; Prognosis; Maintenance

1. INTRODUCTION

Concrete is ubiquitous in construction and is manufactured by water, sand, stone and cement. And cement manufacturing is one of the most energy intensive industries among mineral process industries. It is manufactured by three stages: raw materials preparation and homogenization, burning at about 1450°C to produce clinker, and then grinding the clinker with additions to a fineness of about 3000 cm²/g to produce it. Finally, water, sand, stone and cement according to the proportion are mixed to manufacture concrete.

In modern, concrete and reinforced concrete structures are expected to face larger and more number of cyclic stresses than before. Examples of applications include gravity oil platforms, nuclear reactor pressure vessels and containment and heavy machinery platforms. Such heavy applications with demanding requirements have made concrete a hot research topic in materials science and engineering [1, 2]. The increasing use of reinforced concrete has once again shifted focus on understanding the fatigue life of concrete.

The fatigue life prognosis of concrete is very important to understand fatigue failure in reinforced concrete and pre-stressed reinforced concrete. Generally, the fatigue life can be estimated based on the *S-N* curve [3]. Generation of *S-N* curve requires data from a large number of fatigue tests. For concrete with low stress levels and increased fatigue life, the fatigue tests will be a very complex process. There is therefore a need to develop new easier methods for fatigue life prognosis of concrete.

The grey model has been applied to solve these problems because the original data has better smoothness [4], but the prognostic accuracy is not high. GM has been extended to EGM to improve prognostic accuracy. And the Markov model has been applied to problems involving data with random fluctuations. So EGM has

also been integrated with the Markov model, and the EGMM has been established. This model greatly improves the accuracy.

Kumar et al. applied grey Markov model to forecast crude-petroleum consumption of conventional energy in India [5]. Li et al. proposed a new dynamic analysis model which combined the first-order one-variable grey differential equation model from grey system theory and Markov chain model from stochastic process theory [6] and reference [7] also used the Grey-Markov model for international aviation quantitative prediction. Jiang et al. applied system cloud grey model (SCGM(1,1)) to fit the development tendency of the few time series based on grey system and Markov chain, its error index was stochastically fluctuated [8]. Hsu et al. presented an integration prediction method including grey model (GM), Fourier series, and Markov state transition, to predict the turning time of Taiwan weighted stock index for increasing the forecasting accuracy [9] and reference [10] also used the grey Markov model for stock price prediction. However, little work has been done on the application of the extended grey Markov model in the area of fatigue prognosis.

This paper discusses the basic concepts of GM and the PSOA. Based on GM and PSOA, the EGM and the extended grey Markov model are presented, respectively. An improvement of PSOA and a case study for fatigue life prognosis of concrete is presented.

2. PROGNOSTIC MODEL

2.1. Extended model of GM

In 1982, the grey system theory was developed by Deng [4]. In paper, the grey model can be applied to fatigue life prognosis of concrete. In order to improve the prognostic accuracy of the concrete's fatigue life, the EGM of GM has been proposed and applied for fatigue

life prognosis of concrete. However, the most challenging problem faced by EGM is parameter optimization. The PSOA exhibits relatively simpler and faster convergence than genetic algorithm and other methods. So the PSOA is used to solve the optimal parameter value for EGM, and the EGM based on PSOA is applied for fatigue life prognosis of concrete.

In the grey model, the smoothness of values of the original data series influences the accuracy of grey prognosis. When the values of the original data series have larger fluctuations, the values of original data series require pre-treatment to improve the smoothness of the original data series. The original data series can also be processed by either power transformation or logarithmic transformation to improve the accuracy.

The background value $z^{(1)}(k)$ of GM has great influence on the prognostic accuracy. So the differential equation of GM will be changed into the following forms.

The forward form on difference is:

$$x^{(1)}(t+1) - x^{(1)}(t) + ax^{(1)}(t) = u \quad (1)$$

The backward form on difference is:

$$x^{(1)}(t+1) - x^{(1)}(t) + ax^{(1)}(t+1) = u \quad (2)$$

Then summation of ((equation (1))· λ_1 and ((equation (2))· λ_2 is

$$z^{(1)}(k) = \lambda_1 \times x^{(1)}(k) + \lambda_2 \times x^{(1)}(k) \quad (3)$$

where $x^{(1)}(k)$ is the k^{th} value of the grey generation sequence. EGM becomes the GM when the values of both λ_1 and λ_2 equal 0.5. By solving the values of λ_1 and λ_2 using the optimization method (PSOA), the EGM can have higher accuracy than GM.

