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GAME THEORY FOR SPECULATIVE DERIVATIVES: A POSSIBLE STABILIZING REGULATORY MODEL

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ABSTRACT. The aim of this paper is to propose a methodology to stabilize the financial markets using Game Theory, specifically the Complete Study of a Differentiable Game. Initially, we intend to make a quick discussion of peculiarities and recent development of derivatives, and then we move on to the main topic of the paper: forwards and futures. We illustrate their pricing and the functioning of markets for this particular derivatives type. We also will examine the short or long hedging strategies, used by companies to try to cancel the risk associated with market variables. At this purpose, we present a game theory model. Specifically, we focus on two economic operators: a real economic subject and a financial institute (a bank, for example) with a big economic availability. For this purpose, we discuss about an interaction between the two above economic subjects: the Enterprise, our first player, and the Financial Institute, our second player. We propose a tax on financial transactions with speculative purposes in order to stabilize the financial market, protecting it from speculations. This tax hits only the speculative profits and we find a cooperative solution that allows, however, both players to obtain a gain.

1. Introduction

In the last 30 years, derivatives have become increasingly important in the world of finance. Their frequent use, however, has caused instability on financial markets, so as to leave ample opportunities for profit to speculators with large available capitals. Just the continuous price fluctuations and the strong speculative pressures have exacerbated the crisis in which has plunged the world economy.

Present financial crisis. Regrettably, the current crisis is of a different nature from those of the past, and therefore unknown and (may be) more dangerous. In fact, the cause of the crisis is the capitalistic system itself, which was imploded on itself under the hits of the great speculators, which as selfish bettors have profited, without any ethic, taking advantage of normative loopholes present in today's financial system.

Consequences. Thus, while the rich have become ever richer, the speculative pressures, orchestrated by financial giants, crushed the small investors. The results are immediately visible to all: businesses failing, prices rising, families in bankrupt, whole States on the brink of default. And the speculators themselves are likely to remain victims of the system that created them and fed them.

Credit crunch and welfare systems. The current financial system is based on virtual money, which does not confluence into the real economy, remaining stuck in the finance world (so called phenomenon of the "credit crunch"). Possible solutions? Before arriving to a point of no return (which is actually not so far), it becomes appropriate, and perhaps vital, to establish the "rules of the game", in order to redistribute the social wealth in a way at least close to the concept of equity (purpose that the so desired Welfare State claims, at least in their intentions, to pursue).

Necessity of a strong normative intervention. Indeed, it is impossible to imagine a well system where there is no an "arbiter" (and if it exists, it is not efficient), there are no "rules" (and if they exist, they are neither respected nor effective), there are no "penalties" (and if they exist, they are not satisfactorily implemented).

How do we face the problem? Just this, as much as possible, we propose to do in this work: we study an exemplary game in which we see as a simple and trivial method (the application of a tax on speculative financial transactions) can cure a sick financial system. The objective could be realized, as we shall see, without inhibiting the opportunity for profit, nor for businesses nor for speculators. In fact, the speculators, precisely because of this tax, can no longer act alone without any risks of debacle in the financial markets, and therefore they are forced (if they do not want to risk seeing blur the gains) to interact with the real economic entities, and to split the profit with them. This model acts as a deterrent to easy and socially unfair profits.

2. Derivative contracts

In recent years, the derivative contracts have had an increasing role in the financial world, coming to be an integral part of our financial system. Their simplicity of understanding, their facility of use and the possibility to make a derivative contract on practically everything, have made them one of more used financial instruments.

But what exactly are derivatives?

Definition of derivative contract. It is a financial instrument (entered into between two parties) whose value depends on the values of underlying variables.

Very often the underlying variables are the prices of traded assets, but the derivatives can derives from almost any variable. The majority of the derivatives treated in the financial world are futures and forwards.

2.1. Futures contracts. Definition of futures contract. It is a standardized agreement between two parties to exchange a certain asset of standardized quantity for a certain determined price (futures price), with delivery at a specified future date, the delivery date.

Futures positions. The party that purchases the underlying asset in the future assumes a "long" position, and the party that sells the asset in the future assumes a "short" position.

The terminology indicates the expectations of the parties:

(1) the buyer hopes that the asset spot price will increase (for this reason it is said "long");

(2) the seller expects that the asset spot price will decrease (for this reason it is said "short").

Futures are traded on regulated markets, as very common actions, and their price depends on the laws of supply and demand:

(1) if a futures contract is very much required or supply is not adequate to the demand, the price goes up;
(2) if a futures contract is not required or the supply is higher than demand, the price goes down.

2.2. Forward contracts. Definition of forward contract. It is a non-standardized agreement between two parties to exchange a certain asset for a certain determined price (forward price), with delivery at a specified future date, the delivery date.

Forward positions. The party that purchases the underlying asset in the future assumes a "long" position, and the party that sells the asset in the future assumes a "short" position.

