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EQUIVALENCE PRINCIPLE, HIGGS BOSON AND COSMOLOGY

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ABSTRACT. We discuss here possible tests for Palatini $f(\mathscr{R})$-theories together with their implications for different formulations of the Equivalence Principle. We shall show that Palatini $f(\mathscr{R})$-theories obey the Weak Equivalence Principle and violate the Strong Equivalence Principle. The violations of the Strong Equivalence Principle vanish in vacuum (and purely electromagnetic) solutions as well as on short time scales with respect to the age of the universe. However, we suggest that a framework based on Palatini $f(\mathscr{R})$-theories is more general than standard General Relativity (GR) and it sheds light on the interpretation of data and results in a way which is more model independent than standard GR itself.

1. Introduction

The Higgs Boson seems to have been finally detected (ATLAS Collaboration 2012; CMS Collaboration 2012). If this will be eventually confirmed and its structure will be properly understood, for example finding it to be the first fundamental scalar particle ever detected, this will close the chapter of Physics known as *the Standard Model* (of particle physics). This probably does not mean that the Standard Model will be the final word on particle physics but in any case that will be a remarkable event in which by a more or less coherent theory we are able to compute the outcome of any conceivable experiment in a specific context, and we can support by experiments any detail of the model.

As is well known, the Higgs Boson provides a mechanism to spontaneously break electroweak symmetry and to give (or one should say *'by'* giving) to $W^{\pm}$ and Z_0 Bosons (as well potentially to other observed particles) their observed inertial rest mass. This sounds however strange in view of the Equivalence Principle that is one of the fundamental principles of any conceivable theory of Gravity (recall that *mass* is the *gravitational charge* of a particle. We shall review below different formulations of the Equivalence Principle; for the moment we are satisfied with the formulation which states the equality between inertial and gravitational masses.

If inertial masses are *'decided'* by the Higgs Boson and the details of the Standard Model when the Standard Model itself is not about Gravity, how can be claimed that inertial masses are equal to gravitational masses?

One possible answer could be that the Standard Model *is* about Gravity even if it does not appear to be so (see also Capozziello, Basini, and Laurentis 2011). This is probably

what Superstring Theory would expect. The Standard Model would be a low energy limit of a more general, still unknown, framework in which Gravity is included. If this were the case, the Higgs mechanism would be the particle side of a more general unknown mechanism which acts also on the gravitational side by making the gravitational mass and inertial mass to be equal. This is a possibility. Actually a possibility which is not supported by any experiment, but still a possibility.

Another possibility is that there is nothing (or nothing we can guess now) hidden behind the Higgs mechanism and that it tells us something about the Equivalence Principle and gravitational theories. For example, there are several formulations of the Equivalence Principle and maybe some of them are more friendly than others with respect to the Higgs mechanism. This is particularly interesting to be discussed since it is known that some formulation of the Equivalence Principle is stricter than others in constraining gravitational dynamics. For example, it is commonly believed that the Strong Equivalence Principle allows only standard General Relativity (GR). If the Higgs mechanism is found to go along with the Weak Equivalence Principle and contradict the Strong Equivalence Principle, then this might be the indication that more general dynamics than standard GR should be considered for Gravity.

We shall discuss below what we really know about gravitational masses and argue that while there is a strong evidence in favor of the Weak Equivalence Principle there is no much evidence about the Strong Equivalence Principle. Accordingly, while there is a strong experimental and theoretical support to general relativistic theories, standard General Relativity is considerably less supported. It is just one possible theory among many possibilities and, moreover, it is degenerate under many viewpoints (Capozziello, De Laurentis, Fatibene, *et al.* 2012; Di Mauro *et al.* 2010; Fatibene *et al.* 2010; Olmo and Singh 2009).

2. Formulations of the Equivalence Principle

As we said, there are different formulations of the Equivalence Principle (EP). They are all concerned with *local experiments*. By a local experiment one means any experience which relies on observations within a region of spacetime which is *small*, i.e. in which the gravitational field is (approximately) constant. Traditionally, one discusses experiments in a free falling lift, the lift being small enough to allow neglecting tidal forces (and the experiment being short enough so that the lift keeps being freely falling).

