

## NEAR FIELD COMPUTATION OF THE EXTINCTION OF ELECTROMAGNETIC WAVES IN MULTIPARTICLE SYSTEMS

JAN SCHÄFER<sup>a,\*</sup> AND ALWIN KIENLE<sup>a</sup>

**ABSTRACT.** In this contribution extinction of electromagnetic waves inside a medium consisting of cylindrical absorbing particles is considered. Near fields are calculated using a numerical solution of Maxwell's equations and compared to results given by Lambert-Beer's law.

### 1. Introduction

The transmittance of light passing through an absorbing material can be described by Lambert-Beer's law which is widely used for various applications [1]. Lambert-Beer's law is valid for far field investigations of the transmittance of light through absorbing media where the width of the incident light beam is small compared to the illuminated sample. One constraint of Lambert-Beer's law is the fact that it is not applicable for strongly scattering media. In this conference contribution the validity of Lambert-Beer's law is investigated in the near field for samples which are small compared to the width of the incident light beam. Therefore, the extinction of a plane electromagnetic wave during the passage through a small slab consisting of absorbing particles dispersed in a bulk medium has been calculated in the near field using a numerical solution of Maxwell's equations.

### 2. Method

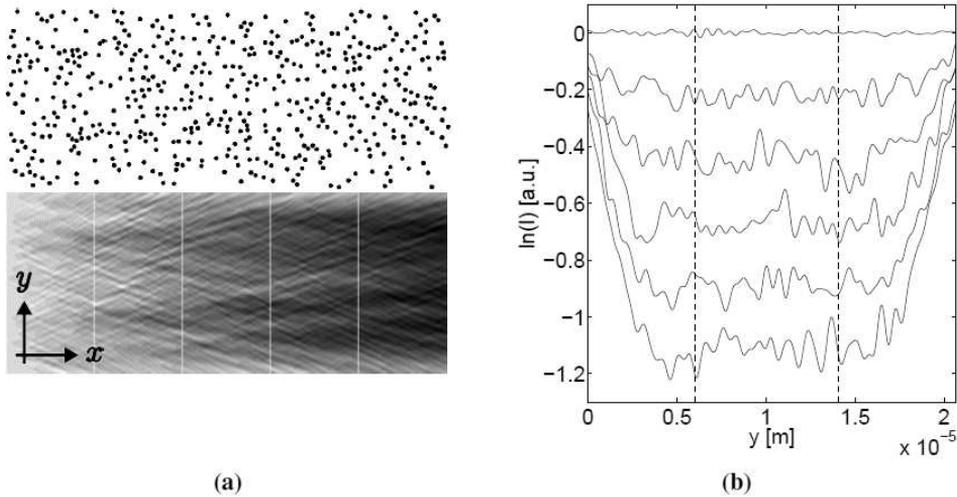
Lambert-Beer's law can be formulated for light with an intensity  $I_0$  passing through a medium of thickness  $x$  and consisting of monodispersive absorbing particles as follows:

$$I = I_0 e^{-\mu_a x}, \quad (1a)$$

$$\mu_a = f_v \frac{C_{abs}}{V_s}. \quad (1b)$$

The absorption coefficient  $\mu_a$  is calculated using the volume fraction  $f_v$  of the particles, as well as the absorption cross section  $C_{abs}$  and the volume  $V_s$  of a single particle. Equation (1) is also applicable for media consisting of infinite parallel cylindrical scatterers in a bulk medium at perpendicular incidence.

For the calculation of the intensity distribution in the near field we utilized a self-implemented two-dimensional finite-difference time-domain (FDTD) method. A pulsed



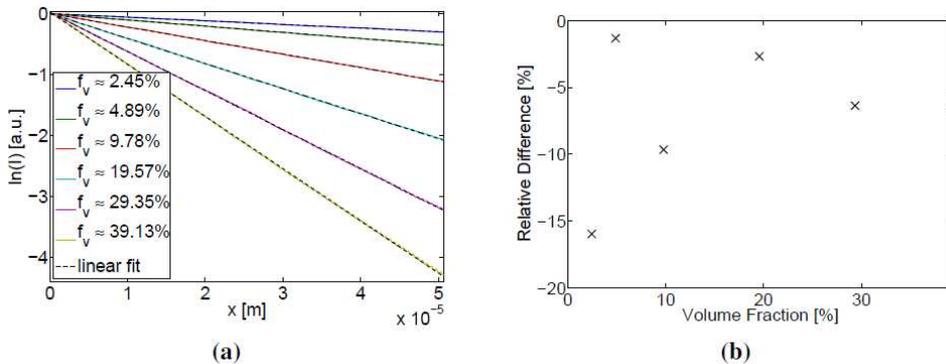
**Figure 1.** (a) Near field intensity distribution (shown below) of light propagating (coming from the left) through a slab of randomly distributed cylindrical scatterers (shown above). (b) Averaged near field results for different slices perpendicular to the propagation direction.

