DOI: 10.1478/C1V89S1P026

AAPP | Atti della Accademia Peloritana dei Pericolanti
Classe di Scienze Fisiche, Matematiche e Naturali
ISSN 1825-1242

Vol. 89, Suppl. No. 1, C1V89S1P026 (2011)

HEIGHT AND THICKNESS OF A SCATTERING (AEROSOL OR CLOUD) LAYER FROM SPACE-BASED OXYGEN A-BAND SPECTROSCOPY: AN ANALYTICAL APPROACH

ANTHONY B. DAVIS*

ABSTRACT. Simplified radiative transfer modeling is used to show that both altitude and thickness of a scattering layer can be inferred from satellite observations in the O_2 A-band at high-enough spectral resolution, preferably with prior information about optical thickness. For simplicity, the surface is assumed black (water-like). This confirms previous claims but using closed-form analytical methods. For optically thin aerosol, the single-scattering limit is used; for optically thick clouds, a diffusion-type model is invoked.

1. Introduction and outline

Many authors have studied the possibility of estimating cloud top height/pressure from space using differential optical absorption spectroscopy (DOAS) in oxygen features, particularly the prominent A-band (0.759–0.772 μm). A few of these authors [1] have also examined the possibility of also inferring cloud (pressure) thickness.

Here, we generalize the concept of pressure thickness determination to optically thin aerosol layers. We also have a closer look at the balance of the components in the measured signal coming from the deterministic path borrowed by the sunlight to and from the top of the scattering layer and the random element that comes from transport inside it.

The modeling approach (§**2**) is deliberately simplified enough to enable the derivation of closed-form expressions. We use this framework to analyze sensitivity to scattering layer height and thickness in both optically thin (§**3.1**) and thick (§**3.2**) limits. The embedding molecular atmosphere is assumed, for simplicity, to overlay a black surface.

2. Forward radiative transfer (RT) model

The semi-infinite region of interest in the purely-absorbing molecular atmosphere is divided into two parts: "above-layer" ($z_\mathrm{t} < z < \infty$, $P_\mathrm{t} > P_z > 0$); "inside-layer" ($0 \leq z_\mathrm{b} < z < z_\mathrm{t}$, $P_0 \geq P_\mathrm{b} > P_z > P_\mathrm{t}$). Due to the absorbing surface, we are not interested here in the "below-layer" section. Variables z and P_z are altitude and pressure, respectively, and P_0 is surface pressure (set to 1013.25 mbar). Optical depth for absorption by oxygen from the top-of-atmosphere (TOA) to pressure level P_z is approximated by $\tau_{O_2} \times (P_z/P_0)$ where τ_{O_2} is the O_2 optical depth for one airmass, measured vertically.

Let $I(\tau_{\mathrm{O}_2}, \mu_0, \mu_\mathrm{v}; P_\mathrm{t}, \Delta P, \tau, \mathrm{pf})$ be the at-sensor radiance that we foresee as dependent on: τ_{O_2}, which will stand in here for wavelength λ; μ_0 and μ_v, the cosines of the solar zenith and viewing angles, respectively; P_t; $\Delta P = P_\mathrm{b} - P_\mathrm{t}$; τ, the optical thickness of the scattering layer (assumed non-absorbing); and "pf," standing for whatever parameters are needed to define the phase function of the particles in the scattering layer.

Introducing the effective bidirectional reflectance function (BRF) of the scattering layer alone, $R(\mu_0, \mu_\mathrm{v}; \tau, \varpi_0, \mathrm{pf})$, and the incoming spectral irradiance at the TOA for $\lambda \approx 0.765$ μm ($F_0 = \pi \times 1.5\,10^{21}$ photons/s/m^2/μm), we have

$$\pi I(\tau_{\mathrm{O}_2}, \mu_0, \mu_\mathrm{v}; P_\mathrm{t}, \Delta P, \tau, \mathrm{pf}) \approx \mu_0 F_0\, \mathrm{e}^{-\tau_{\mathrm{O}_2} \frac{P_\mathrm{t}}{P_0} \left(\frac{1}{\mu_0} + \frac{1}{\mu_\mathrm{v}}\right)} R(\mu_0, \mu_\mathrm{v}; \tau, \varpi_0, \mathrm{pf}), \quad (1)$$

where

$$\varpi_0(\tau_{\mathrm{O}_2}^{(\mathrm{s})}, \tau) = \tau/(\tau + \tau_{\mathrm{O}_2}^{(\mathrm{s})}) \quad (2)$$

with

$$\tau_{\mathrm{O}_2}^{(\mathrm{s})}(\tau_{\mathrm{O}_2}; P_\mathrm{t}, \Delta P) = \tau_{\mathrm{O}_2} \times \Delta P/(P_\mathrm{t} + \Delta P/2). \quad (3)$$

We have made the assumption here that the geometrical thickness $H = z_\mathrm{t} - z_\mathrm{b}$ of the scattering layer is small compared to the scale height of the molecular atmosphere, $H_\mathrm{m} \approx 8$ km. From there, we assume that the gaseous absorption is uniform inside the aerosol or cloud layer, and can thus be captured by a constant single scattering albedo, ϖ_0.