2.2. Particle swarm optimization algorithm

PSO algorithm is an evolution method developed by the Kennedy and Eberhart [11, 12]. Due to its simple concept and calculations involved, it has been applied successfully in many fields within a short time of its development.

In the D -dimension search space, assuming that m particles form a particle group, the i^{th} particle's space is $X_i (X_i=[x_{i1}, x_{i2}, \dots, x_{iD}], i=1, 2, \dots, m)$. It is also a potential solution to the optimization problem. Each particle includes a location and a speed, the location of the i^{th} particle is defined as $P_i (P_i=[p_{i1}, p_{i2}, \dots, p_{iD}], i=1, 2, \dots, m)$, and the flight speed of the i^{th} particle is defined as $V_i (V_i=[v_{i1}, v_{i2}, \dots, v_{iD}], i=1, 2, \dots, m)$. The particle's location is used to compute the fitness value of the objective function. The fitness value corresponding to the best history location of overall particle group is taken as the best history fitness value of objective function. In order to obtain the best history fitness value of objective function, each particle is iterated based on equations (4) and (5).

$$\begin{aligned} V_i(t+1) &= w \times V_i(t) + c_1 \times rand() \times \\ &[P_{sb}^i(t) - P_i(t) + c_2 \times rand()] \\ &\times [P_{gb}(t) - P_i(t) \end{aligned} \quad (4)$$

$$P_i(t+1) = P_i(t) + V_i(t+1) \quad (5)$$

where, $v_i(t)$ is the speed vector of the i^{th} particle in the t^{th} iteration, and $p_i(t)$ is the location of the i^{th} particle in the t^{th} iteration. c_1 and c_2 are the learning factors and obtain values from (0, 2). The $rand()$ is the random function that takes values from [0, 1]. The inertia weight w is a dynamic variable and controls the search capability of all particles. $P_{sb}^i(t)$ denotes the best history location of the i^{th} particle in the t^{th} iteration. $P_{gb}(t)$ denotes the best history location of overall particle group in the t^{th} iteration.

2.3. Extended GM based on PSO

Based on the extended grey model and PSOA, values of a and u can be computed once the values of λ_1 and λ_2

are determined. So values of $\hat{x}^{(0)}(k)$ are computed based on the EGM, and the ratio of the posterior difference can be obtained as follows.

$$\begin{aligned} & \left(\sum_{i=2}^N [x^{(0)}(i) - \hat{x}^{(0)}(i) - s_2]^2 \right) / \left(\sum_{i=1}^N [x^{(0)}(i) - s_1]^2 \right) \times (N / (N - 1)) \end{aligned} \quad \text{where,}$$

$$s_1 = \frac{1}{N} \sum_{i=1}^N x^{(0)}(i),$$

$$s_2 = \frac{1}{N-1} \sum_{i=1}^N [x^{(0)}(i) - \hat{x}^{(0)}(i)].$$

It can be seen from equation (3), the relationship between λ_1 , λ_2 and the posterior difference is non-linear and is difficult to analyze. So the minimum ratio of the posterior difference is regarded as the objective function, and the PSO algorithm can be developed to solve the optimal values of λ_1 and λ_2 .

2.4. Markov model

Markov chain is a type of time series. Based on the concepts of system "state" and "state transition", the Markov prognosis model is established and it is a type of dynamic random mathematical model. The model assumes a random process $x_n (x_n, n \in T$ and T denotes the time set) and a discrete state set $I (I=(i_0, i_1, i_2, \dots, i_{n+1}))$. Moreover, for any integer $n (n \in T)$, the conditional probability $P(x_{n+1}=i_{n+1}/x_0=i_0, x_1=i_1, \dots, x_n=i_n)$ equals to $P(x_{n+1}=i_{n+1}/x_n=i_n)$. $x_n (n \in T)$ is a Markov chain and denoted as follows.

$$P_{ij}^{(k)} = P\{x_{m+k}=j/x_m=i\}, (i, j \in I)$$

where, $P_{ij}^{(k)}$ denotes the probability that system status is state j at the $m+k$ -th time while system status is state i at the m -th time.

