

# An Image Denoising Threshold Estimation Method

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**Abstract** – In this paper, we propose an image denoising threshold method that exploits the subband dependency of the wavelet coefficients to estimate the signal variance using the local neighboring coefficients. The VisuShrink, SureShrink, and BayesShrink denoising methods are important methods for denoising, but these methods remove too many coefficients, leading to poor image quality. The proposed method retains the modified coefficients significantly that result good visual quality. The experimental results show that our method outperforms the VisuShrink, SureShrink, and BayesShrink denoising methods.

**Keywords** – Thresholding; Image Denoising; Peak Signal-to-Noise Ratio (PSNR)

## 1. Introduction

A digital image is more often degraded by noise during its acquisition and/or transmission. It is necessary to remove noise from the image to main its visual quality and it can be done by applying a suitable denoising method. The aim of an image denoising algorithm is to recover the clean image from its noisy version by removing the noise and retaining the maximum possible image information. In the recent years, there has been a fair amount of research on thresholding and threshold selection procedures for image denoising [9-13]. The threshold selection plays an important role in image denoising because the large value of the threshold kills the image data, while the small value of threshold keeps the noisy data [1]. The VisuShrink [1-2], SureShrink [3-4], and BayesShrink [5-6] methods are the most commonly used threshold selection methods. The VisuShrink threshold is a function of noise variance and the number of samples [1-2]. The SureShrink threshold is considered to be optimal in terms of the Stein's Unbiased Risk Estimator (SURE) [3-4]. This threshold is determined in BayesShrink through modeling the coefficients as Gaussian distribution function [5]. These denoising methods have been improved by our proposed method that follows term-by-term threshold estimation. The rest of the paper is organized as follows. Section 2 gives the overview of the related work. Section 3 describes the proposed denoising method. Experimental results are given in section 4 that is followed by the conclusion in section 5.

## 2. Related Work

There are many threshold selection methods such as VisuShrink, SureShrink, and BayesShrink. The very first

time, Donoho and Johnstone gave a mechanism to find the threshold value which is known as VisuShrink [1-2]. The VisuShrink threshold is evaluated by the following expression:

$$T_{\text{Visu}} = \sigma \sqrt{2 \log M} \quad (1)$$

where  $M$  is the number of pixels in the image and  $\sigma$  is the noise variance that is defined as:

$$\sigma^2 = [(\text{median} | y(i, j) |) / 0.6745]^2 \quad (2)$$

here  $y(i, j) \in HH_1$  subband coefficients that are obtained by applying the wavelet transform to the image.

The VisuShrink has been found to yield an overly smoothed image since the estimate is derived under the constraint with high probability. The SureShrink was proposed by Donoho and Johnstone in which the above problem was overcome using the combination of both the Universal and SureShrink thresholds [3-4]. The SureShrink threshold,  $T_{\text{Sure}}$ , is defined as:

$$T_{\text{Sure}} = \min(t_j, \sigma \sqrt{2 \log M}) \quad (3)$$

where  $t_j$  represents the threshold value at  $J^{\text{th}}$  decomposition level in wavelet domain.

One of the most popular methods namely, BayesShrink was proposed by Chang et al. in which the threshold was derived from Bayesian method [5-6]. This method has better performance than the SureShrink in terms of mean square error (MSE). The BayesShrink threshold for every subband is given as follows:

$$T_{\text{Bayes}} = \frac{\sigma^2}{\hat{\sigma}_y} \quad (4)$$

where  $\hat{\sigma}_y$  is the noise free signal variance.

### 3. Proposed Denoising Method

A noisy image with additive noise is modeled by:

$$y(i, j) = x(i, j) + n(i, j) \quad 0 \leq i, j \leq M - 1 \quad (5)$$

where  $y(i, j)$ ,  $x(i, j)$ , and  $n(i, j)$  denote the observed noisy image, the unknown original image, and an independent identically distributed (i.i.d) random white Gaussian noise with zero mean and finite variance  $\sigma^2$ , respectively. Our goal is to recover  $x(i, j)$  from the noisy observation  $y(i, j)$ .

The wavelet coefficients of the noisy image after applying a wavelet transform on (5), are given by:

$$Y(i, j) = X(i, j) + N(i, j) \quad (6)$$

where  $Y(i, j)$ ,  $X(i, j)$ , and  $N(i, j)$  are the wavelet transform coefficients of  $y(i, j)$ ,  $x(i, j)$ , and  $n(i, j)$ , respectively [7-8].