We have yet to obtain closed-form expressions for $R(\mu_0, \mu_\mathrm{v}; \tau, \varpi_0, \mathrm{pf})$, the scattering layer's BRF in (1), where we take $\mu_\mathrm{v} = 1$ (nadir-looking satellite sensor) in the remainder of the study. They are derived in the asymptotic limits of RT: single scattering for optically thin layers (i.e., background aerosol) and a diffusion for optically thick layers (i.e., clouds).

3. Dissection of the O_2 A-band signal

We now show that the A-band signal for a space-based sensor has two main components that vary with τ_{O_2}, our surrogate for λ: one dependent only on P_t and on known quantities, namely, μ_0 and μ_v; and another dependent on those known quantities, on ΔP (and P_t), and on *inherent* layer properties that can be determined independently, namely, τ and "pf." Moreover, we show that by exploiting this dichotomy one can derive both P_t and ΔP by considering different regimes in τ_{O_2}, iteratively if necessary.

3.1. Optically thin case (aerosol layers and thin cirrus). If $\tau \ll 1$, we have

$$R(\mu_0, \mu_\mathrm{v}; \tau, \varpi_0, \mathrm{pf}) \approx \pi\, \varpi_0(\tau_{\mathrm{O}_2}; P_\mathrm{t}, \Delta P, \tau)\, p(\mu_\mathrm{s}; \mathrm{pf})\, \tau/\mu_0\mu_\mathrm{v}, \quad (4)$$

where $p(\mu_\mathrm{s}; \mathrm{pf})$ is the phase function. μ_s denotes the cosine of the scattering angle, $-\mu_0\mu_\mathrm{v} + \sqrt{1-\mu_0^2}\sqrt{1-\mu_\mathrm{v}^2}\cos\Delta\phi$, $\Delta\phi$ being the viewing-solar azimuthal angle difference. This linear model for reflectivity growth can be used for τ up to a few tenths.

It is reasonable to assume a double Henyey–Greenstein phase function for the (often dominant) fine aerosol mode:

$$p(\mu_\mathrm{s}; f_1, g_1, g_2) = \frac{1}{4\pi}\left(f_1 \frac{1-g_1^2}{(1+g_1^2-2g_1\mu_\mathrm{s})^{3/2}} + (1-f_1)\frac{1-g_2^2}{(1+g_2^2-2g_2\mu_\mathrm{s})^{3/2}}\right). \quad (5)$$

We will take $(f_1, g_1, g_2) = (0.9, +0.7, -0.3)$ for specificity and, generally speaking, we assume that the aerosol phase function can be inferred from independent observations.

Figure 1 shows three sensitivities of the A-band spectral signal represented here as a monotonically decreasing function of τ_{O_2}, for $\mu_0 = 1/2$. The reference case is a conservatively scattering layer with $\tau = 0.1$, $\Delta P = 200$ mbar, and $P_\mathrm{t} = 700$ mbar (hence z_b = 1 km, z_t = 3 km). From left to right, we vary those three basic layer properties within more-or-less expected ranges. In the leftmost panel, and in view of (1), we see that P_t can