Practically, a forward contract is a futures contract with two important differences:

(1) contractual terms are not standardized, and then the two parties may negotiate and close the deal with the conditions that they prefer (for example they can trade a different quality of the asset, or even they can base the contract on different underlying variables than exchange-traded asset).
(2) the forwards are traded in unregulated markets, the so-called over-the-counter market (OTC). Generally, the size of the trading taking place on the OTC markets is much larger than that one of the negotiations taking place in exchanges.

3. Pricing forward and futures contracts

The futures price is linked to the underlying spot price (see [1]).

We assume that:

(1) money can be borrowed or loaned at the same risk-free interest rate r;
(2) there are no transaction costs;
(3) the underlying asset does not offer dividends;
(4) the underlying asset has not storage costs and has not convenience yield to take physical possession of the goods rather than futures contract.

Then, the general relationship linking the futures price F_t and spot price S_t is necessarily

$$F_t = S_t(1 + r/m)^{mT},$$

where

- F_t is the futures price at time t;
- S_t is the spot price at time t;
- $(1 + r/m)^{mT}$ is the factor of interest capitalization at risk-free rate r, calculated m times each year for T years.

If not, the arbitrageurs would act on the market until futures and spot prices return to levels indicated by the above relation.

In fact:

(1) if $F_t > S_t(1 + r/m)^{mT}$, arbitrageurs can borrow money at the riskless rate r, buy the asset at the spot price S_t and sell it at the forward price F_t, so obtaining a sure profit.
(2) if $F_t < S_t(1 + r/m)^{mT}$, arbitrageurs can short sell the asset at the spot price S_t, invest the proceeds of sale at the risk-free rate r and then repurchase the asset at the forward price F_t.

4. Collateral deposits

In futures and forward market, the deposits of collateral have a very important role. In fact, when the two parties reach an agreement, there is no certainty that at the expiry of the contract both parties respect the commitment (for example at one of the two parties may lack the financial resources)

In order to avoid the rise of repeated insolvencies, regulatory authorities seek to adopt systems that can combine the speed and agility of business with a reasonable assurance that the contract be honored without the occurrence of defaults, both in the exchanges than in OTC markets (where the risks are greater).

4.1. Collateral deposits in the exchanges. In order to enter into a futures contract must make contacts with a broker, who asked the trader to make a deposit to guarantee its commitments. This deposit is called initial margin, and the minimum level is set by the exchanges.

Every day on collateral deposits are charged or accredited some money:

(1) if the settlement price (the price at which the contract is negotiated at the close of trading) of day $t + 1$ is greater than the price of day t, at day $t + 1$ the traders that at time t have assumed a short position lose the difference between the two prices (difference charged on the deposit account of the trader), while the traders that have assumed a long position gain the difference between the two prices (difference accredited on the deposit account of the trader);
(2) if the settlement price of day $t + 1$ is lower than the price of day t, at day $t + 1$ the traders that at time t have assumed a short position gain the difference between the two prices (difference accredited on the deposit account of the trader), while the traders that have assumed a short position lose the difference between the two prices (difference charged on the deposit account of the trader).

This procedure of daily settlement of profits and losses is known as "marking to market".

A maintenance margin is usually set to 75% of the initial margin. If the balance of the collateral deposit drops below the maintenance margin, the trader receives a request for integration (a so-called margin call) by the broker. Then, within the next day, the trader must make a deposit to bring balance to the level of initial margin. If this payment is not made, the trader's position is closed out.

4.2. Collateral deposits in the OTC markets. Like in the regulated markets, in the OTC markets the collaterals are also becoming increasingly important. Credit risk is very high and for this reason has been adopted a procedure known as "collateralization", which imitates the system present in the exchanges.

Collateralization system. It consists on the daily collateral integration. In a word, the party that suffers a loss because of a decrease in contract value (according to its short or long position) has to integrate the collateral. On balances of collateral deposits is usually paid an interest.

5. Market traders

In derivatives market there are three main categories of operators, depending on the purpose with which use the derivative contract: hedgers, speculators and arbitrageurs.

(1) ***Hedgers*** use forwards and futures to reduce the risks resulting from their exposures to market variables. Forward hedges eliminate the uncertainty on the price to pay for the purchase (or receivable for the sale) of the underlying asset, but they not necessarily lead to a better result. The use of the derivative allows to neutralize the adverse trend of the market, offsetting losses/gains on the price of the underlying asset with the gains/losses obtained on the derivatives market.

(2) ***Speculators*** realize investment strategies, buying (or selling) futures and then sell (or buy) them at a price higher (or lower). Who decides to speculate assumes a risk about the favorable or unfavorable trend of the futures market. The futures market offers a financial leverage to speculators, which are able to take relatively large positions with a low initial outlay.

(3) ***Arbitrageurs*** take the offsetting positions of two or more contracts to lock in a risk-free profit, and they take advantage of a price difference between two or more markets. The arbitrageurs exploit a temporary mismatch between the performance (intended to coincide when the contract expires) of the futures market and the underlying market.

6. Futures (or forward) hedging operations

Among the participants in the futures markets there are many hedgers. Their purpose is to use these markets to reduce exposure to specific risks, which can refer to any variable. A hedge is said perfect when it completely eliminates the risk, but unfortunately the perfect hedges are unusual.