Einstein (1922) originally stated the EP as follows:

> *A little reflection will show that the law of the equality of the inertial and gravitational mass is equivalent to the assertion that the acceleration imparted to a body by a gravitational field is independent of the nature of the body. For Newton's equation of motion in a gravitational field, written out in full, it is:*
>
> $$m_{in}\vec{a} = m_{gr}\vec{g} \tag{1}$$
>
> *It is only when there is numerical equality between the inertial and gravitational mass that the acceleration is independent of the nature of the body.*

This is of course slightly overstated since what matters is that the inertial and gravitational mass are proportional (and of course the proportionality constant G can be merged into the definition of the intensity of the gravitational force $\vec{g}$) provided that the proportionality constant is universal.

The *weak form of the Equivalence Principle* (wEP) can be stated as follows

> *The free falling trajectory of a point mass in a gravitational field depends only on its initial position and velocity, and it is independent of its composition and its rest mass.*

or

> *The outcome of any local gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.*

The weak EP claims the universality of free falling and thence is a motivation for the geometrization of Gravity. When all test particles fall in the same way one can regard Gravity as a property of spacetime (hence a geometrical property) and test particles fall only feeling the structures which describe the geometry of spacetime itself.

One can show that under few general physical assumptions about free falling, the geodesic trajectories of some projective structure of connections easily follows (Fatibene, Francaviglia, and Magnano 2012; Schouten 1954).

A similar result was found by Ehlers, Pirani and Schild (EPS) in a seminal paper in 1972 (Ehlers, Pirani, and Schild 1972). By assuming reasonable properties about light rays and free falling particles one can show that spacetime geometry is described by a conformal class $[g]$ of Lorentzian metrics and a projective class $[\Gamma]$ of (linear) connections, i.e. a projective structure $\mathfrak{P}$. These two structures are loosely related by a compatibility conditions (which comes from the evidence that light rays themselves are free falling as well). Then one can gauge fix the projective gauge freedom by choosing a suitable representative connection $\tilde{\Gamma}$ in the class $[\Gamma]$.

However, the connection which determines free falling is related to but not uniquely induced by the metric structure (as in standard GR). Even once the metric structure has been fixed in the conformal class one still has a residual freedom in choosing compatible free falling. This freedom is encoded in a covector A_λ and the connection $\tilde{\Gamma}$ is in the form

$$\tilde{\Gamma}^\alpha_{\beta\mu} = \{g\}^\alpha_{\beta\mu} + \left(g^{\alpha\lambda} g_{\beta\mu} - 2\delta^\alpha_{(\beta}\delta^\lambda_{\mu)}\right) A_\lambda \tag{2}$$

where $\{g\}^\alpha_{\beta\mu}$ denotes the Levi-Civita connection of the metric g. Notice that the connection $\tilde{\Gamma}$ is not even required to be metric (and in fact it is not if the covector A is not exact). Then according to EPS analysis the geometric structure of spacetime should be described as a *Weyl geometry* $(M, [g], \tilde{\Gamma})$.

There are quite general classes of dynamics such that assuming independent metric and connection as variables then the dynamics itself enforces the EPS-compatibility condition (2) to be fulfilled at least *a posteriori* if not *a priori* as it happens in standard GR. These are called *extended theories of gravitation*. If dynamics also enforces A to be exact then the theory is called an *extended metric theory of gravitation* (Capozziello and De Laurentis 2011; Nojiri and Odintsov 2011).

In standard GR one not only assumes *a priori* that free falling is metric, but one also assumes that it is determined by the original metric used in the variational principle g, which is the same metric that determines also light cones and which is also used to define distances. Accordingly, standard GR is not generic as an extended theory of gravitation; it is not even generic as an extended *metric* theory of gravitation.

In a generic extended metric theory of gravitation one starts from a metric g and a connection $\tilde{\Gamma}$, independent *a priori*. Then dynamics forces the connection $\tilde{\Gamma}$ to be metric for some metric $\tilde{g}$, i.e. $\tilde{\Gamma} = \{\tilde{g}\}$, and compatible with g. Then one finds that $\tilde{g}$ is necessarily conformal to g. It is still true that one can describe free fall and light cones by a single metric $\tilde{g}$; however the dynamics is more general than in standard GR. In fact we know a whole class of extended metric theories, namely $f(\mathscr{R})$-theories in Palatini formulation, which are more general than standard GR, as we shall see in the next Section.

Since in Palatini $f(\mathscr{R})$-theories free fall is universal they obey wEP. Another example of theory satisfying wEP is Brans-Dicke theory. In Brans-Dicke theory one starts from a metric g and a scalar field ϕ. The gravitational dynamics is written in terms of both g and ϕ while g alone determines free falling, light cones and distances. The scalar field ϕ replaces the gravitational constant G which becomes pointwise dependent.

