

EMISSIVE EXCHANGE BETWEEN PARTICLES

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ABSTRACT. A formulation is developed to predict the exchange of emission between discrete points in a particle and between neighboring particles. The formulation relies on the volume integral equation for time harmonic fields, coupled to a VSWF, T matrix representation.

1. Introduction

An emerging topic, in the general field of radiative transport, is that related to "nanoscale" heat transfer. In particular, it has been recognized that near-field coupling of electromagnetic (EM) fields – via a process often referred to as photon tunneling – can significantly augment the rate of thermal emission between two objects when the distance separating the objects becomes on the order of nanometers[1]. In the past, prediction of near-field emission has employed the fluctuation–dissipation theory of Rytov, in which the emitted EM field is modeled as that produced by phase–uncorrelated, randomly oriented dipole sources within the emitting medium [2]. Application of the theory to a specified geometrical system, e.g., a sphere or a pair of spheres, involves solution of time harmonic yet inhomogeneous Maxwell's equations for the system, and the inhomogeneity of the problem – which accounts for the emissive source function – results in a solution that is more complex than that for the corresponding homogeneous problem, i.e., the conventional plane wave scattering problem for the system [3].

The purpose of the presented work is to demonstrate that the volume integral formulation (VIE), coupled with a T matrix representation, provides sufficient information to predict emissive exchange, on both a point–to–point and a particle–to–particle basis. The work is an extension and a generalization of our previous formulation for emissive exchange between spheres [4].

2. Formulation

The space constraints of the abstract prevent a complete and detailed mathematical exposition, and we will attempt to convey the salient features of the formulation. The foundation of our formulation lies in the volume integral equation (VIE) representation of EM

scattering from particles [5, 6]. We will use the VIE to 1) develop the basic T matrix relationships for a particle, and 2) identify cross sections describing the exchange of emissive energy among different points within the same particle, and among different particles.

The VIE formulation is made more convenient for the problem at hand by transforming it so that the cartesian vector components (x, y, z) of the relevant quantities (fields, dyadic operators) are replaced by dipole azimuthal degree components $(-1, 0, 1)$. By doing so, the field external to the particle, denoted as $\mathbf{E}(\mathbf{r})$, resulting from an exciting field $\mathbf{E}_{exc}(\mathbf{r})$, can be described by a vector spherical wave function (VSWF) expansion,

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{exc}(\mathbf{r}) + a_\mu \mathbf{N}_\mu^{(3)}(\mathbf{r}), \quad \mathbf{r} \in V_{ext,C} \quad (1)$$

$$a_\mu = \int_{V_{int}} J_{\mu m}(-\mathbf{r}') \int_{V_{int}} T_{m k}^{(2)}(\mathbf{r}', \mathbf{r}'') \mathbf{P}_k^* \cdot \mathbf{E}_{exc}(\mathbf{r}'') d^3 \mathbf{r}'' d^3 \mathbf{r}' \quad (2)$$

in which $\mathbf{N}^{(3)}$ is the outgoing VSWF, J is the regular translation matrix for the VSWF, and $T_{m k}^{(2)}(\mathbf{r}, \mathbf{r}')$ is the two-point T matrix, which is basically the azimuth-transformed dyadic transition operator. In the above and what follows, Greek subscripts are shorthand for the triplet of degree, order, and mode ($= 1$, TM, $= 2$, TE), and the Roman subscripts m and k (and their primes) imply the dipole-level harmonics, for which order $= 1$, mode $=$ TM, and $m, k = -1, 0, 1$. A tensorial convention is also adopted, for which summation over subscripts not appearing in the left-hand-side of the equation is implied. The quantity \mathbf{P}_k is a cartesian vector which performs the cartesian-to-azimuthal degree transformation; the 3×3 matrix formed by $(\mathbf{P}_{-1}, \mathbf{P}_0, \mathbf{P}_1)$ is unitary. The regions V_{int} and $V_{ext,C}$ refer the points within the particle and points outside of the circumscribing sphere surrounding the particle.