There are two types of hedges:

(1) a short hedge may be appropriate when the hedger has a business that he has to sell in the future. The short hedge could be used by a hedger who does not currently have the asset, but who knows that he will acquire it;
(2) the hedges that are based on the purchase of futures are called long hedges. A long hedge may be appropriate when the hedger knows that has to buy a certain asset and he tries to lock in the purchase price.

In practice, the risk of an adverse change in the price is canceled. The potential loss that is obtained on the spot market (the market at current prices) was offset by the gain on futures contracts.

Remark. The use of hedging operation does not necessarily implie a better economic results, because the hedges cancel both losses and profits that would be obtained on the spot market.

7. A game theory financial model

Now we pass to the presentation of our financial model (for the complete study of a game theory model see [2–11]), based on the introduction of a tax on financial transactions with speculative purposes.

7.1. Methodologies. The strategic game G that we propose for modeling our financial interaction requires a construction on 3 times, say time 0, 1 and 2.

(0) At time 0, the Enterprise knows the quantity of goods that it needs to conduct its business. The Enterprise can choose to buy futures contracts in order to hedge the market risk on the goods price.

(1) At time 1, on the other hand, the Financial Institute acts with speculative purposes on the spot market (buying or short-selling at time 0) and on the futures market (by the opposite action of that one performed on the spot market) of the asset that interests to the Enterprise. The Financial Institute may so take advantage of the temporary misalignment of the spot and futures prices of the asset, created by the hedging strategy of the Enterprise.

(2) At the time 2, the Financial Institute cashes or pays the sum determined by its behavior in the futures market at time 1.

8. The description of the game

We assume that our first player is an Enterprise that may choose to buy futures contracts to hedge against an upwards change of price for the underlying asset; the Enterprise should have at time 1 a certain quantity M_1 of this asset, in order to conduct its business.

Therefore, the Enterprise can choose a strategy $x \in [0, 1]$, which represents the percentage of the underlying quantity M_1 that the Enterprise purchases through futures, depending on it wants:

(1) to not hedge ($x = 0$),
(2) to hedge partly ($0 < x < 1$),
(3) to hedge totally ($x = 1$).

On the other hand, our second player is a Financial Institute operating on the spot market of the underlying asset that the Enterprise knows it should have at time 1. The Financial Institute works in our game also on the futures market:

- taking advantage of possible gain opportunities - given by misalignment between spot prices and futures prices of the asset;
- or accounting for the loss obtained, because it has to close the position of short sales opened on the spot market.

These actions determine the payoff of the Financial Institute.

The Financial Institute can choose a strategy $y \in [-1, 1]$, which represents the percentage of the underlying quantity M_2 that it can buy (in algebraic sense) with its financial resources, depending on it intends:

(1) to purchase the underlying on the spot market ($y > 0$);
(2) to short sell the underlying on the spot market ($y < 0$);
(3) to not intervene on the market of the underlying ($y = 0$).

In fig. 1 we illustrate graphically the bi-strategy space $E \times F$ of the game G.

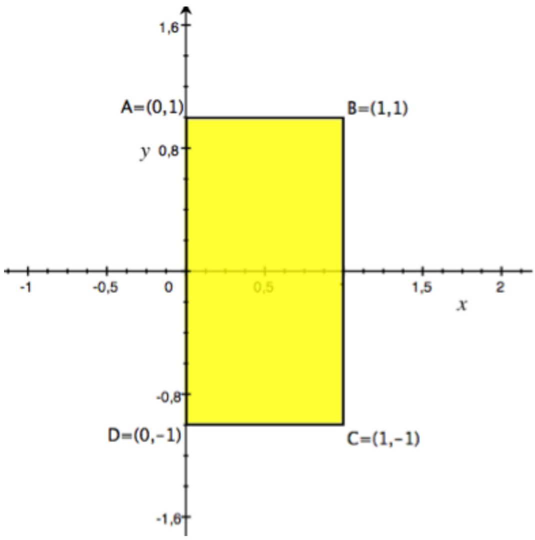

Figure 1. The bi-strategy space of the game

9. The payoff function of the Enterprise

The payoff function of the Enterprise, which is the function that represents the quantitative loss or win of the Enterprise, referred to time 1, is given by the the net gain obtained on not hedged asset $x'M_1$ (here $x' := 1 - x$).

The gain related with the not hedged asset is given by the not hedged goods quantity

$$(1 - x)M_1,$$

multiplied by the difference

$$F_0 - S_1(y),$$

between the futures price at time 0 (the term F_0) - which the Enterprise should pay, if it decides to hedge - and the spot price $S_1(y)$ at time 1, when the Enterprise actually buys the goods that it did not hedge.