Any Palatini $f(\mathscr{R})$-theory can be shown to be *mathematically* equivalent to a Brans-Dicke theory through a conformal transformation. However, the two theories are *physically* inequivalent since free fall is assumed to be described by different metrics. In Palatini $f(\mathscr{R})$-theory free falling is described by $\tilde{g}$, while in the corresponding Brans-Dicke theory it is described by g. In both cases it is universal and geometric, though (Fatibene *et al.* 2012).

Let us now come to the Strong Equivalence Principle. The *strong form of Equivalence Principle* (sEP) can be stated as follows

> *The outcome of any local experiment (gravitational or not) in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.*

Strong EP implies that the gravitational interaction is the same everywhere in the universe and it has always to be the same. For this reason it is commonly believed that sEP implies standard GR. If Gravity were mediated by a metric together with some other universal interaction (sometimes called some *fifth force*) Gravity would fail to obey sEP. Accordingly, most of the literature about testing sEP is concerned with discussing Brans-Dicke theories in which gravitational interaction is described by a metric *and* a scalar field.

There is some evidence in favor of sEP from nucleosynthesis, which seems to imply that the universal constant G cannot be varied for more than 10% (Capozziello *et al.* 2009a; Capozziello and Francaviglia 2008). However, 10% is not a strong constraint. Moreover, although these claims are extremely important from a heuristic point of view, it is difficult to draw general consequences from this kind of claims which are not stated as mathematical propositions.

3. Palatini $f(\mathscr{R})$-theories

Let us here review Palatini $f(\mathscr{R})$-theories. They are extended metric theories of gravitation and we shall discuss various forms of the Equivalence Principle in this case. Let M be

a spacetime manifold (of dimension 4) and let us consider a Lagrangian in the form

$$L = \sqrt{g} f(\mathscr{R}) + L_m(\varphi, g) \tag{3}$$

where f is a generic (analytic or *sufficiently regular*) function, φ is a collection of matter fields and we set $\mathscr{R}(g,\tilde{\Gamma}) := g^{\mu\nu}\tilde{R}_{\mu\nu}$ where $\tilde{R}_{\mu\nu}$ is the Ricci tensor of the independent connection $\tilde{\Gamma}$. Let us remark that *a priori* the Ricci tensor $\tilde{R}_{\mu\nu}$ of the connection $\tilde{\Gamma}$ is not necessarily symmetric since the connection is not necessarily metric.

With this choice we are implicitly assuming that matter fields φ minimally couple with the metric g, which in turn encodes electromagnetic fundamental structures (photons and light cones). Since gravity, according to EPS formalism, is mostly inherent with the Equivalence Principle and free fall, that are encoded in the projective structure $\mathfrak{P}$, one should better assume that matter couples *also* with $\tilde{\Gamma}$ and investigate the more general case in which the matter Lagrangian has the more general form $L_m(\varphi, g, \tilde{\Gamma})$. However, this case in much harder to be investigated since it entails that a second stress tensor is generated by the variational derivative

$$\sqrt{g} T_\alpha^{\mu\nu} = \frac{\delta L_m}{\delta \tilde{\Gamma}^\alpha_{\mu\nu}} \tag{4}$$

No relevant general progress in this direction is still at hands, although it should correspond to an even more physically reasonable situation. However, even if there is no general understanding of this larger class of theories, a few concrete and significative examples have been worked out insofar (Fatibene *et al.* 2010; Fatibene, Francaviglia, and Mercadante 2010; Olmo and Singh 2009).

As we said the matter Lagrangian L_m is here assumed to depend only on matter and metric (together with their derivatives up to order 1). Thus if one needs covariant derivatives of matter fields they are explicitly defined with respect to the metric field g. Requiring that the matter Lagrangian does not depend on the connection $\tilde{\Gamma}$ is a standard requirement to simplify the analysis of field equation below, although (as we said above) it would correspond to more reasonable physical situations. Let us notice here that what follows can be in fact extended to a more general framework; there are in fact matter Lagrangians depending on the connection $\tilde{\Gamma}$ in which field equations still imply the EPS-compatibility condition (2) (Capozziello *et al.* 2010; Capozziello, De Laurentis, Fabbri, *et al.* 2012; Fatibene *et al.* 2010; Sotiriou 2009; Sotiriou and Liberati 2007). Field equations of (3) are