#### 3.1. Parameters

VisuShrink, SureShrink and BayesShrink denoising methods sometimes blur and loose some details due to their thresholds since the constructed wavelet coefficients are smaller than their threshold values. Finding optimized value of thresholding is a major problem. A small threshold surpasses all the noisy coefficients and the resultant i.e. the denoised signal is still noisy, while a large threshold value makes more number of coefficients as zero which leads to smooth signal and destroys the details that may cause blurs and artifacts. So, we try to find out suitable threshold function by analyzing the parameters of the wavelet coefficients in each subband in order to have an optimal value.

We describe steps to shrink the noisy coefficients using soft thresholding. We compute the new threshold that takes the noisy coefficients in each subband, adaptive parameter, and noise variance of the noisy coefficients into account. We define a new threshold  $T_{\text{NEW}}$  as follows:

$$T_{\text{NEW}} = (1 - e^{-t_l}) \frac{y^2(i, j)}{t_l} \quad (7)$$

where the adaptive parameter  $t_l$  is defined for each subband at each decomposition level  $l$  as follows:

$$t_l = \sigma \left( 2 \frac{v_l}{\sigma} \right) \left( \sqrt{R(n) \left( \frac{\log \hat{M}}{l} \right)} \right) \quad (8)$$

where,  $R(n) = \left( \frac{2n}{2n+1} \right)$ , is called as the noise reduction factor;  $n$  is any positive integer i.e.  $n > 0$  and  $1 \leq l \leq J$ ;  $J$

represents the number of decomposition levels and  $\hat{M} = M/2^l$ .

The choice of  $n$  is not dependent on the scale, subband, noise, and image; it has also been observed that we have good quality of the image i.e. high PSNR for the higher values of  $n$  in case of high noise level on the average since the value of new threshold function given in (7) increases for all values of  $n$  and hence the method performs significantly better for higher noise value.

Another parameter  $V_s$  is defining by

$$V_s = \max \left( 0, \frac{\sum_{i,j=0}^{M-1} y(i, j)}{\hat{M}} - \sigma \right) \quad (9)$$

where,  $y(i, j) \in$  details subband coefficients in  $HH_l$ ,  $HL_l$ , and  $LH_l$ .

Now, we shrink the noisy coefficients using soft-thresholding method as follows:

$$\hat{x}(i, j) = \begin{cases} \text{sign}(Y(i, j)) ( |Y(i, j)| - T_{\text{NEW}} ), & |Y(i, j)| > T_{\text{NEW}} \\ 0, & |Y(i, j)| \leq T_{\text{NEW}} \end{cases} \quad (10)$$

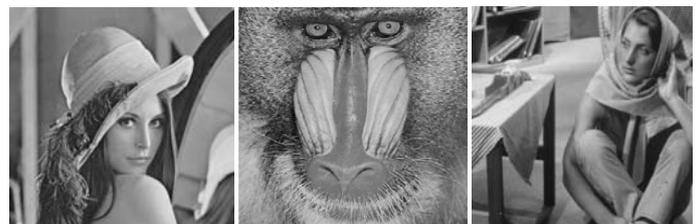
$$\text{here, sign}(x) = \begin{cases} 1 & \text{for } x > 0 \\ -1 & \text{for } x < 0 \\ 0 & \text{for } x = 0 \end{cases} \quad (11)$$

#### 3.2. Image Denoising Algorithm Structure

This section discusses the image denoising algorithm which achieves near optimal threshold in the wavelet domain for recovering the original image from the noisy one. This algorithm is quite simple to implement.

Following are the steps performed in this method.

- i. Perform multiscale decomposition on the image corrupted by Gaussian noise using 2-D wavelet transform.
- ii. Estimate the noise variance  $\sigma^2$  (i.e., robust median estimator) using (2).
- iii. For each subband (except the low pass residual)
  - a) Compute the new threshold  $T_{\text{NEW}}$  using (7).
  - b) Apply soft-thresholding using (10) to the noisy coefficients in order to get the noiseless coefficients.
- iv. Finally, apply the inverse wavelet transform to the modified coefficients to get the denoised estimate image  $\hat{x}$ .