source field is generated using the total-field/scattered-field (TFSF) approach which allows gathering of results for multiple monochromatic wavelengths in a single simulation run [2]. In this approach near field is separated in an inner total field region containing the scatterers, and in an enclosing scattered field region [3]. The scattered field is important for far field computation, if the near field is considered the scattered field is superposed with the incident field which gives the total field as relevant quantity. The simulation region is surrounded by perfectly matched layer (PML) absorbing boundary conditions [3] in order to prevent confusing reflections at the borders. In the FDTD method pulse propagation in the simulation model is computed time-resolved using a leap-frog iteration scheme [3]. For the calculation of the near field solution at a desired wavelength, discrete Fourier transforms have to be performed for the time-dependent total field solutions over the whole simulation time. FDTD results have been validated using the near field solution based on the analytical theory of scattering of electromagnetic waves by an infinite cylinder [4].

Simulation models composed of randomly positioned cylinders were generated using a Metropolis Monte-Carlo method [5]. Starting from an ordered distribution of a specific number of cylinders in a pre-defined area, cylinder positions are subsequently randomly moved in an iterative procedure. After an adequate number of iterations an uncorrelated random cylinder distribution is achieved.

### 3. Results

In this section results of near field extinction for light propagating in a medium consisting of multiple absorbing infinitely long cylinders are presented. The cylinder center



**Figure 2.** (a) Averaged intensity distribution plotted against the propagation direction for different concentrations. (b) Relative difference of the absorption coefficient obtained by Maxwell and Lambert-Beer solutions.

positions were randomly distributed in a  $20 \times 50 \mu\text{m}^2$  area. The narrow side was illuminated by a plane electromagnetic wave of wavelength  $\lambda = 600 \text{ nm}$ . The diameter of the cylinders was set to  $d = 0.5 \mu\text{m}$ . The relative refractive index of the cylinders compared to the surrounding non-absorbing medium was assumed to be  $m = 1 + 0.01i$ . Scattering of the particles was minimized by matching the real part of the refractive index of the cylinders to that of the outer media. However, scattering is also induced by a difference in the imaginary part of the refractive index as well. This can be seen when calculating the light scattering by a single cylinder which results in a large absorbing cross section of  $C_{abs} = 3.938 \cdot 10^{-8} \text{ m}$  but also a small scattering cross section of  $C_{sca} = 8.115 \cdot 10^{-10} \text{ m}$  ( $C_{ext} = 4.019 \cdot 10^{-8} \text{ m}$ ).

Different concentrations of the cylindrical particles have been considered. The near field results for a selected cylinder distribution are shown in Figure 1a for a volume fraction of about  $f_v \approx 9.8\%$ . A decay of light intensity can be observed along the propagation direction. Intensity distribution is dependent on particle position and scattering occurs, particularly, at locations characterized by high local cylinder concentrations.

For better comparison a random medium has to be emulated by averaging the results over many simulations for different random cylinder distributions. In Figure 1b the natural logarithm of the intensity in different slices perpendicular to the propagation direction and averaged over 10 random distributions are shown. At the top the intensity distribution of the light is shown before entering the medium and at the bottom the intensity after leaving the medium. In between, slices at positions as depicted by vertical white lines in Figure 1a are shown. Intensity is to a greater or lesser extent equal in the middle part of the slices and increasing at the borders. These border effects become stronger as light propagates further through the sample. It is assumed that Lambert-Beer's law will only be valid inside the sample where border effects are not present.

For the comparison with Lambert-Beer's law intensity results were averaged along the y-axis and border values were neglected (only intensity inside the region depicted by

dashed vertical black lines in Figure 1b was averaged). The natural logarithm of the averaged intensity is plotted against the propagation direction in Figure 2a. A linear decrease can be observed, results of linear fits are also shown. The slope of the fitted curves is proportional to the absorption coefficient when considering equation (1a). Fitted solutions were compared to theoretical results using equation (1b) for the calculation of the absorption coefficient. The relative error of the absorption cross section obtained by fitting the Maxwell results in comparison to the absorption cross section of Lambert-Beer's law is shown in Figure 2b. For most concentrations the absorption cross sections can be evaluated using numerical Maxwell solutions with less than 10 % deviation from those predicted by Lambert-Beer's law. For small concentrations larger deviations are observed. This can be explained by a retarded onset of absorption. For low density cylinder distributions it is more likely that the first cylinder is reached by the incoming light wave after a certain passage of undisturbed propagation which distorts the fitted curves.

#### 4. Conclusion

Near field intensity distribution of light propagating through a medium containing cylindrical scatterers has been calculated applying a numerical solution of Maxwell's equations. Results have been averaged over multiple random cylinder distributions to emulate random media and compared to results of Lambert-Beer's law. Border effects have been observed which falsify results, but good conformance is achieved when neglecting the border regions.

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<sup>a</sup> Institut für Lasertechnologien in der Medizin und Medizintechnik  
Helmholtzstr. 12  
89081 Ulm, Germany

\* To whom correspondence should be addressed | Email: jan.schaefer@ilm.uni-ulm.de

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