$$\begin{cases} f'(\mathscr{R})\tilde{R}_{(\mu\nu)} - \frac{1}{2} f(\mathscr{R}) g_{\mu\nu} = \kappa T_{\mu\nu} \\ \tilde{\nabla}_\alpha \left(\sqrt{g} f'(\mathscr{R}) g^{\beta\mu} \right) = T_\alpha^{\beta\mu} = 0 \end{cases} \tag{5}$$

where $f'(\mathscr{R})$ denotes the derivative of the function $f(\mathscr{R})$ with respect to its argument $\mathscr{R}$.

We do not write explicitly the matter field equations which are standard and will be considered as matter equations of state. The constant $\kappa = 8\pi G/c^4$ is the coupling constant between matter and Gravity. The second stress tensor $T_\alpha^{\beta\mu}$ vanishes since the matter Lagrangian is assumed to be independent of the connection $\tilde{\Gamma}$. The first stress tensor $T_{\mu\nu}$ arises since the matter Lagrangian is a function of the metric

$$\sqrt{g} T_{\mu\nu} = \frac{\delta L_m}{\delta g^{\mu\nu}} \tag{6}$$

Notice that $T_{\mu\nu}$ depends both on the matter fields and the metric g.

Under these simplifying assumptions the second field equations can be solved explicitly. Let us consider in fact the conformal transformation $\tilde{g}_{\mu\nu} = f'(\mathscr{R}) \cdot g_{\mu\nu}$. Notice that the conformal factor encodes free fall, since it depends explicitly on $\mathscr{R}$, which is linear in g, linear in the first derivatives of $\tilde{\Gamma}$ and quadratic in $\tilde{\Gamma}$. One has

$$\tilde{g}^{\mu\nu} = (f'(\mathscr{R}))^{-1} \cdot g^{\mu\nu} \qquad \sqrt{\tilde{g}} = (f'(\mathscr{R}))^2 \sqrt{g} \tag{7}$$

and then

$$\sqrt{\tilde{g}}\tilde{g}^{\beta\mu} = \sqrt{g} f'(\mathscr{R}) \cdot g^{\beta\mu} \tag{8}$$

Thus the second field equation in (5) can be recast as

$$\tilde{\nabla}_\alpha \left(\sqrt{g} f'(\mathscr{R}) g^{\beta\mu} \right) = \tilde{\nabla}_\alpha \left(\sqrt{\tilde{g}} \tilde{g}^{\beta\mu} \right) = 0 \tag{9}$$

where $\tilde{\nabla}$ denotes the covariant derivative with respect to $\tilde{\Gamma}$. The condition (9) by the Levi-Civita theorem implies then

$$\tilde{\Gamma}^\alpha_{\beta\mu} = \{\tilde{g}\}^\alpha_{\beta\mu} \tag{10}$$

i.e. the connection $\tilde{\Gamma}$ is forced by dynamics to be the Levi-Civita connection of the conformal metric $\tilde{g}$ (Capozziello *et al.* 2009a). Thus in these theories the connection is *a posteriori* metric and the geometry of spacetime is described by a metric Weyl geometry. As a consequence the Ricci tensor $\tilde{R}_{\mu\nu}$ is symmetric being the Ricci tensor of the metric $\tilde{g}$.

The first field equation now reads as

$$f'(\mathscr{R})\tilde{R}_{\mu\nu} - \frac{1}{2} f(\mathscr{R}) g_{\mu\nu} = \kappa T_{\mu\nu} \tag{11}$$

The trace of this equation (with respect to $g^{\mu\nu}$) is so important in the analysis of these models that it has been called the *master equation* (Borowiec *et al.* 1998). It reads as

$$f'(\mathscr{R})\mathscr{R} - 2f(\mathscr{R}) = \kappa T := \kappa g^{\mu\nu} T_{\mu\nu} \tag{12}$$

For any given (analytic) function f the master equation establishes an *algebraic* (i.e. not differential) relation between $\mathscr{R}$ and T, that can, at least in principle, be solved. More precisely, we see that the function $F(\mathscr{R}, T) := f'(\mathscr{R})\mathscr{R} - 2f(\mathscr{R}) - \kappa T$ is also analytic and, excluding few degenerate cases (namely when $f(\mathscr{R}) = C_1\mathscr{R}^2 + C_2$), it can be generically solved for $\mathscr{R} = r(T) = \kappa \hat{r}(T)$. Many of the subsequent results still hold for non-analytic functions f, provided that they are sufficiently regular to avoid essential singularities and allow applying the implicit function theorems.