So the payoff function of the Enterprise is defined by

$$f_1(x, y) = M_1 x'(F_0 - S_1(y)), \tag{1}$$

where

- M_1 is the amount of goods that the Enterprise should have at time 1;
- $x' := (1 - x)$ is the percentage of the underlying asset that the Enterprise buys on the spot market at time 1 without any hedge (and therefore exposed to the fluctuations of the spot price of the goods);
- F_0 is the futures price at time 0. It represents the price established at time 0 that the Enterprise has to pay at time 1 in order to buy the goods. By definition, assuming the absence of dividends, known income, storage costs and convenience yield to keep possession of the underlying, the futures price is given by

  $$F_0 = S_0 u^T, \tag{2}$$

  where $u = (1+i)^T$ is the unique capitalization factor at time T with rate i. By i we mean the risk-free interest rate charged by banks on deposits of other banks, the so-called "LIBOR" rate. S_0 is, on the other hand, the spot price of the underlying asset at time 0. The value S_0 is a constant because it is not influenced by our strategies x and y.
- $S_1(y)$ is the spot price of the underlying at time 1, after that the Financial Institute has implemented its strategy y. It is given by

  $$S_1(y) = S_0 u + nuy \tag{3}$$

  where n is the marginal coefficient representing the effect of y on $S_1(y)$. $S_1(y)$ depends on y because, if the Financial Institute intervenes in the spot market by a strategy $y \neq 0$, then the price $S_1(y)$ changes because any demand change has an effect on the asset price. We are assuming a linear dependence. The value S_0 and the value ny should be capitalized, because they should be "transferred" from time 0 to time 1.

The payoff function of the Enterprise. Therefore, recalling the Eq.3, that is

$$S_1(y) = (S_0 + ny)u,$$

and the Eq.2, that is

$$F_0 = S_0 u^T,$$

and replacing them in the Eq.1, that is

$$f_1(x, y) = M_1 x'(F_0 - S_1(y),$$

we have:

$$f_1(x, y) = M_1((1 - x)[S_0 u - (S_0 + ny)u]. \tag{4}$$

After the appropriate simplifications, here is represented the payoff function of the Enterprise:

$$f_1(x, y) = M_1(1 - x)(-nuy). \tag{5}$$

From now the value nu will be indicated by ν.

10. The payoff function of the Financial Institute and the stabilizing proposal

The payoff function of the Financial Institute at time 1, which is the algebraic gain function of the Financial Institute at time 1, is the multiplication of the quantity of goods bought on the spot market, that is yM_2, by the difference between the futures price $F_1(x,y)$ (it is a price established at time 1 but cashed at time 2) transferred to time 1, that is

$$F_1(x,y)u^{-1},$$

and the purchase price of goods at time 0, say S_0, capitalized at time 1 (in other words we are accounting for all balances at time 1).

Stabilizing strategy of normative authority. In order to avoid speculations on spot and futures markets by the Financial Institute, which in this model is the only one able to determine the spot price (and consequently also the futures price) of the underlying commodity, we propose that the normative authority imposes to the Financial Institute the payment of a tax on the speculative transactions. So the Financial Institute cannot take advantage of price swings caused by itself.

We assume that this tax is fairly equal to the incidence of the strategy of the Financial Institute on the spot price, so the price effectively cashed or paid for the futures by the Financial Institute is

$$F_1(x,y)u^{-1} - \nu y,$$

where νy is the tax paid by the Financial Institute, referred to time 1.

Remark. We note that if the Financial Institute wins, it acts on the futures market at time 2 to cash the win, but also in case of loss it must necessarily act in the futures market and account for its loss because at time 2 (in the futures market) it should close the short-sale position opened on the spot market.

The payoff function of the Financial Institute is defined by:

$$f_2(x,y) = yM_2[F_1(x,y)u^{-1} - \nu y - S_0 u], \tag{6}$$

where:

- y is the percentage of goods that the Financial Institute purchases or sells on the spot market of the underlying;
- M_2 is the maximum amount of goods that the Financial Institute can buy or sell on the spot market, according to its economic availability;
- S_0 is the price paid by the Financial Institute in order to buy the goods. S_0 is a constant because our strategies x and y do not influence it.
- νy is the normative tax on the price of the futures paid at time 1. We are assuming the tax is equal to the incidence of the strategy y of the Financial Institute on the spot price S_1.
- $F_1(x,y)$ is the price of the future market (established) at time 1, after that the Enterprise has played its strategy x.

  The function price $F_1(x,y)$ is given by

$$F_1(x,y) = S_1(y)u + mux, \tag{7}$$

where $u := (1+i)$ is the factor of capitalization of interests. By i we mean risk-free interest rate charged by banks on deposits of other banks, the so-called LIBOR rate. With m we intend the marginal coefficient that measures the influence of x on $F_1(x,y)$. $F_1(x,y)$ depends on x because, if the Enterprise buys futures with a strategy $x \neq 0$, the price F_1 changes because an increase of future demand influences the futures price. The value S_1 should be capitalized because it follows the relationship between futures and spot prices expressed in Eq.1. The value mx is also capitalized because the strategy x is played at time 0 but has effect on the futures price at time 1.
- $u^{-1} := (1+i)^{-1}$ is the discount factor. $F_1(x,y)$ must be actualized at time 1 because the money for the sale of futures will be cashed at time 2.

