

## WHITE AREAS ON THE MAP OF APPLYING NON-EQUILIBRIUM THERMODYNAMICS: ON THE SELF ACCELERATING ELECTRON

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ABSTRACT. Classical electrodynamics displays a formula for the equation of the motion of charged particles that has run away solutions. Gyarmati's wave approach leads to a solution fitting to the classical term in the low frequency regime. A shortage of the solution is that the characteristic time in the equation depends on the mass of the charged particle. An abstract model with two dynamic degrees of freedom results in a solution free of the above problem. For complete compatibility with electrodynamics, the need for a generalization of Maxwell's equations is probable.

### 1. Introduction

Non-equilibrium thermodynamics has been applied with success to a lot of phenomena even in the form that is referred to nowadays as “classical irreversible thermodynamics”—in the form that was accepted just after the first papers of Onsager [1–4] and is presented in the classical monographs [5–8]. It concerned the principle of local equilibrium. The latter turned out to be too tight and was generalized [9–14]. The departure from local equilibrium was fruitful and opened a broad perspective of applications [15–25]. Nevertheless, the variety of the explored fields non-equilibrium thermodynamics applies to is large there are lots of white area on the map. One of them concerns electromagnetic radiation.

Classical electrodynamics results a formula for the dipole radiation leading to violation of the second law of thermodynamics. The emitted energy is given by

$$P = \frac{\mu_0}{6\pi c} \langle \ddot{\mathbf{p}}^2 \rangle, \quad (1)$$

where  $P$  is the radiated power,  $\mu_0$  the magnetic permeability of the vacuum,  $c$  the velocity of light,  $\ddot{\mathbf{p}}$  the second time derivative of the dipole moment, and  $\langle \dots \rangle$  denotes time average. The SI units are used; Maxwell's equations read

$$\begin{aligned} \operatorname{rot} \mathbf{E} + \dot{\mathbf{B}} &= 0, & \operatorname{div} \mathbf{B} &= 0, \\ \operatorname{rot} \mathbf{H} &= \mathbf{j} + \dot{\mathbf{D}}, & \operatorname{div} \mathbf{D} &= \rho_e, \\ \mathbf{D} &= \varepsilon_0 \mathbf{E}, & \mathbf{B} &= \mu_0 \mathbf{H}. \end{aligned} \quad (2)$$

The usual equation of motion derived from eq. (1) for a classical point charge

$$m\dot{\mathbf{v}} = \mathbf{F} + \frac{\mu_0 e^2}{6\pi c} \ddot{\mathbf{v}} \quad (3)$$

leads to run away solution; rest or rectilinear uniform motion is not stable. The calculations leading from Maxwell's equations to equation of motion (3) via eq. (1) is complicated and make use of several hypothesis and, usually, approximations. The solution of the discrepancy was sought with several argumentation; the shape of the charge, relativistic effects, and quantum effects were considered. Relativistic effects can not eliminate the problem as the instability of rest appears even if the motion is slow, on the other hand, formula (1) is valid for macroscopic bodies; quantum considerations are not probable to help, at least, classical and macroscopic considerations have to display any solution for the contradiction. The shape of the charge has no effect on the dipole approximation and any charge distribution from far enough can be approximated well by dipole for any required accuracy. For the latter reason the shape of the charge can modify the equation of motion but with a powerless force. The acceptable solutions argued with irreversibility. The present author [26] regarded the moving charge and the electromagnetic field radiated by it as a thermodynamic system and applied Gyarmati's wave approach [9]. The equation of motion

$$\tau \dot{\mathbf{F}} + \mathbf{F} = m\dot{\mathbf{v}} \quad (4)$$

was obtained. As the considerations started from the first and the second law of thermodynamics run away solutions were excluded a priori. For agreement with the classical radiation formula in the low frequency regime,

$$\tau = \frac{\mu_0 e^2}{6\pi c m} \quad (5)$$

must hold. The same equation was obtained by G. W. Ford and R. F. O'Connell [27–31] assuming also irreversibility; they assumed interaction with a “blackbody radiation field”. The formula (4) is not satisfactory. The main deficiency is that  $\tau$  depends on the mass of the particle, whereas it ought to depend only on the charge. Interaction with a “blackbody radiation field” is not probable as  $\tau$  must not depend on temperature unless formula (5) is dropped. Onsager's thermodynamics supplemented with dynamic variables gives better models.