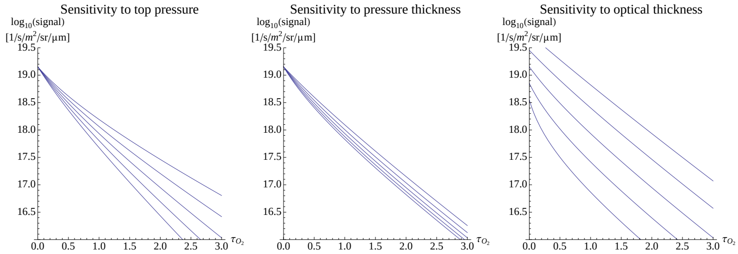

Figure 1. *A-band responses for aerosol layers from* (1), (4). The reference layer has $(P_\mathrm{t}, \Delta P, \tau) =$ (700 mbar, 200 mbar, 0.1). **Left**, bottom to top: $P_\mathrm{t} =$ (800, 700, 600, 500, 400) mbar, equiv. $z_\mathrm{t} = H_\mathrm{m} \log P_0/P_\mathrm{t} =$ (1.9, 3.0, 4.4, 5.7, 7.4) km. **Middle**, top to bottom: $\Delta P =$ (100, 150, 200, 250, 300) mbar, equiv. $H \approx H_\mathrm{m}/(P_\mathrm{t}/\Delta P + 1/2) =$ (1.1, 1.5, 2.0, 2.4, 2.8) km. **Right**, bottom to top: $\tau =$ (0.025, 0.05, 0.1, 0.2, 0.4). The cutoff at $I \approx 10^{16}$ photons/s/m^2/sr/μm, hence at $\tau_{\mathrm{O}_2} \approx 3$, is dictated by signal-to-noise considerations (SNR ≈ 1) while SNR ≈ 200 at $I \approx 10^{19}$ ($\tau_{\mathrm{O}_2} = 0$, continuum) [R. Pollock, pers. comm.] using specs from the Orbiting Carbon Observatory [3].

be inferred from the asymptotic *slope* of log(signal) vs. τ_{O_2}, ignoring ϖ_0's relatively weak dependence on P_t in (2)–(3). From the middle panel, and in view of (2)–(4), we see that we can infer ΔP (via ϖ_0) from the *intercept* of the same asymptotic trend. This last procedure assumes that we know τ and pf (i.e., the phase function beyond its dependence on μ_s) as well as P_t (estimated from the slope). If there is a bias in τ it will immediately lead to one in ϖ_0 (hence in ΔP). This is a classic degeneracy between scattering and absorption in all predicted radiances based on single scattering theory.

3.2. Optically thick case (thick aerosol and cloud layers). If $\tau \gg 1$, then one can assume $I(z, \vec{\Omega}) \approx [J(z) + 3\vec{F}(z) \cdot \vec{\Omega}]/4\pi$ inside the plane-parallel layer, where $\vec{\Omega}$ is the direction of propagation; J ($\vec{F}$) denotes the scalar (vector) flux at level z, equivalently total (scattering+absorption) optical depth $\tau_z = (\tau' + \tau_{\mathrm{O}_2}) \times (z_\mathrm{t} - z)/H$ into the cloudy layer from the top. In short, we assume an isotropic radiance field ($\propto J$) modulated by a dipole pattern ($\propto F_z$). Accordingly, we adopt a δ-Eddington model for the phase function [4]:

$$p(\mu_\mathrm{s}; f, g') = [2f\delta(1-\mu_\mathrm{s}) + (1-f)(1+3g'\mu_\mathrm{s})]/4\pi, \tag{6}$$

leading in the absence of droplet absorption to $g'(g,f) = (g-f)/(1-f)$ and $\tau'(\tau, g, f) = \tau(1-g)/(1-g'(g,f))$. It is commonly assumed that $f = g^2$, hence $g' = g/(1+g)$.

This leads to diffusion-type RT described by the following coupled ODEs [2]:

$$\mathrm{d}F_z/\mathrm{d}\tau_z = -(1-\varpi_0)J + \varpi_0 \mathrm{e}^{-\tau_z/\mu_0}, \quad \mathrm{d}J/\mathrm{d}\tau_z/3 = -(1-\varpi_0 g)F_z + \varpi_0 g \mu_0 \mathrm{e}^{-\tau_z/\mu_0}, \tag{7}$$

subject to boundary conditions $(J + 2F_z)|_{\tau_z=0} = (J - 2F_z)|_{\tau_z = \tau'(\tau,g,g^2)+\tau_{\mathrm{O}_2}} = 0$. Then

$$R(\mu_0, \mu_\mathrm{v}; \tau, \varpi_0, \mathrm{pf}) = (J - 2F_z)|_{\tau_z=0}/4 = J(\tau_z)|_{\tau_z=0}/2. \tag{8}$$

This leads to a complex expression where there is in fact no dependence on μ_v since only the outgoing (hemispherical) flux is predicted in diffusion theory, but a reasonable Lambertian assumption is readily made.