The payoff function of the Financial Institute. Recalling the Eq.7, that is

$$F_1(x,y) = S_1(y)u + mux,$$

and replacing it in the Eq.6, that is

$$f_2(x,y) = yM_2[F_1(x,y)u^{-1} - \nu y - S_0 u],$$

we have:

$$f_2(x,y) = yM_2 mx \tag{8}$$

The payoff function of the game is so given, for every $(x,y) \in E \times F$, by:

$$f(x,y) = (-\nu y M_1(1-x), yM_2 mx) \tag{9}$$

11. The payoff functions in presence of collaterals

In this game we do not consider the presence of collateral. But:

- even if the price F_0 will be paid at time 1, the Enterprise could deposit, already at time 0, the sum F_0 as guarantee that (at the expiry) the contract will be respected;
- even if the price F_1 is paid at time 2, the Financial Institute could deposit, already at time 1, the sum F_1 as guarantee that (at the expiry) the contract will be respected.

Proposition 1. *Let F_0 be the futures price of the asset at time 0 and let $u := (1+i)$ be the capitalization factor. Then, the payoff function of the Enterprise in presence of collateral is the same of the payoff function f_1 of the Enterprise without collateral.*

Proof. In order to calculate the win of the Enterprise at the time 1, we recall its payoff function

$$f_1(x,y) = -\nu y M_1(1-x).$$

In presence of collaterals, at the sum F_0 (that is paid as collateral at time 0 and for this reason it has to be capitalized) must be subtracted the interests $F_0 i$, cashed by the Enterprise on the deposit of collateral.

So, in the payoff function of the Enterprise we have to put the value

$$F_0 u - F_0 i \tag{10}$$

in place of the futures price F_0.

We will show that the value obtained in the Eq.10 is equal to the value in place of which must be replaced, that is the asset futures price F_0. So we want show that

$$F_0u - F_0i = F_0.$$

Recalling that $u := (1 + i)$, we have

$$F_0(1 + i) - F_0i = F_0.$$

This completes the proof. ■

Remark. So we have shown that, in presence of collaterals, the payoff function f_1 of the Enterprise that we have found before without considering eventual collateral, results valid also with guarantee deposits.

Proposition 2. *Let*

$$F_1(x, y) = S_1(y)u + mux$$

be the futures price of the asset at time 0 and let $u := (1 + i)$ *be the capitalization factor. Then, the payoff function of the Financial Institute in presence of collateral is the same of the payoff function* f_2 *of the Financial Institute without collateral.*

Proof. In order to calculate the win of the Financial Institute at the time 1, we recall its payoff function $f_2(x, y) = yM_2mx$.

In presence of collaterals, at the value F_1 (that is paid as collateral at time 1) we must subtract the interests (actualized at time 1) on the deposit of collateral cashed at time 2 by the Financial Institute.

The interests cashed by the Financial Institute are given by $F_1(x, y)iu^{-1}$. So, in the payoff function of the Financial Institute we have to put the value

$$F_1(x, y) - F_1(x, y)iu^{-1} \tag{11}$$

in place of the actualized futures price F_1u^{-1}.

We will show that the value obtained in the Eq.11 is equal to the value in place of which must be replaced, that is the actualized futures price $F_1(x, y)u^{-1}$ of the asset. So we want show that $F_1(x, y) - F_1(x, y)iu^{-1} = F_1(x, y)u^{-1}$.

Recalling that $F_1(x, y) = S_1(y)u + mux$, we obtain

$$S_1(y)u + mux - (S_1(y) + mx)uu^{-1}i = (S_1(y)u + mux)u^{-1},$$

and therefore

$$S_1(y)u + mux - (S_1(y) + mx)i = S_1(y) + mx.$$

Recalling that $u = (1 + i)$, we have

$$S_1(y)(1 + i) + mx(1 + i) - S_1(y)i + mxi = S_1(y) + mx.$$

This completes the proof. ■

Remark. So we have shown that, in presence of collaterals, the payoff function of the Financial Institute that we have found before without considering eventual collateral, results valid also with guarantee deposits.

12. Critical space of the game

Since we are dealing with a non-linear game, it is necessary to study in the bi-win space also the points of the critical zone, which belong to the bi-strategy space. In order to find the critical area of the game we consider the Jacobian matrix and we put its determinant equal 0.

For what concern the gradients of f_1 and f_2, we have

$$\text{grad } f_1(x, y) = (M_1 y\nu, -\nu M_1(1-x))$$

$$\text{grad } f_2(x, y) = (M_2 my, M_2 mx).$$

The determinant of the Jacobian matrix is

$$\det J_f(x, y) = M_1 M_2 \nu ymx + M_1 M_2 m(1-x)\nu y.$$

Therefore the critical space of the game is

$$Z_f = \{(x, y) \in \mathbb{R}^2 : M_1 M_2 \nu ymx + M_1 M_2 m(1-x)\nu y = 0\}.$$

Dividing by $M_1 M_2 \nu m$, which are all positive numbers (strictly greater than 0), we have:

$$Z_f = \{(x, y) \in \mathbb{R}^2 : yx + (1-x)y = 0\}.$$

Finally we have:

$$Z_f = \{(x, y) \in \mathbb{R}^2 : y = 0\}.$$

The critical area of our bi-strategy space is represented in fig. 2 by the segment $[H, K]$.