The $P_{ij}^{(k)}$ is successively ranked, and the matrix $P^{(k)}$ is obtained as follows.

$$P^{(k)} = \begin{bmatrix} P_{11}^{(k)} & P_{12}^{(k)} & \dots & P_{1n}^{(k)} \\ P_{21}^{(k)} & P_{22}^{(k)} & \dots & P_{2n}^{(k)} \\ \dots & \dots & \dots & \dots \\ P_{n1}^{(k)} & P_{n2}^{(k)} & \dots & P_{nn}^{(k)} \end{bmatrix} \quad (6)$$

where, $P_{ij}^{(k)} \geq 0$ and $\sum_{j \in I} P_{ij}^{(k)} = 1$. When k equals 1,

assumed that $P_{ij}^{(k)}$ equals to P_{ij} , then, one-step transition matrix $P^{(1)}$ equals to P . The values of $P_{ij}^{(0)}$ are given as follows.

$$P_{ij}^{(0)} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Based on the Markov chain theory, the data sequence can be divided into a number of states and the states are denoted as x_1, x_2, \dots, x_n , respectively. $P_{ij}^{(k)}$ denotes the transition probability from state x_i to state x_j by k steps, and is shown as follows.

$$P_{ij}^{(k)} = n_{ij}^{(k)} / N_i \quad (7)$$

where, $n_{ij}^{(k)}$ denotes the number of transitions from state x_i to state x_j by k steps, and N_i denotes the total number of state x_i appeared.

Based on the results of sections 2.1, 2.2, 2.3 and 2.4, the relationship among GM, EGM, PSO algorithm and Markov model is illustrated in Figure 1.

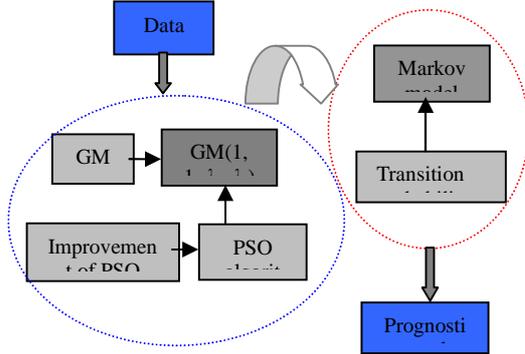


Fig 1. Structure of the extended grey Markov model.

3. A Case Study

3.1. Improvement of PSO

In equation (4), the inertia weight w is an important parameter for searching capabilities of the particle swarm optimization algorithm. The specific computation equation is as follows.

$$w = w_{max} - (T/T_{max}) \times 0.8$$

where, T is the current number of iterations, T_{max} is the largest cutoff number of iterations and w_{max} is the initial value of w .

In order to improve the evolution speed of PSO, w is given several different initial values that equal to 1.2, 0.8 and 0.6, respectively, based on the particle swarm size M . When the value of w_{max} becomes larger, the PSO improves the global search-ability, and gradually moves closer to the local region when w_{max} becomes smaller. When the value of w_{max} becomes smaller, the PSO has better local search ability, and the local search is implemented within the region of the optimal solution. This strategy can increase the probability of finding the optimal solution and avoid premature convergence to a large extent.

3.2. Selection of experimental data

In this paper, the fatigue life experimental results of bi-axial amplitude tension compression fatigue experiment of concrete under certain pressure are chosen from literature [13] to implement the prognosis procedure of the extended grey Markov model. The specific conditions of the experiment are shown as follows.

The strength grade of concrete is C30 and the proportions of cement, sand, stone and water are 1, 1.73, 3.01 and 0.5, respectively. Under the circumstances of lateral pressure level ($|\sigma_2/f_c|$) of 0.25 (σ_2 and f_c represent the lateral pressure and compressive strength of concrete, respectively), when the maximum pressure level S_{max} (i.e. σ_{max}/f_c) equals to 0.7, 0.65, 0.6, 0.55 and 0.45, respectively, the corresponding test results of fatigue life are shown in Table 1.

Table 1. Test results of fatigue life of literature.

Lateral pressure level	Maximum pressure level (S_{max})	Fatigue life (N_f)					Average of lognormal distribution
0.25	0.7	54	92	125	318	341	2.1675
	0.65	158	545	1312	6326	7736	3.1418
	0.6	2374	5427	13529	20637	71847	4.0825
	0.55	10552	11274	50910	130815	294398	4.6736
	0.5	194342	618996	645790	913011	1284998	5.792
	0.45	2500000*	2500000*				

(**) shows the sample that doesn't occur fatigue damage. The minimum stress level (S_{min}) is -0.2

3.3. Extended grey model

Under the circumstances of lateral pressure level of

0.25, the lognormal fatigue life sequence ($x^{(0)}(k)$) is cumulated once to obtain the cumulative sequence of the first order ($x^{(1)}(k)$). The results are shown in Table 2.

Table 2. Values of $x^{(0)}(k)$ and $x^{(1)}(k)$ under confined pressure (0.25 f_c).