(a)

(b)

(c)

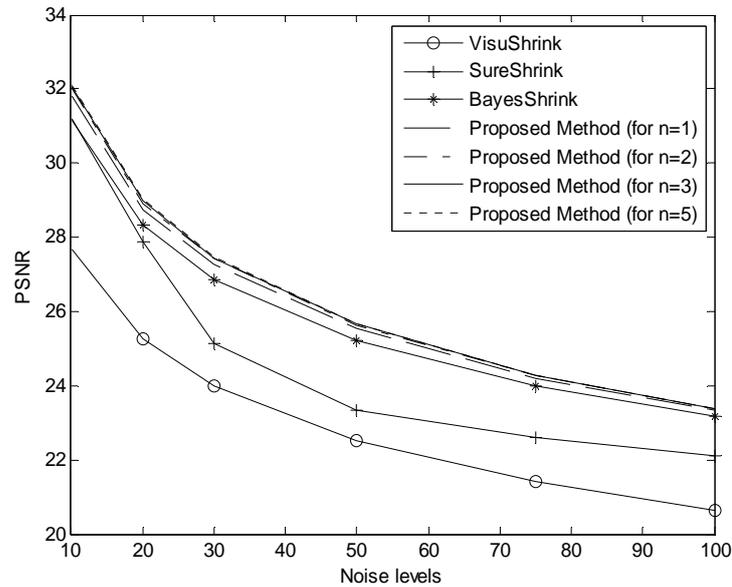


**Fig. 1.** Original test images: (a) Lena (b) Mandrill (c) Barbara (d) Goldhill and (e) Cameramen with the size 512×512

### 4. Experimental Results

In order to analyze the performance of our proposed denoising method, we take four original test images: Lena, Mandrill, Barbara, Goldhill, and Cameramen, each of size 512×512 pixels (refer **Fig. 1**). The discrete wavelet transformation (DWT) used is Symlet least asymmetric compactly supported wavelet with eight vanishing moments up to four decomposition levels [7-8, 14-15]. These images are contaminated with Gaussian noise with noise levels: 10, 20, 30, 50, 75, and 100. Our results are measured in terms of PSNR (dB). The denoised image is said to be very much closer to the original image when the PSNR is higher. We have taken the four different values of  $n$  ( $n=1, 2, 3,$  and  $5$ ) in our experiments. We have compared the experimental results of the proposed method with that of the SureShrink, VisuShrink, and BayesShrink methods.

Denoising using proposed method for  $n=1$ , (g) Denoising using proposed method for  $n=2$ , (h) Denoising using proposed method for  $n=3$ , (i) Denoising using proposed method for  $n=5$  of Goldhill

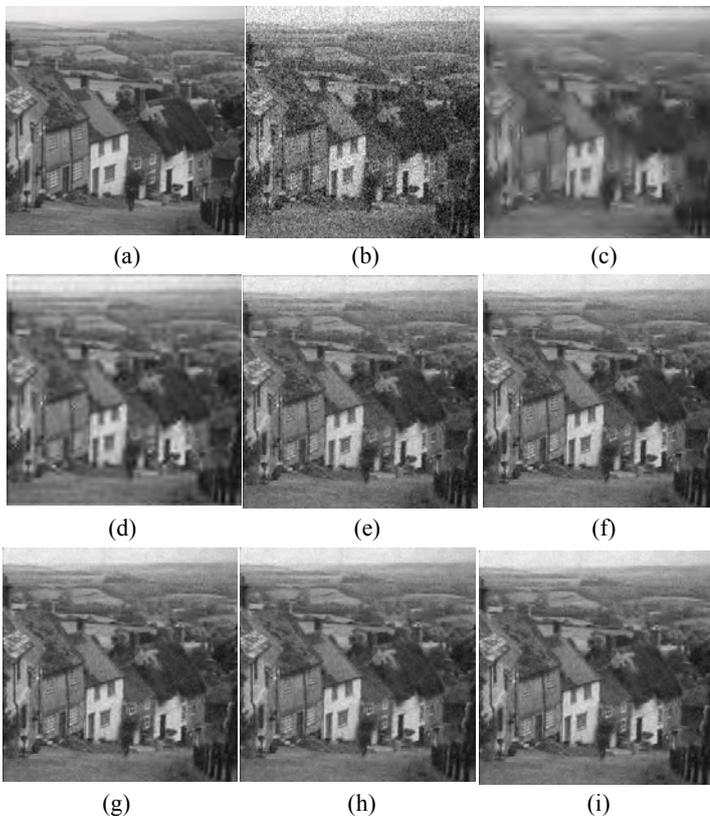


**Fig. 3.** PSNR vs. Noise level of VisuShrink, SureShrink, BayesShrink, and proposed methods with  $n=1, 2, 3,$  and  $5$  (for Goldhill)

