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APPLICATION OF THE VECTORIAL COMPLEX RAY MODEL TO THE SCATTERING OF AN ELLIPSOID PARTICLE

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ABSTRACT. We have developed a novel model – Vectorial Complex Ray Model (VCRM) – for the scattering of a smooth surface objet of arbitrary shape [Opt. Lett. 36(1)]. In this model, the wave is described by bundles of rays, and a ray is characterized not only by its direction and amplitude but also the curvature and the phase of the wave. These new properties allows to take into account the phase shift due to the focal lines of an arbitrary shaped wave and the amplitude due to the divergence of the wave. The interferences can therefore be calculated correctly for an arbitrarily shaped particle of smooth surface. In this communication, we present an application of the VCRM in the 2D scattering of a plane wave by a homogeneous ellipsoid at oblique incidence. The transversal convergence effect of the wave will be discussed.

1. Introduction

In the study of electromagnetic and light scattering, the variable separation methods based on the solution of Maxwell equations (or its equivalents) are limited to objects that can be described in a coordinate system of the same geometry, such as sphere, spheroid, ellipsoid, and circular or elliptical cylinder. Even in these 'simple' cases, the numerical calculation remains another obstacle. Except for the sphere and the circular cylinder, the size of the scatterer can hardly exceed a few tens of wavelengths. Numerical methods such as T matrix, discrete-dipole approximation (DDA), etc., can be applied to nonspherical particles, but the size parameter of the scatter is also severely limited [1].

Geometrical optics is a simple and intuitive method for treating the interaction of an object with electromagnetic or light waves when the dimension of the object is much larger than the wavelength [2, 3]. One of its main advantages over other methods is that it can be applied to objects of complex shape, which are hard or even impossible to be dealt with by rigorous theories or most numerical techniques. Many researchers have contributed to the improvement of geometrical optics. Some take into account the forward diffraction or other particular wave effects (Airy theory for the rainbow [4] and Marston's model for the critical scattering [5]). Others combine directly geometrical optics with the electromagnetic wave method [6]. However, in these studies interference effects of all order rays are rarely taken into account. On the other hand, when ray optics is extended to a three dimensional (3D) object of irregular shape, it becomes a heavy task (see [7, 8] and references therein) because of the difficulties in the determination of reflection and refraction

angles, the calculation of local divergence factors and the phase shift due to focal lines. To overcome these obstacles, we have developed a so-called vectorial complex rays model (VCRM) [9] that consists of three points: the rays are dealt with by vectors; the divergence and the focal line phase shifts are calculated by differential geometry; and the total scattered field is the superposition of the contributions of all complex rays. This model makes it possible to calculate the divergence factor of a single ray bundle and is easy to extend to irregularly shaped 3D objects. In this communication, we present an application of the VCRM in the 2D scattering of a plane wave by an ellipsoid at oblique incidence. The effect of the transversal conversion of the wave will be discussed.

2. Description of the model

In VCRM, the wave is considered as bundles of vectorial complex rays. Each ray is characterized by four parameters: amplitude, phase, direction of propagation and polarization state (see [9] for details).

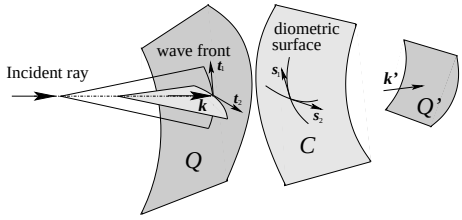

Figure 1. Schema of the fronts of waves and the dioptric surface.

Consider an arbitrary wave of wave front described by its curvature matrix Q impinging on a dioptric surface of curvature matrix C (Fig. 1). Then the curvature matrix Q' of the wave after refraction/reflection is given by the wave front matrix equation:

$$(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{n}C = k'\Theta'^{T}Q'\Theta' - k\Theta^{T}Q\Theta \tag{1}$$

where the letters with prime represents the quantities after refraction/reflection, ^{T} the transpose of the matrix, Θ the projection matrix between the unitary vectors of the coordinates systems on the planes tangent to the wave front $(\mathbf{t}_1, \mathbf{t}_2)$ and the dioptric surface $(\mathbf{s}_1, \mathbf{s}_2)$:

$$\Theta = \begin{pmatrix} \mathbf{t}_1 \cdot \mathbf{s}_1 & \mathbf{t}_1 \cdot \mathbf{s}_2 \\ \mathbf{t}_2 \cdot \mathbf{s}_1 & \mathbf{t}_2 \cdot \mathbf{s}_2 \end{pmatrix}$$

Knowing the wave vector $\mathbf{k}$ of the incident ray, the wave vector of refracted/refracted ray $\mathbf{k}'$ is determined by the vector Snell law :

$$(\mathbf{k}' - \mathbf{k}) \times \hat{\mathbf{n}} = \mathbf{0} \tag{2}$$

where $\mathbf{n}$ is the normal of the dioptric surface.