## 2. Thermodynamic models of moving charges.

To do with the problem chalked out above assume a macroscopic body with charge. The thermodynamic system is the body and a proper part of the electromagnetic field around. The essential properties of the system are its energy, and some others that are determined by the energy if at rest but if moving. Suppose that all properties can be given by state variables, one of which is the energy  $U$ . The first law now reads

$$\dot{U} = I_q + \mathbf{F}\mathbf{v}. \quad (6)$$

The entropy depends on  $U$  and some independent vector variables  $\xi_i$ . The  $\xi$  variables can be chosen several way, but a special choice is comfortable. It is based

on the Morse–lemma [32], the second law, and that the entropy in equilibrium is a monotonous increasing function of energy. The entropy function is given as

$$S = S \left( U - \frac{1}{2} \sum_{i=1}^n \xi^2 \right). \quad (7)$$

The balance of entropy is

$$\dot{S} = \frac{1}{T} I_q + P_s, \quad P_s > 0. \quad (8)$$

If we do not want to waste time for studying the intricate details of heat exchange we had better adopt

$$\frac{1}{T} = \frac{\partial S}{\partial U}. \quad (9)$$

Combining the above relations, we get the correlation

$$TP_s = \mathbf{F}\mathbf{v} - \sum_{i=1}^n \xi \dot{\xi} \quad (10)$$

for the energy dissipated in unite time; from it, Onsager's equations are obtained. They read

$$\begin{aligned} \mathbf{F} &= R_{00}\mathbf{v} + \sum_{k=1}^n R_{0k}\dot{\xi}_k \\ -\xi_i &= R_{i0}\mathbf{v} + \sum_{k=1}^n R_{ik}\dot{\xi}_k \quad \{i = 1, 2, \dots, n\}. \end{aligned} \quad (11)$$

The coefficients satisfy the Onsager–Casimir reciprocal relations. If we suppose that all  $\xi_i$  is reversed with time inversion, i.e., they are of  $\beta$ -type the Onsager–Casimir reciprocal relations read

$$R_{0i} = -R_{i0}, \quad R_{ik} = R_{ki}. \quad (12)$$

As a consequence of the second law the coefficients in the main diagonal are positive;

$$R_{00} > 0, \quad R_{ii} > 0. \quad (13)$$

The Morse–lemma does not determines the choice of the dynamic state variables even in the linear approximation; an orthogonal transformation leaves the form of the entropy function (7) invariant. This fact makes possible to transform the matrix of the  $R_{ik}$  coefficients into main diagonal and Onsager's equations turn into

$$\begin{aligned} \mathbf{F} &= R_{00}\mathbf{v} + \sum_{k=1}^n R_{0k}\dot{\xi}_k \\ -\xi_i &= R_{i0}\mathbf{v} + R_{ii}\dot{\xi}_i \quad \{i = 1, 2, \dots, n\}. \end{aligned} \quad (14)$$

To make the content of the equations transparent introduce the new variables

$$\beta_i = R_{0i}\xi_i \quad \{i = 1, 2, \dots, n\} \quad (15)$$

and substitute into eqs. (14). We get

$$\mathbf{F} = R_{00}\mathbf{v} + \sum_{i=1}^n \dot{\boldsymbol{\beta}}_i \quad (16)$$

$$R_i \dot{\boldsymbol{\beta}}_i + \boldsymbol{\beta}_i = R_{i0}^2 \mathbf{v}. \quad (17)$$

The first term on the right hand side of the eq. (16) is obviously drag, which tends to zero if inertial motion is possible. The quantities  $\boldsymbol{\beta}_i$  are parts of the linear momentum, each of which follows the velocity with relaxation as eqs. (17) show. The quantities  $R_i$  are relaxation times and  $R_{i0}^2$  parts of the mass. Introducing new notations

$$\mathbf{I} = \sum_{i=1}^n \boldsymbol{\beta}_i, \quad \tau_i = R_i, \quad m_i = R_{i0}^2, \quad (18)$$

and dropping the drag eqs. (16) and (17) turn to

$$\mathbf{F} = \dot{\mathbf{I}} \quad (19)$$

$$\tau_i \dot{\boldsymbol{\beta}}_i + \boldsymbol{\beta}_i = m_i \mathbf{v}. \quad (20)$$

The quantity  $\mathbf{I}$  is, obviously, the linear momentum and eq. (19) is Newton's second law. The parts of the linear momentum— $\boldsymbol{\beta}_i$ —follow the velocity changes with some delay.

Each  $\boldsymbol{\beta}_i$  is modeled by a mass with a viscously stretching handle. The whole is a bunch of masses with common handle but with individual stretching bounds as shown with three elements in figure 1.

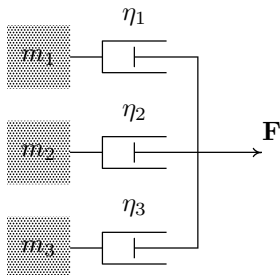


Figure 1.