Sequence	1	2	3	4
Pressure level S	0.7	0.65	0.6	0.55
Logarithmic fatigue life sequence $x^{(0)}(k)$	2.1675	3.1485	4.0825	4.6736
Cumulative sequence (one time) $x^{(1)}(k)$	2.1675	5.3142	9.3967	14.0703

Table 3. Prognostic results of fatigue life by GM and EGM.

Sequence	Experiment results	GM		EGM	
		Fitted values	Error	Fitted values	Error
1	2.1657	2.1657	0	2.1657	0
2	3.1418	3.2363	-0.0878	3.2246	-0.0761
3	4.0825	3.9087	0.1738	3.9007	0.1818
4	4.6736	4.7212	-0.0476	4.6598	0.0138

Based on Table 2, the prognostic results of fatigue life by GM and EGM are shown in Table 3. According to the results in Table 3, the mean error of the GM is 0.0773, and the mean error of the EGM is 0.0679. So the prognostic accuracy of the EGM based on PSOA is superior than that of the GM. The comparative results between GM and EGM are shown in Figure 2.

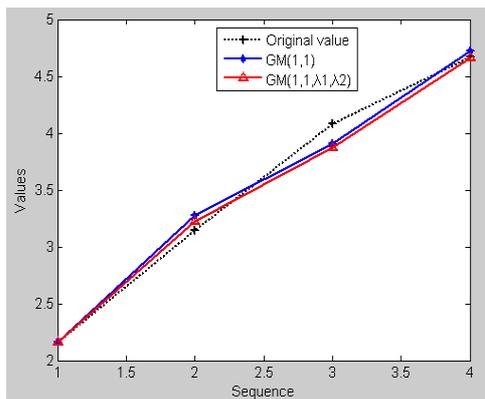


Figure 2. The comparison of fitted values between GM(1, 1) and GM(1, 1, λ_1, λ_2).

3.4. Extended grey Markov model

Because the prognostic accuracy of EGM is superior than that of the GM. The extended grey Markov model is applied for fatigue life prognosis of concrete. Based on Table 3, the relative errors of the fitted values of EGM are obtained and the relative errors are amplified 100 times. The results are shown in Table 4.

Based on the relative errors (δ) in Table 4, the extended grey Markov model of the case is established. The available data for each state is limited, so only two states are divided in this study. The state's standard is shown in Table 5, and the state transition probability matrix is computed based on equation (6) and (7).

Table 4. The result's relative error of EGM.

	Sequence	Fitted values	Relative error (δ)	State
	EGM	1	2.1657	0
2		3.2246	-2.417	1
3		3.9007	4.4531	2
4		4.6598	0.2952	2

Table 5. The state's standard.

State	State boundaries
S_1	$\delta=(0, 4.5)$
S_2	$\delta=(-4.5, 0)$

3.5. Result analysis

The probability that the data series transfers to states S_1 and S_2 is compared based on the state transition probability matrix. And it can be seen, under the conditions of 0.25 times lateral pressure, the bias between experimental values and fitted values of concrete's fatigue life logarithm value is most likely in state S_2 . So the prognostic values obtained by EGM can be used for Markov prognosis. The prognostic values of various models are shown in Table 6,

Table 6. Prognostic results of different model.

Maximum stress	EGM	Extended Markov model	Grey	S-N curve	Actual testing data
0.5	5405 12	653681		25637 7	619455
0.45	7285 976	5302063		90419 06	2500000*

It can be seen from Table 6, the EGMM prognosis of concrete matches fairly well with experimental values. This is explained by a proportion of concrete in the range of 1-3 kg/m³ in case of quantity of water. The proportion of concrete in respect of ash stone is in the range of 6-17 kg/m³. The proportion of concrete in respect of quantities of cement is a bit higher and stands in the range of 4-12 kg/m³. The prognostic accuracy of EGMM is superior than that of EGM and S-N curve, and more reliable prognostic results can be obtained through the EGMM. With the increase of measurement data, the prognostic accuracy of the EGMM also increases.

4. CONCLUSIONS

In this paper, we have presented a prognostic framework and methodology for fatigue life prognosis of concrete. Through the comparison of the performances between GM and EGM, the proposed EGMM framework combines EGM and Markov model in an integrated

manner. The prognostic curve for fatigue life prognosis of concrete is modeled by EGM and PSOA. The health states of concrete are obtained based on the prognosis curve and the state transition probability is computed by the Markov model. As a whole, the fatigue life prognosis of concrete are modeled as the EGMM. Finally, a case study is used to illustrate the procedure of the proposed prognostic methodology and shows that EGMM-based prognosis has a much better performance than the grey Markov model and EGM based prognosis.

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