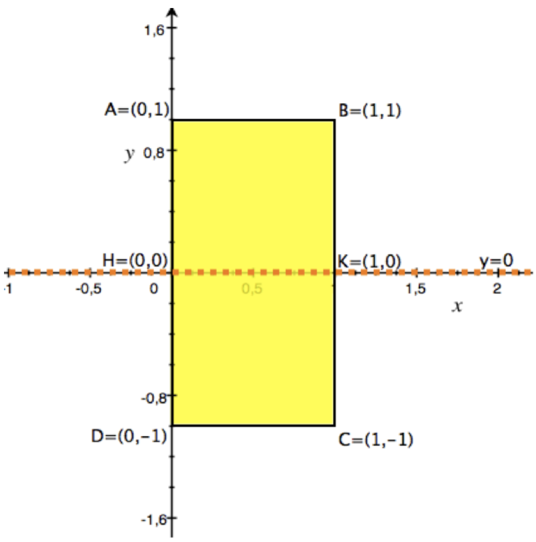

Figure 2. The critical space of the game

13. Payoff space

In order to represent graphically the payoff space $f(E \times F)$, we transform, by the function f, all the sides of bi-strategy rectangle $E \times F$ and the critical space Z of the game G.

The segment $[A, B]$ is the set of all the bi-strategies (x, y) such that $y = 1$ and $x \in [0, 1]$. Calculating the image of the generic point $(x, 1)$, we have:

$$f(x, 1) = (M_1[-\nu(1 - x)], M_2 mx).$$

Therefore setting

$$X = M_1[-\nu(1 - x)] \text{ and } Y = M_2 mx$$

and assuming $M_1 = 1, M_2 = 2$, and $\nu = m = 1/2$, we have $X = -1/2(1 - x)$ and $Y = x$.

Replacing Y instead of x, we obtain the image of the segment $[A, B]$, defined as the set of the bi-wins (X, Y) such that

$$X = -1/2 + Y \text{ and } Y \in [0, 1/2].$$

It is a line segment with extremes $A' = f(A)$ and $B' = f(B)$.

Following the procedure described above for the other side of the bi-strategy rectangle and for the critical space, that are the segments $[B, C]$, $[C, D]$, $[D, A]$ and $[H, K]$, we get the payoff space $f(E \times F)$ of our game G (fig. 3 and fig. 4 in 3D).

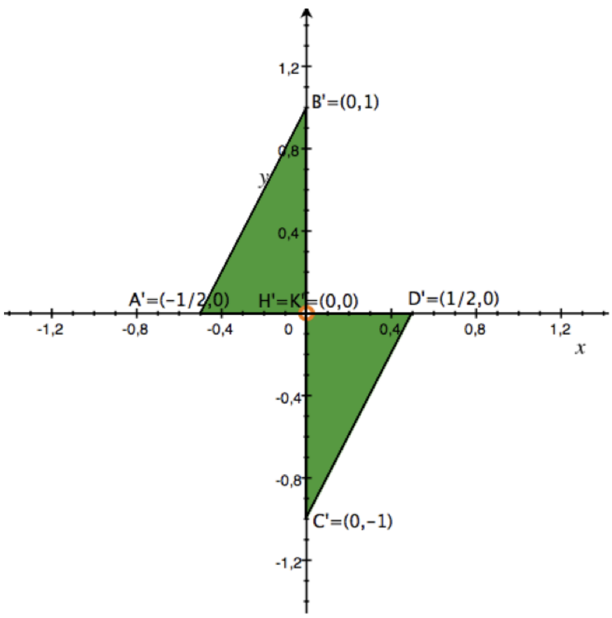

Figure 3. The payoff space of the game

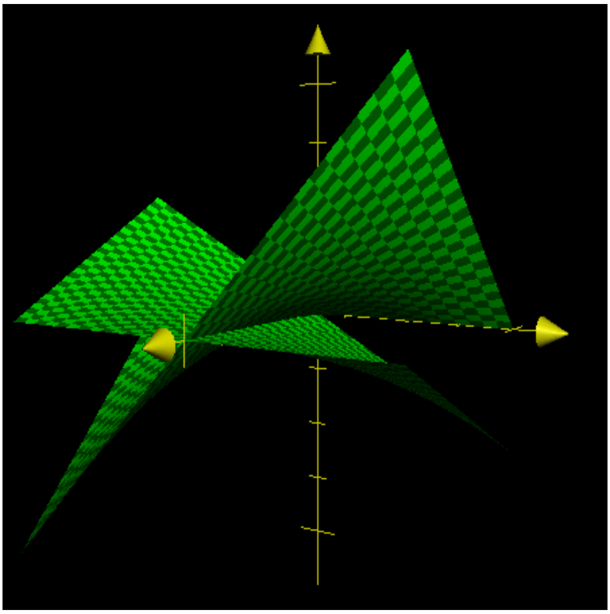

Figure 4. The payoff space of the game

We can see that the set of possible winning combinations of the two players takes a curious butterfly shape that promises the game particularly interesting.

