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SOME RESULTS IN THE NONLINEAR STABILITY FOR ROTATING BÉNARD PROBLEM WITH RIGID BOUNDARY CONDITION

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ABSTRACT. The scope of this article is to expose the stabilizing properties of rotation and solute gradient for the Bénard problem with (at least one-sided) rigid boundary conditions. We perform a linear investigation of the critical threshold for the rotating Bénard problem with a binary fluid, and we also make an investigation with a Lyapunov function for the particular problem of a rotating single fluid. In all the these cases an increase of the Taylor number has stabilizing effects.

In onore di Giuseppe Grioli
per i suoi 100 anni, con riconoscenza.

1. Introduction

The Bénard problem has attracted the attention of scientists since the early 1900. The first systematic treatment of the problem can be found in the book of Chandrasekhar [1], where the stability of the motionless solution is discussed in detail. As it follows from his and later analyses, the instability threshold strongly depends on the boundary conditions on the velocity field $\mathbf{U}$, the temperature Θ, and the concentration field Γ.

While Chandrasekhar's linear analysis allowed to determine the thresholds above which the rest solution is unstable and convection sets in, Joseph [9] approached the question of stability for the classic Bénard problem by choosing an appropriate Lyapunov functions (which in this case is the $L^2(\Omega)$ norm, also called *energy norm*), and by proving the nonlinear stability of the equilibrium. Joseph's analysis not only approaches the Bénard problem from a different angle, but also allows to show that the motionless solution is stable precisely below the thresholds obtained by Chandrasekhar, which gives a converse of Chandrasekhar's instability result. The result obtained by Joseph can be easily generalized to other problems whenever the linear operator is symmetric with respect to the scalar product [4, 13]. This technique of showing that nonlinear stability holds precisely up to the point in which linear instability sets in, is now referred to as *proving the coincidence between linear instability and nonlinear stability thresholds*, and has been used successfully in many articles.

The classical Bénard problem can be enriched with a variety of physical effects: uniform rotation of the fluid, presence of a magnetic field, presence of a solute in the fluid. All such effects change the dynamical equations by adding skew-symmetric terms in the linear operator, which typically stabilize the equilibrium solution [4, 19], i.e. raise the stability threshold.

At the end of the eighties, Straughan, Rionero, Galdi, Mulone, Padula [5, 6, 17] affined the technique of finding a Lyapunov function, and applied it to many of the above variations of the classical Bénard problem. It turned out that there are techniques to compute Lyapunov functions that show the stabilizing effects of skew-symmetric terms, and sometimes even prove the coincidence of linear instability and nonlinear stability (and in this last case the Lyapunov functions are called *optimal*). All the investigations above are made under the hypothesis that the boundary conditions are of stress-free type. Such assumption allows an a-priori choice of Fourier series expansion of the solutions, and permits an analytic approach to the problem.

Rigid boundary conditions, which are physically much more reasonable than stress-free ones, do not allow such analytic approach, and they hence make it much more difficult to investigate the stability of the motionless solution. While in the similar yet simpler problem of fluid flows in a porous medium, the coincidence between linear instability and nonlinear stability has been solved by Straughan [22] (see also [11]), in the fluid dynamics Bénard problem with stress-free boundary conditions the coincidence is only partially solved [17], and it seems possible that the coincidence can be proven only using a family of different Lyapunov functions tailored on the eigenvectors of the linearized system. On the other hand, results obtained with rigid boundary conditions are mostly numerical [2, 3], and the construction of an *optimal* Lyapunov function that yields coincidence of linearly unstable and nonlinearly stable thresholds is still an open question.

In this article, as far as we know for the first time, we show the stabilizing effect of the rotation when one or both the boundaries are rigid. The problem is complicated by the fact that the techniques developed in [17] cannot be used when the analytic expression of critical eigenvalues and relative eigenvectors is impossible to obtain. As a matter of fact, the construction of an optimal Lyapunov function for the rotating Bénard problem is a still open problem for general boundary conditions.

In Section 2 we give an overview of known linear results and some new results as well. In particular we perform a numerical linear analysis of the Bénard problem with rotation and solute, and plot the critical threshold of heat Rayleigh number depending on Taylor number and solute Rayleigh number. In Section 3 we provide a Lyapunov function for the purely rotating Bénard problem, and we use it to prove numerically the stabilizing effect of the rotation. For the construction of such a Lyapunov function, we have been guided by the stress-free case [16, 17], where analytic computations can be performed.