The two-point T matrix is a solution to the VIE

$$\frac{1}{\alpha} T_{m k}^{(2)}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \delta_{m-k} + \int_{V_{int}} H_{m m'}(\mathbf{r} - \mathbf{r}'') T_{m' k}^{(2)}(\mathbf{r}'' \mathbf{r}') d^3 \mathbf{r}'', \quad \mathbf{r}, \mathbf{r}' \in V_{int} \quad (3)$$

$$\alpha = \frac{i k^3}{6\pi} (m^2 - 1) \quad (4)$$

where m is the particle refractive index and $H_{m k}(\mathbf{r} - \mathbf{r}')$ is the outgoing translation matrix for dipole-dipole interaction; this quantity results from the azimuth transformation of the dyadic Green's function.

When the exciting field originates from sources located solely in $V_{ext,C}$ – as would be the case for a plane wave – it is possible to represent the exciting field for all points $\mathbf{r} \in V_{int}$ as a regular VSWF expansion centered about the particle origin. For this case, application of the VSWF translation theorem to Eq. (1) will show that

$$a_\mu = T_{\mu \nu} f_\nu \quad (5)$$

$$T_{\mu \nu} = \int_{V_{int}} J_{\mu m}(-\mathbf{r}) \int_{V_{int}} T_{m k}^{(2)}(\mathbf{r}, \mathbf{r}') J_{k \nu}(\mathbf{r}') d^3 \mathbf{r}' d^3 \mathbf{r} \quad (6)$$

$$= \int_{V_{int}} J_{\mu m}(-\mathbf{r}) T_{m \nu}^{(1)}(\mathbf{r}) d^3 \mathbf{r} \quad (7)$$

where f_ν denote the expansion coefficients for the exciting field. The above formulas define two additional operator matrices, being the one point matrix $T_{m\nu}^{(1)}(\mathbf{r})$ and the standard $T_{\mu\nu}$ matrix, via successive integrations of the two point matrix (i.e., the transition dyad) over particle volume. In this respect, the T matrix for the particle can be viewed simply as an integral transform of the operator-based solution to the VIE.

By using the $T^{(1)}$ and T formulas in Eq. (3), and applying the properties of the translation matrices, it is possible to derive the important energy conservation statement of

$$C_{ext} = C_{abs} + C_{sca} \quad (8)$$

$$C_{ext} = -\frac{2\pi}{k^2} \operatorname{Re} T_{\mu\mu}, \quad C_{sca} = \frac{2\pi}{k^2} |T_{\mu\nu}|^2 \quad (9)$$

$$C_{abs} = -\frac{2\pi}{k^2} \operatorname{Re} \left(\frac{1}{\alpha} \right) \int_{V_{int}} \left| T_{m\mu}^{(1)}(\mathbf{r}) \right|^2 d^3\mathbf{r} \quad (10)$$

When the particle is non absorbing, for which $\operatorname{Re} \alpha = 0$, the T matrix will be unitary.

An additional, point-level energy conservation statement can be obtained, again by starting with Eq. (3). This results in

$$C'_{sor}(\mathbf{r}) = C'_{emi}(\mathbf{r}) + \int_{V_{int}} C''_{xch}(\mathbf{r}, \mathbf{r}') d^3\mathbf{r}', \quad \mathbf{r} \in V_{int} \quad (11)$$

$$C'_{sor}(\mathbf{r}) = \frac{2\pi}{k^2} \operatorname{Re} \left(\frac{1}{\alpha} \right) \operatorname{Re} T_{mm}^{(2)}(\mathbf{r}, \mathbf{r}), \quad C'_{emi}(\mathbf{r}) = -\frac{2\pi}{k^2} \operatorname{Re} \left(\frac{1}{\alpha} \right) \left| T_{m\mu}^{(1)}(\mathbf{r}) \right|^2 \quad (12)$$

$$C''_{xch}(\mathbf{r}, \mathbf{r}') = \frac{2\pi}{k^2} \left[\operatorname{Re} \left(\frac{1}{\alpha} \right) \right]^2 \left| T_{mk}^{(2)}(\mathbf{r}, \mathbf{r}') \right|^2 \quad (13)$$