The validation of the model has been given in our previous publication [9] by comparison of numerical results with rigorous theories (Mie theory and classical geometrical optics) in simple cases (homogeneous sphere).

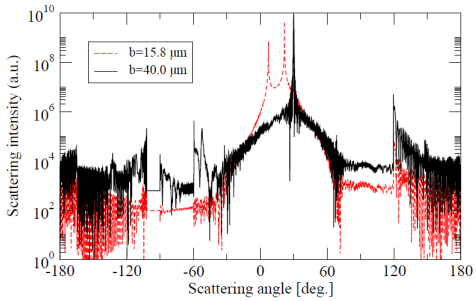

Figure 2. Scattering diagram of an ellipsoid of water $a = 50$ μm, $c = 30$ μm illuminated by a plane wave of wavelength $\lambda = 0.6328$ μm with an incident angle $\theta_0 = 30^\circ$ and b as parameter.

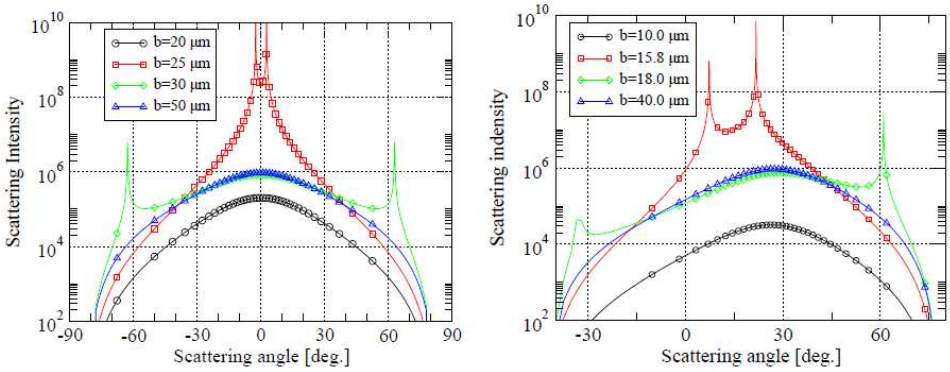

Figure 3. Convergence of the ray $p = 1$ of an ellipsoid of water ($m = 1.333$) illuminated by a plane wave of wavelength $\lambda = 0.6328$ μm with b as parameter. left: $\theta_0 = 0$, $a = c = 50$ μm; right: $\theta_0 = 30^\circ$, $a = 50$ μm, $c = 30$ μm.

3. Application to the scattering of an ellipsoid

We consider a homogenous ellipsoid particle illuminated by a plane wave of propagating in the plane defined by two axis of the ellipsoid and we are interested only by the scattering wave in this plane. The problem is therefore a 2D scattering since the rays remain always in this plane. Suppose that the three radii of the ellipsoid in x, y and z direction are respectively a, b and c, and the incident plane wave is in the xz plane with an incident angle θ_0 respective to z axis. Due to the symmetry of the problem, the wave front matrix equation (1) can be written in two scalar equations:

$$\frac{k' \cos^2 \beta}{R_1'} = \frac{k \cos^2 \alpha}{R_1} + \frac{k' \cos \beta - k \cos \alpha}{\rho_1} \tag{3}$$

$$\frac{k'}{R_2'} = \frac{k}{R_2} + \frac{k' \cos \beta - k \cos \alpha}{\rho_2} \tag{4}$$

where α and β are respectively the incident and refraction angles. Note that (3) and (4) are also valid for reflection by taking $k' = -k$. The two main curvature radii of the ellipsoid at $y = 0$ plane is [10]:

$$\rho_1 = \frac{b^2}{a}\left[1 + (a^2/c^2 - 1)z^2/c^2\right]^{1/2} \quad (5)$$

$$\rho_2 = \frac{c^2}{a}\left[1 + (a^2/c^2 - 1)z^2/c^2\right]^{3/2} \quad (6)$$

Figure 2 presents the total scattering diagrams of an ellipsoid $a = 50$ μm, $c = 30$ μm of water $m = 1.333$ illuminated by a plane wave of wavelength $\lambda = 0.6328$ μm with an incident angle $\theta_0 = 30^\circ$ and b as parameter. We find that besides the rainbows due to the convergence in the scattering plane around, for example, -100°, -60° and 120°, two peaks appear 7° and 21° when $b = 15.8$ μm but not such peaks don't exit for $b = 40$ μm. This phenomenon is due to the transversal convergence of the wave in the direction perpendicular to the scattering plane. To show clearly this effect, we present in Figure 3 the scattering intensity of the first refractive rays ($p = 1$) with different size parameters. We find that this convergence is very sensible to the radius of the ellipsoid in the transversal direction. It is worth to note that this model is also applied to the scattering of bubbles[11].

Acknowledgments

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