To get a simpler model, we suppose that the relaxation times  $\tau_i$  are short and negligible except the first. It means that the coefficients  $R_i$  in eqs. (20) are equal to zero except the first. If so, the  $\boldsymbol{\beta}_i$   $\{i = 2, 3, \dots, n\}$  are proportional; the model contains two independent dynamic variables  $\boldsymbol{\xi}_0$  and  $\boldsymbol{\xi}_1$ , or equivalently  $\boldsymbol{\beta}_0$  and  $\boldsymbol{\beta}_1$  for which the equations are

$$\mathbf{I} = m_0 \mathbf{v} + \boldsymbol{\beta}_1, \quad \tau_1 \dot{\boldsymbol{\beta}}_1 + \boldsymbol{\beta}_1 = m_1 \mathbf{v}, \quad (21)$$

where

$$m_0 = \sum_{i=2}^n m_i.$$

Eliminating  $\beta_1$  from the equations results

$$\tau_1 \dot{\mathbf{I}} + \mathbf{I} = m(\tau_2 \dot{\mathbf{v}} + \mathbf{v}), \quad (22)$$

where the notations

$$m = m_0 + m_1, \quad \tau_2 = \tau_1 \frac{m_0}{m_0 + m_1}$$

have been introduced. The model in figure 1 reduces to the simpler one shown in figure 2.

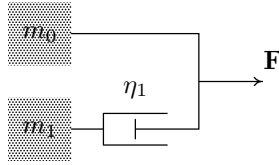


Figure 2.

Figure 3 shows the same model but it may be easier to understand.

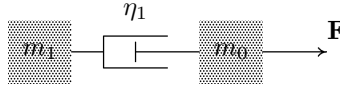


Figure 3.

The amount of the dissipated energy is calculated from eq. (10);

$$TP_s = \frac{(\tau_1 \dot{\mathbf{I}} - \tau_2 m \dot{\mathbf{v}})^2}{\tau_1 - \tau_2}, \quad (23)$$

which turns into

$$\langle TP_s \rangle = \frac{(\tau_1 - \tau_2)m}{1 + \tau_1^2 \omega^2} \langle \dot{\mathbf{v}}^2 \rangle \quad (24)$$

for harmonic oscillation. The denominator is approximately one for low frequency; the result compares to the classical formula (1) if and only if

$$(\tau_1 - \tau_2)m = \tau_1 m_1 = \frac{\mu_0 e^2}{6\pi c}. \quad (25)$$

This formula is nearly the same as eq. (5) derived from eq. (4) but not the whole mass of the particle has to be put into the denominator;  $m_1$  is only some part of it. Thus  $\tau_1$  may be independent of the mass of the particle. One can say that  $m_1$  is the “electromagnetic mass”, nevertheless, it may be only a part of it.

Now turn attention to the dissipated and the radiated energy. Introducing (25) into eq. (24) and the dipole moment of an oscillating charge into eq. (1), we get

$$\langle TP_s \rangle = \frac{\mu_0 e^2}{6\pi c(1 + \tau_1^2 \omega^2)} \langle \dot{\mathbf{v}}^2 \rangle, \quad P = \frac{\mu_0 e^2}{6\pi c} \langle \dot{\mathbf{v}}^2 \rangle. \quad (26)$$

The comparison results a very bad discrepancy. The radiated power is bigger than the dissipated one, which is nonsense. This shortcoming has to be eliminated.

In the formulae (26) the symbol  $\mathbf{v}$  stands for the velocity of the handle of the particle in the formula of energy dissipation— $m_0$  in figure 3—and for the velocity of the charge in the formula for the radiated energy. If the charge is assumed to move

together with the field— $m_1$  in figure 3—the agreement is perfect. This assumption is rather trivial, if charges are regarded as the singularities of the electromagnetic field. On the other hand, the charged particle can be driven by electromagnetic forces and one can hardly find an explanation why the force acts on the handle.

The thermodynamic model has eliminated the runaway solution, nevertheless a new contradiction emerged; the discrepancy has changed dress. To touch Maxwell's equations seems unavoidable, but the requirements are hard. First of all, electrodynamics is the most successful framework of theoretical physics and the results are verified immeasurably many times in practice, moreover, 20th century's physics is rather based on it. On the other hand, an irreversible electrodynamics is really attractive and challenging. E.g., a semitransparent vacuum could explain—within the equations—why the retarded potential is exclusively used for the radiation of an aerial in infinite space, moreover, it would give an alternative and natural explanation why the sky in the night is dark.

For such a generalization, more complicated thermodynamic models have to be assumed. The  $\beta$ -type variables alone may be insufficient and Onsager's equations for a system with both  $\alpha$ - and  $\beta$ -type dynamic variables, are less transparent and studying them needs help from other branches of science.

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