Equation (11) can be interpreted as a spectral emissive energy balance for points within an isothermal particle. When multiplied by the spectral blackbody function, the source density function $C'_{sor}(\mathbf{r})$ (units of area/volume) would describe the emissive power density from point \mathbf{r} . Part of this energy is absorbed at all other points in the particle, and the remainder escapes the particle; these two components are represented by the two terms on the right hand side. The emission cross section of the particle is obtained from the volume integral of $C'_{emi}(\mathbf{r})$, and per Eq. (10), is equal to the absorption cross section.

Via the principle of detailed balancing, we submit that the exchange and emission density functions could be used to describe the net transfer of emitted EM energy in a non-isothermal particle. For the case of a cold environment (i.e., no external fields), the net spectral transfer of energy at point \mathbf{r} , per unit volume, would be described by

$$Q'_\lambda(\mathbf{r}) = \left(\int_{V_{int}} C''_{exc}(\mathbf{r}, \mathbf{r}') (I_{b\lambda}(\mathbf{r}') - I_{b\lambda}(\mathbf{r})) d^3\mathbf{r}' + C'_{emi}(\mathbf{r}) I_{b\lambda}(\mathbf{r}) \right) d\lambda \quad (14)$$

where $I_{b\lambda}(\mathbf{r})$ denotes the spectral blackbody intensity evaluated at the local temperature of point \mathbf{r} . When integrated over volume ($d^3\mathbf{r}$), the term involving the exchange function will cancel out, and the net emission from the particle would be obtained from the volume integral of $C'_{emi}(\mathbf{r}) I_{b\lambda}(\mathbf{r})$.

The T matrix formulation enables a straightforward extension of the analysis to multiple, interacting particles. Providing the particles are separated by at least their circumscribing radii – so that a T matrix formulation can be used to describe the scattered field from

each particle – the starting point for analysis is the set of multipole interaction equations that describe the scattered field coupling among the particles. The equations are analogous to Eq. (3), and appear as

$$T_{\mu\nu}^{(2)}(i, j) = T_{\mu\mu'}(i) \left(\delta_{i-j} \delta_{\mu'-\nu} + \sum_{i'} H_{\mu'\nu'}(i - i') T_{\nu'\nu}^{(2)}(i', j) \right) \quad (15)$$

where $T_{\mu\nu}(i)$ is the T matrix for particle i (i.e., Eq. (6) for particle i) and $T_{\mu\nu}^{(2)}(i, j)$ relates the scattering coefficients for particle i due to an incident field at particle j . And as before, source and exchange cross sections can be derived from energy conservation principles applied to Eq. (15), and are given by

$$C_{sor}(i) = \frac{2\pi}{k^2} D_{\mu\nu}(i) \operatorname{Re} T_{\nu\nu}^{(2)}(i, i) \quad (16)$$

$$C_{exc}(i, j) = \frac{2\pi}{k^2} D_{\mu\nu}(i) D_{\mu\nu'}(j) \left| T_{\nu'\nu}^{(2)}(i, j) \right|^2 \quad (17)$$

where

$$D_{\mu\nu}(i) = -\operatorname{Re} \left([T(i)]_{\mu\nu}^{-1} + \delta_{\mu-\nu} \right) \quad (18)$$

For isothermal particles, $C_{sor}(i) I_{b\lambda}(T_i) d\lambda$ will be the spectral emitted energy leaving particle i , and $C_{exc}(j, i) I_{b\lambda}(T_i) d\lambda$ will be the amount of this energy that is absorbed by particle j .

Issues related to the calculation of the exchange cross sections, as well as an examination of the effects of field coupling among closely-spaced particles, will be presented at the meeting.

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