2. The problem and known results

Consider an incompressible newtonian fluid filling $\Omega = \mathbb{R}^2_{x,y} \times (-d/2, d/2)_z$, an infinite layer of thickness d. Denoting by $\mathbf{i}, \mathbf{j}, \mathbf{k}$ the unit vectors of the reference frame, we assume that the fluid is subject to the vertical action of gravity $\mathbf{g} = g\,\mathbf{k}$ and is uniformly rotating around the vertical axis with angular velocity $\hat{\Omega}\,\mathbf{k}$. The state of the fluid is determined by the velocity field $\mathbf{U}$, the pressure Π, the temperature Θ, and the solute concentration Γ. Denoting by $\rho(T, C) = \rho_0(1 - \alpha_T(T - T_0) + \alpha_C(C - C_0))$, the equations of the fluid in the Boussinesq approximation are

$$\begin{cases} \mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} = -\nabla\Pi/\rho_0 + \rho(T,C)\mathbf{g}/\rho_0 - 2\,\hat{\Omega}\,\mathbf{k}\times\mathbf{U} + \nu\Delta\mathbf{U} \\ \nabla\cdot v = 0 \\ \Theta_t + \mathbf{U}\cdot\nabla\Theta = \kappa_\Theta\Delta\Theta \\ \Gamma_t + \mathbf{U}\cdot\nabla\Gamma = \kappa_\Gamma\Delta\Gamma. \end{cases}$$

(For the validity of Boussinesq approximation see the very recent results of Gouin, Muracchini and Ruggeri [7].)

These equations are typically associated to boundary conditions $\mathbf{U} = 0$, called *rigid* boundary conditions (that we indicate with R) or $\mathbf{k}\cdot\mathbf{U} = 0$, $\partial_z(\mathbf{i}\cdot\mathbf{U}) = \partial_{\mathbf{z}}(\mathbf{j}\cdot\mathbf{U}) = \mathbf{0}$, called *stress-free* boundary conditions on the velocity (that we indicate with F). Such conditions can be given independently on each of the two boundaries. On the temperature and solute concentration typical conditions are $\Theta = \bar{\Theta}$ or $\Theta_z = \bar{F}$ (with $\bar{\Theta}$, $\bar{F}$ given real numbers), respectively $\Gamma = \bar{\Gamma}$ or $\Gamma_z = \bar{G}$. We indicate the above conditions on temperature respectively with T and H, on concentration the conditions are indicated respectively with C and K. The rigid boundary condition is physically more reasonable than stress-free since it corresponds to a solid boundary. On the other hand stress-free boundary conditions allow an analytical approach to the problem.

Let us now consider the stationary solution, that has the form $\mathbf{U} = 0$, $\Theta = \Theta_0 - \bar{F}z$, $\Gamma = \Gamma_0 - \bar{G}z$, and Π a quadratic polynomial in z. Denoting by $\mathbf{u} = (u, v, w)$ the disturbance to velocity, p the disturbance to pressure, ϑ and γ the disturbances to temperature and solute concentration, the evolution equations of a nondimensional disturbance of such motionless solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + (R\vartheta - C\gamma)\mathbf{k} + T\mathbf{u}\times k + \Delta u \\ \nabla\cdot\mathbf{u} = 0 \\ P_r(\vartheta_t + \mathbf{u}\cdot\nabla\vartheta) = Rw + \Delta\vartheta \\ P_c(\gamma_t + \mathbf{u}\cdot\nabla\gamma) = Cw + \Delta\gamma. \end{cases} \tag{1}$$

As is well known, applying curl and double-curl one can reduce the problem to

$$\begin{cases} \Delta w_t = \Delta^*(R\vartheta - C\gamma) - T\zeta_z + \Delta^2 w + \mathcal{N}_w \\ \zeta_t = Tw_z + \Delta\zeta + \mathcal{N}_\zeta \\ P_r\vartheta_t = Rw + \Delta\vartheta + \mathcal{N}_\vartheta \\ P_c\gamma_t = Cw + \Delta\gamma + \mathcal{N}_\gamma, \end{cases} \tag{2}$$

where $\zeta = (\nabla\times\mathbf{u})\cdot\mathbf{k}$ is the third component of the vorticity of $\mathbf{u}$, and where

$$\begin{aligned} &\mathcal{N}_w = -[\nabla\times(\nabla\times(\mathbf{u}\cdot\nabla\mathbf{u}))]\cdot\mathbf{k}, \\ &\mathcal{N}_\zeta = -[\nabla\times(\mathbf{u}\cdot\nabla\mathbf{u})]\cdot\mathbf{k}, \\ &\mathcal{N}_\vartheta = -P_r(\mathbf{u}\cdot\nabla\vartheta), \quad \mathcal{N}_\gamma = -P_c(\mathbf{u}\cdot\nabla\gamma) \end{aligned}$$

are nonlinear terms.