The computed PSNR values are given in **Table 1** for four denoising methods: SureShrink, VisuShrink, BayesShrink, and proposed method. It is evident from the results given in **Table 1** that our proposed method outperforms remarkably over the VisuShrink, SureShrink, and BayesShrink for all test images. In other words, our method removes the noise praiseworthy. We have applied proposed method, VisuShrink, SureShrink, and BayesShrink methods to the Goldhill for the noise level 30. The resultant denoised images are shown in **Figs. 2(c)-(i)**. It is evident from **Figs. 2(c)-(i)** that our proposed technique produces much brighter and smoother denoised images than the VisuShrink, SureShrink, and BayesShrink. We have also shown that the PSNR values graphically for the four mentioned methods in **Fig. 3**. As it is evident from this graph, the PSNR gain of our proposed method is better than the VisuShrink, SureShrink, and BayesShrink taking  $n$  as 1, 2, 3, and 5 in our experiments. We have compared the experimental results of the proposed method with the SureShrink, VisuShrink, and BayesShrink methods. Similar results were obtained for other test images also. Because of repetitive nature, graphs for other images have been omitted.

### 5. Conclusions

In this paper, we have developed an image denoising threshold estimation method which is completely data-driven and utilizes the information about subband coefficients. This method succeeds in removing a large amount of additive noise and also preserves most of the



**Fig. 2.** (a) Original images, (b) Noisy image with noise level 30, (c) Denoising using VisuShrink, (d) Denoising using SureShrink, (e) Denoising using BayesShrink, (f) Denoising using proposed method for  $n=1$ , (g) Denoising using proposed method for  $n=2$ , (h) Denoising using proposed method for  $n=3$ , (i) Denoising using proposed method for  $n=5$  of Goldhill

edges and visual quality of the image. Our proposed method gives better performance than the VisuShrink, SureShrink, and BayesShrink.

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**Table 1.** PSNR results (in db) for test images: Lena, Mandrill, Barbara, Goldhill and Cameraman with noise levels: 10, 20, 30, 50, 75, and 100

Image Name	Noise levels	Visu Shrink	Sure Shrink	Bayes Shrink	Proposed Method			
					n=1	n=2	n=3	n=5
Lena	10	29.34	30.96	31.34	32.78	33.04	33.13	33.21
	20	26.40	26.43	29.09	29.92	30.17	30.27	30.34
	30	24.83	25.21	27.73	28.39	28.61	28.68	28.74
	50	23.00	23.84	25.91	26.48	26.60	26.64	26.67
	75	21.66	22.95	24.57	24.88	24.94	24.95	24.96
	100	20.76	22.38	23.67	23.73	23.77	23.78	23.78
Mandrill	10	23.74	27.76	27.75	30.03	29.95	29.92	29.89
	20	21.21	24.43	24.54	25.90	25.84	25.81	25.79
	30	20.20	21.57	22.79	23.81	23.75	23.72	23.70
	50	19.34	19.62	21.02	21.70	21.65	21.62	21.60
	75	18.84	19.29	20.07	20.50	20.46	20.45	20.43
	100	18.51	19.09	19.54	19.85	19.83	19.82	19.82
Barbara	10	26.40	29.33	29.22	32.15	32.30	32.36	32.40
	20	23.39	24.13	26.36	28.37	28.47	28.50	28.53
	30	22.07	22.28	24.66	26.23	26.26	26.26	26.26
	50	20.75	21.32	22.84	23.83	23.82	23.81	23.80
	75	19.80	20.72	21.84	22.37	22.37	22.36	22.35
	100	19.11	20.33	21.22	21.48	21.48	21.47	21.47
Goldhill	10	27.67	31.20	31.17	31.83	32.00	32.06	32.11
	20	25.25	27.86	28.33	28.75	28.91	28.97	29.01
	30	23.99	25.12	26.85	27.28	27.41	27.45	27.49
	50	22.51	23.32	25.21	25.55	25.63	25.65	25.67
	75	21.41	22.58	24.01	24.21	24.26	24.28	24.29
	100	20.64	22.10	23.18	23.32	23.37	23.38	23.39
Cameraman	10	27.96	31.13	30.78	32.33	32.58	32.67	32.74
	20	24.93	27.71	27.87	29.13	29.33	29.39	29.45
	30	23.39	24.10	26.04	27.35	27.48	27.53	27.56
	50	21.62	22.20	24.04	25.27	25.33	25.35	25.36
	75	20.32	21.18	22.78	23.56	23.56	23.56	23.55
	100	19.44	20.55	21.93	22.28	22.26	22.24	22.23

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