Figure 2 shows the same sensitivities of the A-band spectral signal as in Fig. 1 but for cloud layers. In the top row, we see little sensitivity to ΔP or even τ. However, in the

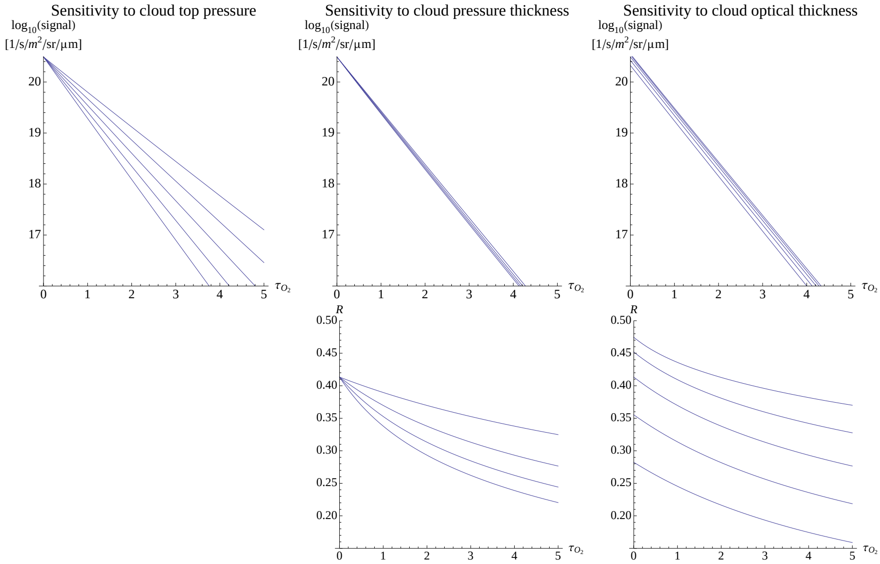

Figure 2. *A-band responses for cloud layers from* (1), (8). **Top row:** As in Fig. 1 but the reference layer has $g' \approx 0.46$, from $g \approx 0.85$ for liquid clouds, and $(P_\mathrm{t}, \Delta P, \tau') =$ (800 mbar, 100 mbar, 10). **Left**, bottom to top: $P_\mathrm{t} =$ (900, 800, 700, 600, 500) mbar, equiv. $z_\mathrm{t} =$ (1.0, 1.9, 3.0, 4.4, 5.7) km. **Middle**, top to bottom: $\Delta P =$ (50, 100, 150, 200) mbar, equiv. $H =$ (0.5, 0.9, 1.4, 1.8) km. **Right**, top to bottom: $\tau' =$ (2.5, 5, 10, 20, 40), hence $\tau =$ (9, 18, 36, 71, 143). The ($I \approx 10^{16}$/s/m^2/sr/μm) cutoff for SNR $\approx$ 1 is now at $\tau_{\mathrm{O}_2} \approx 5$ and SNR $\approx$ 20,000 is reached at the continuum radiance $I \approx 10^{21}$/s/m^2/sr/μm [R. Pollock, pers. comm.]. **Bottom row:** $R(\mu_0; \tau, \varpi_0, \mathrm{pf})$ estimated from the observed value of πI in (1), given P_t, where ϖ_0 is a function of $(\tau_{\mathrm{O}_2}; P_\mathrm{t}, \Delta P, \tau)$, and the two rightmost top panels, given the estimate of P_t from the leftmost top panel.

bottom row, these sensitivities are revealed by removing the dominant term in (1) for the return path of the sunlight to the cloud, which yields an estimate of $R(\mu_0; \tau, \varpi_0, \mathrm{pf})$ in (8), where ϖ_0 in (2)–(3) depends explicitly on $(\tau_{\mathrm{O}_2}; P_\mathrm{t}, \Delta P, \tau)$. It can be shown that $|\partial R/\partial \tau_{\mathrm{O}_2}|_{\tau_{\mathrm{O}_2}=0}$ is proportional to $(\mu_0 + 2/3)\Delta P$, with a prefactor weakly dependent on τ [2]. After determination of P_t from the leftmost top panel (as for aerosols), and given τ from independent measurements, this procedure (executed in the part of the top middle panel the least affected by noise) provides a robust method for estimating ΔP.

References

[1] See citations listed in §5.8.1 of Ref. [2]; preprint available from http://science.jpl.nasa.gov/people/ADavis.
[2] A.B. Davis, et al., "Space-time Green functions for diffusive radiation transport, in application to active and passive cloud probing," in *Light Scattering Reviews, 4* (ed. A. Kokhanovsky, Springer-Praxis, Heidelberg, 2009).
[3] D. Crisp, et al., "The Orbiting Carbon Observatory (OCO) mission," *Adv. Space Res.* **34**, 700 (2004).
[4] J.H. Joseph, et al., "The δ-Eddington approximation for radiative flux transfer," *J. Atmos. Sci.* **33**, 2452 (1976).

* Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Drive, Mail Stop 169-237, Pasadena, CA 91109, USA

Email: Anthony.B.Davis@jpl.nasa.gov

Paper presented at the ELS XIII Conference (Taormina, Italy, 2011), held under the APP patronage; published online 15 September 2011.