14. Pareto boundaries

The superior extremum of the game is a shadow maximum because it does not belong to the payoff space: $\alpha = (1/2, 1) \notin f(E \times F)$.

The infimum of the game is a shadow minimum because it does not belong to the payoff space: $\beta = (-1/2, -1) \notin f(E \times F)$.

The weak maximal Pareto boundary of the payoff space is $[B', K'] \cup [H', D']$.

The proper maximal Pareto boundary of the payoff space is $\partial^* f(E \times F) = \{B', D'\}$.

The weak minimal Pareto boundary of the payoff space is $[A', H'] \cup [K', C']$.

The proper minimal Pareto boundary of the payoff space is $\partial_* f(E \times F) = \{A', C'\}$.

In fig. 5 we show the previous considerations.

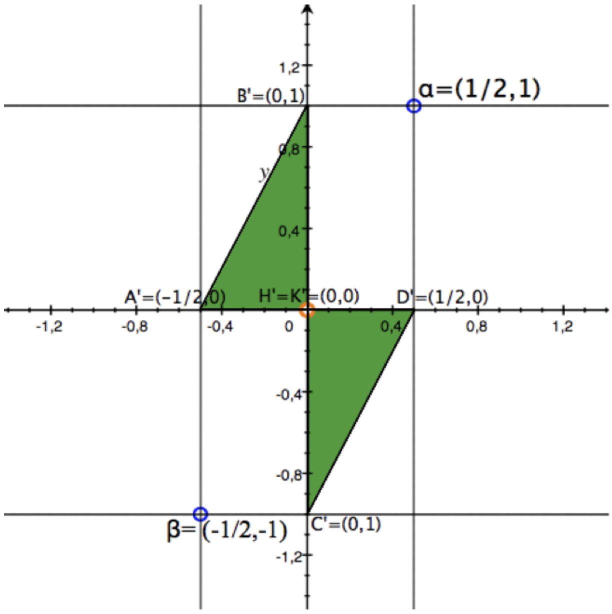

Figure 5. Pareto boundaries and extrema of the game

Control and accessibility of non-cooperative Pareto boundaries. *Definition of Pareto control*. The Enterprise can cause a Pareto bi-strategy x_0 if exists a strategy such that for every strategy y of the Financial Institute the pair (x_0, y) is a Pareto pair.

In this regard, in our game there are no maximal Pareto controls, nor minimal. So neither player can decide to go on the Pareto boundary without cooperation with the other one. The game promises to be quite complex to resolve in a satisfactory way for both players.

15. Equilibria of the game

The different game equilibria (Nash equilibria, conservative equilibria, offensive equilibria, devote equilibria) were already studied in [2]. There are no satisfactory equilibria for both players, and the main point of interest of the payoff space is the point $B' = (0, 1)$, which is the retro-image of the more likely Nash equilibrium and of the most likely conservative meeting. Let us go further.

16. Cooperative solutions

The best way for two players to get both a win is to find a cooperative solution. One way would be to divide the maximum collective profit, determined by the maximum of the collective gain functional g, defined by

$$g(X, Y) = X + Y$$

on the payoffs space of the game G, i.e the profit $W = \max_{f(E\times F)} g$.

The maximum collective profit W is attained (with evidence) at the point B', which is the only bi-win belonging to the straight line with equation $X + Y = 1$ and to the payoff space $f(E \times F)$.

So the Enterprise and the Financial Institute play $x = 1$ and $y = 1$, in order to arrive at the payoff B'. Then, they split the obtained bi-win B' by contract.

Financial point of view. The Enterprise buys futures to create artificially a misalignment between futures and spot prices, misalignment that is exploited by the Financial Institute, which gets the maximum win $W = 1$.

First possible division. For a possible **fair division** of this win $W = 1$, we employ a transferable utility solution: finding on the transferable utility Pareto boundary of the payoff space a *non-standard Kalai-Smorodinsky solution* (non-standard because we do not consider the whole game, but only its maximal Pareto boundary). We find the supremum of maximal Pareto boundary,

$$\sup \partial^* f(E \times F),$$

which is the point $\alpha = (1/2, 1)$, and we join it with the infimum of maximal Pareto boundary,

$$\inf \partial^* f(E \times F),$$

which is $(0, 0)$.

The coordinates of the intersection point P (see fig. 6), between the straight line of maximum collective win (i.e. $X + Y = 1$) and the straight line joining the supremum of the maximal Pareto boundary with the infimum (i.e., the line $Y = 2X$) give us the desirable division of the maximum collective win $W = 1$ between the two players.