In vacuum or for purely electromagnetic matter obeying Maxwell equations one has $T = 0$, i.e. the trace T of the matter stress tensor $T_{\mu\nu}$ is zero so that $\mathscr{R}$ can take just constant values out of a discrete set that depends on f, i.e. $\mathscr{R} \equiv \rho \in \{\rho_0, \rho_1, \dots\}$, this set being uniquely fixed by the choice of the function f. In this vacuum (as well as in the purely electromagnetic) case, Palatini $f(\mathscr{R})$-theories are generically equivalent to Einstein models with cosmological constant and the possible value of the cosmological constant is chosen in a discrete set which depends on the analytic function f, since it depends on ρ. This is known as the *universality theorem* for Einstein equations (Borowiec *et al.* 1998).

Accordingly the Physics described by Palatini $f(\mathscr{R})$-theories in vacuum is not richer than standard GR Physics with cosmological constant. Still one should notice that in these vacuum $f(\mathscr{R})$-theories free fall is given by $\tilde{g}$ while in standard GR it is given by g (while distances are defined by g in both cases); however, the conformal factor $\phi = f'(\rho)$ is

constant in this case so that it does not affect geodesics and can be compensated by just a change of units for time and distances.

However, when *real matter* is present the situation is completely different. In this more general case we have that $\mathscr{R} = r(T)$ is no longer constant and it depends on the spacetime points. The first field equation becomes then

$$\tilde{G}_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{R}\tilde{g}_{\mu\nu} = \kappa\left(\frac{1}{f'(r(T))}\left(T_{\mu\nu} - \frac{1}{4}T g_{\mu\nu}\right) - \frac{1}{4}\hat{r}(T)g_{\mu\nu}\right) = \kappa\tilde{T}_{\mu\nu} \qquad (13)$$

so that a Palatini $f(\mathscr{R})$-theory with *real* matter behaves like standard GR for the conformal metric $\tilde{g}$, that is in turn related to g by $\tilde{g} = f'(\mathscr{R})\cdot g = f'(r(T))\cdot$, but with a strongly modified source stress tensor. Naively speaking, one can reasonably hope that the modifications dictated by the choice of the function f can be chosen to fit observational data and, in particular, they provide alternative views about the fugitive *'dark side'* of our Universe.

In a sense, the presence of standard visible matter φ (assumed to generate, through the matter Lagrangian $L_m(g,\varphi)$, an energy momentum stress tensor $T_{\mu\nu}$) would produce by gravitational interaction with $\tilde{\Gamma}$ (i.e. with the Levi-Civita connection of the conformal metric $\tilde{g} = f'(T)\cdot g$) a kind of *effective* energy-momentum stress tensor $\tilde{T}_{\mu\nu}$ in which whenever $T \neq 0$ standard matter φ is seen to exist together with *dark (virtual)* matter generated by the gauging of the rulers imposed by the T-dependent conformal transformations on g that disappears if and only if $T = 0$. In a sense, the *dark side* of Einstein equations can be mimicked by suitably choosing f and L_m and it can be seen as a curvature effect induced by $T = g^{\mu\nu}T_{\mu\nu} \neq 0$ (Capozziello *et al.* 2009a,b; Capozziello and Francaviglia 2008).

4. Strong Equivalence Principle

Obviously, any Palatini $f(\mathscr{R})$-theory satisfies wEP. Free fall is in fact universally determined by the metric $\tilde{g}$.

Let us now consider a situation to test sEP. For example, one can consider measurements of the (possibly effective) gravitational constant G at different locations and times. We have quite a number of tests on Earth. The constant G has been measured many times since 1798 when Cavendish used the torsion balance (which was invented by Michell) to measure it. A simplified version of Cavendish experiment consists in measuring the acceleration of a test particle free falling within the gravitational field generated by another known mass m. The motion does not depend on the mass of the test particle because of the wEP and the acceleration is correlated to the measurements of distances and to the quantity Gm. Being m known, then that is a measurement of G.

One could say that the same kind of measurement is involved in Astronomy whenever one measures the revolution period T and the radius a of a planet. By the celebrated Kepler third law (which applies approximately but quite well to the solar system) the quantity T^2/a^3 is also correlated to the product $\mu = GM$ of the constant G and the mass M of the Sun. Unfortunately, that is the *definition* used in Astronomy for the *mass of the Sun*.

What one generally does is to measure the orbital constant μ and define the mass of the Sun as $M = \mu/G$ using the value of the constant G measured now here on Earth, namely $G = (6.67384 \pm 0.00080)\cdot 10^{-11} m^3\, Kg^{-1}\, s^{-2}$. Thus to use these situation as a measurement

for G one should have an independent knowledge of the mass of the Sun, which in fact is not the case.

In principle an experiment measuring G very far away and/or very far in the past is simple. *Just* observe the motion of a test particle in the gravitational field of an object of known mass. There are interesting candidates of bodies of known mass. Supernovae of type Ia are one of them. Of course it is probably hard to see a test particle from 10^{10} light years away. However, it is important that tests are in principle possible. Another important issue is to reliably measure the distance between the gravitational source and the test particle.

Let us thence consider a Palatini $f(\mathscr{R})$-theory in a cosmological situation. A solution is a metric $\tilde{g}$ which satisfies Einstein equations (or better Friedmann-Robertson-Walker equations) with extra sources due to the modification of the dynamics (Olmo and Singh 2009). The metric $\tilde{g}$ describes the free falling and the light cones. As we said, the original conformal metric g comes from a gauge fixing of the conformal structure defined on spacetime. A gauge fixing of the conformal structure is, by definition, a protocol to define distances, so that *by construction* distances in spacetime are measured by using g. In standard GR there is no ambiguity since one has only one natural metric. In $f(\mathscr{R})$-theory distances should be measured by g, as in the equivalent Brans-Dicke theory. However, in the equivalent Brans-Dicke theory free falling is described by g, while in $f(\mathscr{R})$-theory it is described by $\tilde{g}$ (Fatibene *et al.* 2012).

The difference among standard GR, Brans-Dicke and $f(\mathscr{R})$-theory is parametrized by the conformal factor φ, which in Brans-Dicke is an independent field, in standard GR is $\varphi \equiv 1$, while in $f(\mathscr{R})$-theory is determined by the master equation as $\varphi = f'(\mathscr{R})$ and it depends on the scalar curvature $\mathscr{R}$.

In vacuum $f(\mathscr{R})$, the scalar curvature $\mathscr{R}$ turns out to be constant by field equations and the modification just amounts to an effective cosmological constant, which is strictly constant, depending on the function f. If one considers any observational constraint from solar system tests, such a constant needs to be very tiny and the Physics described by an $f(\mathscr{R})$-theory is practically identical to standard GR.

This analysis extends to any situation in which one uses vacuum solutions of Einstein equations. Let us remark that there are basically two empirical situations which are modeled by a non-vacuum solution (neglecting the electromagnetic field which leads to $T = 0$ and is somehow trivial). The two situations we mean are galactic dynamics and Cosmology. In studying galactic dynamics the galaxy is modeled by a fluid of radial density $\rho(r)$ (sometimes in Newtonian approximation). In Cosmology, matter is introduced as a fluid as well. Strangely enough these two situations were exactly the first two motivations for introducing dark matter and dark energy (Capozziello and Francaviglia 2008).

In these non-vacuum situations one has non-zero $\mathscr{R}$ as a function of the matter density and the conformal factor is non-constant and depends on the radial distance r or on t, respectively. Let us now consider Cosmology. Here the key issue is:

> *To what extent would we be aware of a conformal factor depending on time it were the case?*

For sure one could reasonably expect solar system effects to vanish or being negligible. For sure the induced modification of the cosmological constant G would be negligible at the Earth scales. For sure one could not expect to see it changing in three centuries on Earth!

On the other hand we remarked that astrophysical observations put quite loose constraints on G since hardly ever one knows independently the mass of very far objects.

As we noticed in Olmo and Singh (2009) a time dependent conformal factor may also affect interpretations of distances and cosmic acceleration which, to the best of our knowledge, have never been considered or taken into account in interpreting data.

In a first approximation one could expect a slowly varying conformal factor (slowly varying at cosmological scale as well, so that the constraints from nucleosynthesis may be too rough to be challenged). At any given time the conformal factor would be approximately constant so that it can be compensated by a suitable rescaling of units for distances and time. In this simplified scenario it would be as if rulers and clocks were slowly deformed during cosmological evolution, their evolution being governed by matter density of ordinary matter.