Second possible division. For another possible **fair division** of the win $W = 1$, we propose a *transferable utility Kalai-Smorodinsky method*. The bargaining problem we face is the pair $(\Gamma, v^\sharp)$, where:

(1) our decision constraint Γ is the transferable utility Pareto boundary of the game (straight line $X + Y = 1$);
(2) we take the conservative bi-value $v^\sharp = (0, 0)$ as threat point of our bargaining problem.

Solution. For what concerns the solution: we join $v^\sharp$ with the supremum

$$\sup(\Gamma \cap [v^\sharp, \rightarrow [),$$

according to the classic Kalai-Smorodinsky method, supremum which is given by $(1, 1)$.

The coordinates of the intersection point P' (see fig. 6), between the straight line of maximum collective gain (i.e. $X + Y = 1$) and the segment joining the $v^\sharp$ and the considered supremum (the segment is part of the line $\mathbb{R}(1, 1)$), give us the desirable division of the maximum collective win $W = 1$, between the two players.

In fig. 6 is showed the situation.

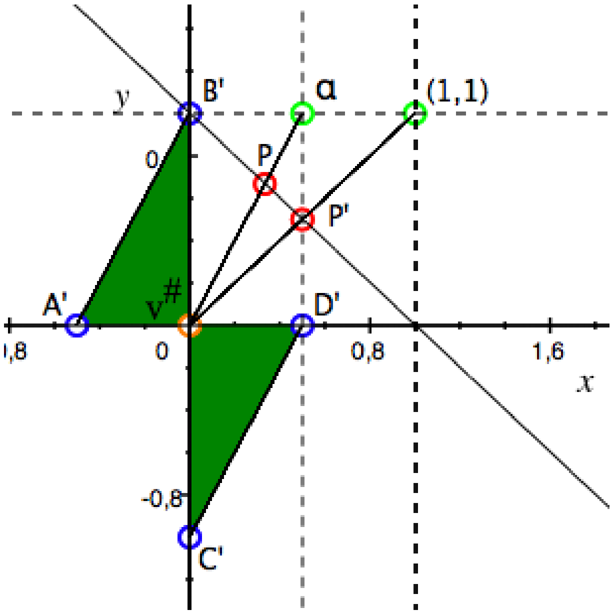

Figure 6. Transferable utility solution: cooperative solutions

Thus $P = (1/3, 2/3)$ and $P' = (1/2, 1/2)$ suggest as solution that the Enterprise receives respectively $1/3$ or $1/2$ by contract by the Financial Institute, while at the Financial Institute remains the win $2/3$ or $1/2$.

Why are there differences between the two possible division of collective profit? The difference between the points P and P' are due to the different method used.

About the point P, we consider as threat and utopia point respectively the $\inf$ and the $\sup$ of the maximal Pareto boundary. Therefore, the division is more profitable for the Financial Institute because it can obtain in the game a higher maximum profit (that is 1) than the Enterprise (that can obtain 1/2).

About the point P', we consider as threat point the conservative bi-value $v^\sharp = (0, 0)$ and its supremum $(1, 1)$, according to the classic Kalai-Smorodinsky method. Therefore, the division is equally profitable for both players because they have an equal conservative value $v_1^\sharp = v_2^\sharp = 0$.

17. Conclusions

The game just studied suggests a possible regulatory model providing the possibility to make more stable the financial market through the introduction of a tax on financial transactions. In fact, in this way, it could be possible to avoid arbitrary and uncontrolled speculative attacks. The banks could equally gain without burdening on the financial system by unilateral manipulations of asset prices: if the Financial Institute wants to obtain a profit, it is forced to an interaction with the Enterprise (which represents any possible real economic subject). Otherwise, without introducing the financial transaction tax, the Financial Institute could exploit the misalignment between spot and futures prices, caused by itself at any time and with any frequency.

The **unique optimal solutions** are cooperative solutions (exposed in the section **16**), otherwise the game appears like a sort of "your death, my life". This type of situation happens often in the economic competition and leaves no escapes if either player decides to work alone, without a mutual collaboration. In fact, all non-cooperative solutions lead dramatically to mediocre results for at least one of the two players.

Another reading key. Now it is possible to provide an interesting key in order to understand the conclusions which we reached using the two transferable utility solutions. Since the point $B = (1, 1)$ is also the most likely Nash equilibrium, the values $1/3$ or $1/2$ (that the Financial Institute pays by contract to the Enterprise) can be seen as the fair price paid by the Financial Institute to be sure that the Enterprise chooses the strategy $x = 1$, so they arrive effectively to more likely Nash equilibrium $B = (1, 1)$, which is also the optimal solution for the Financial Institute.

Recapitulation. In this paper we have showed a normative model where:

- a tax on financial transactions makes more stable the financial markets, limiting the speculative actions of big financial operators, which have not immediate interest to speculate;
- this tax does not inhibit the possibility of profits for the financial subjects, but this possibility is restricted to an interaction with a real economic operator;
- this interaction leads a mutual profit only if the two operators enter into a prior agreement of cooperation;
- in this way, we obtain a possible fair redistribution of the financial profits, which arrive (at least in part) in the real economy.

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