Looking at a far away system we would misjudge times and distances (for example period T and radius a of orbiting test particles) and since third Kepler law is not homogeneous in space and time, this would in turn imply a misjudging of the source mass, for example an extra contribution with respect to light sources to be called *dark matter*.

If no other information about far objects mass is available it would be relatively difficult to realize it. However, at least for strict Palatini $f(\mathscr{R})$-theories, it is not impossible to falsify this scenario. For example, if one wants to explain the whole of dark matter by this kind of scenario, one should expect that far away regions with similar amount of light matter and hence similar $\mathscr{R}$, would present similar amounts of *"observed" dark matter*.

Again to the best of our knowledge data about dark sources stratified by distances from us (and then by age and then by matter density T) are going to be available in the near future only.

5. Conclusions and perspectives

We considered the effects of Palatini $f(\mathscr{R})$-theories on sEP. In principle these models violate sEP, though a realistic test is probably bound to fail in view of the quality of data which are currently available. Some better data may be available in the near future.

Let us be explicit about expectations about data. Besides the fact that one should not have *expectations* about experimental data, we explicitly gave criteria to *falsify* Palatini $f(\mathscr{R})$-theories. It would probably be too much to expect that this simplistic scenario is enough to explain all or part of the effect related to dark sources. First of all some amount of dark matter may be really out there. It would probably naive to assume that we know and understand all fundamental matter; especially few months after that the big puzzle of Standard Model has been completed. Secondly, Palatini $f(\mathscr{R})$-theories are just one simple possibility of extending gravitational theory. One could consider models where dynamics is an arbitrary function F of more general invariants such as $R_{\alpha\beta}R^{\alpha\beta}$. Not very much is known about these more general theories of gravitation, except few specific workout examples (see, e.g., Ferraris 1986). It is already hard enough to obtain general results without specifying the function f, which already contains infinitely many parameters for fitting almost anything. Another option would be to consider non-minimally coupled Palatini gravity (Allemandi *et al.* 2005) which retained most of the structure here considered though allows more complicated conformal factor.

However, we believe that Palatini $f(\mathscr{R})$-theories are worth being considered exactly because they provide a reasonably more general framework than standard GR. Future data may confirm standard GR, dark matter and dark sources and any feature we still do not suspect. Still considering data in a wider framework allows to project experiments and interpret results more freely and allows to regard results more objectively, in such a way which is not too much entangled with a single model and are thus endowed with a more intrinsic and specific meaning, especially in gravitational theories in which observations are hardly ever model independent.

In some sense if Higgs mechanism is something which just concerns particle physics, one could still believe that gravitational free fall is universal; simply gravitational free falling does not depend on the mass. GR is *more fundamental* than Newtonian physics and most of the scandal about wEP comes from the second principle of dynamics (which is part of the Newtonian approximation). In view of the scenario described for $f(\mathscr{R})$-theories near the Earth now one expects $T \sim 0$, universality theorem applies and Newtonian limit is justified so that wEP is satisfied. It is considerably more difficult to imagine sEP together with Higgs mechanism. If the gravitational mass were found to change with time, then sEP would be saved only assuming that the inertial mass would also change in time, i.e. the Higgs coupling constant (which is something which lives in the Standard Model) would depend on time. One could not give any explanation of such a situation if not assuming that the Standard Model does in fact speak about Gravity as well; which we repeat may be the case though we do not have any evidence currently. On the other hand, if sEP is violated this does allow non-standard dynamics. Once again it would be clear that standard GR is one possible theory in a large family which could be better understood when considered in the bigger family. This is a common remark; standard tests of GR are discussed (see Weinberg 1972) in a more general setting (including Brans-Dicke theories) so that tests are found to be compatible with standard GR. Moreover, we do have evidences that the standard scenario about standard GR is contradicted by observations, so that one is forced to introduce dark sources to save standard dynamics. In our opinion, in a theory as GR were almost anything —also observations— is model dependent it is hard to support by experiments standard dynamics if one assumes *a priori* it is correct; only by arguing it may be modified one can test and show that such modifications are not real.

Again in our opinion, if one has to assume some (maybe conjectured but in the end still unknown in detail) hidden connection between Particle Physics and Gravity one should first go along the easy way and try to explain observations by relying on what is relatively better known and tested.

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