2.1. Linear analysis. In this subsection we recall the linear results for the rotating Bénard problem for a binary fluid with a combination of stress-free and rigid boundary conditions on velocity, fixed temperature or fixed heat flux, fixed solute concentration or fixed solute flux. This analysis has been performed e.g. in [2], we briefly recall here its derivation and show in the plots some unpublished results.

Imposing that solutions are in planform, i.e. are of the form

$$f(x, y, z, t) = e^{\sigma t} F(z) g(x, y),$$

the linear system associated to (2) is

$$\begin{cases} \sigma(D^2 - a^2)W = -a^2(R\Theta - C\Gamma) - TDZ + (D^2 - a^2)^2 W \\ \sigma Z = TDw + (D^2 - a^2)Z \\ \sigma P_r \Theta = RW + (D^2 - a^2)\Theta \\ \sigma P_c \Gamma = CW + (D^2 - a^2)\Gamma. \end{cases}$$

Here $g(x, y)$ a function of x, y periodic with wave vector (a_x, a_y) such that $\Delta g + a^2 g = 0$, and $a^2 = a_x^2 + a_y^2$ is the wave number.

In [2] the authors investigate the interaction between the stabilizing effects of rotation and solute concentration. In Figure 1 we complement such results with the analysis of the cases in which the boundary conditions are rigid and fixed heat flux on one side and stress-free and fixed temperature on the other (RH-FT). We impose fixed concentration at the boundaries (C-C).

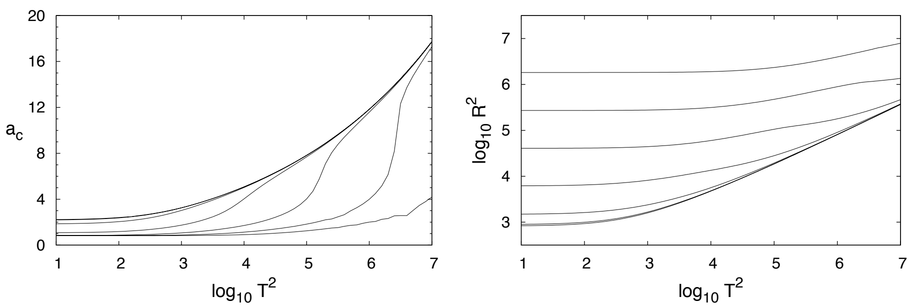

Figure 1. In right panel, the dependence on Taylor number T^2 of the critical thresholds for Rayleigh numbers R^2 in the case of RHC-FTC boundary conditions. In this plot $P_r = P_c = 1, C = 10, 10^2, \ldots, 10^7$ from bottom to top. The left panel represent the dependence of the critical wave number on T^2.

In Figure 2 we plot the effect of solute Rayleigh number in the case of fixed heat flux on both sides (RHC-RHC).

We finally plot in Figure 3 the critical threshold of temperature Rayleigh number for the case of single fluid and $P_r = 1$. In this case the instability can only occur as stationary convection, we can hence pose $\sigma = 0$ and compute the spectrum of the above eigenvalue problem using a Chebyshev-tau algorithm. The plots shown in Figure 4 of [3] can be complemented with such plots, which represent the critical (linear) Rayleigh number R_c as function of the Taylor number T with two possible boundary conditions: rigid and fixed heat flux in both boundaries (which is also shown in the cited article) and rigid with fixed heat flux in one boundary and stress-free and fixed temperature in the other.

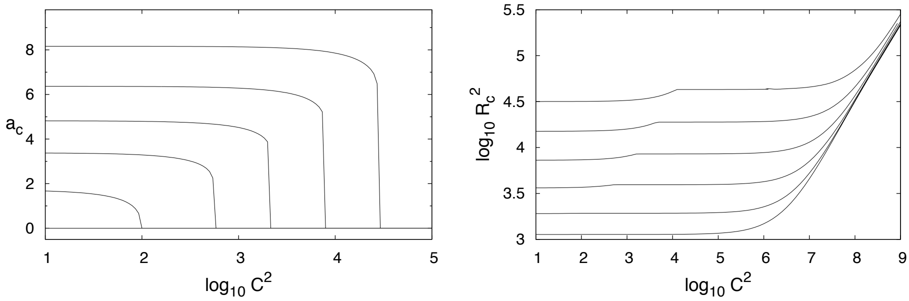

Figure 2. In right panel, the dependence on solute Rayleigh number C^2 of the critical thresholds for Rayleigh numbers R^2 in the case of RHC-RHC boundary conditions. In this plot $P_r = P_c = 1, T = 10^3, 10^{3.5}, 10^4, \ldots, 10^{5.5}$ from bottom to top. The left panel represent the dependence of the critical wave number on C.

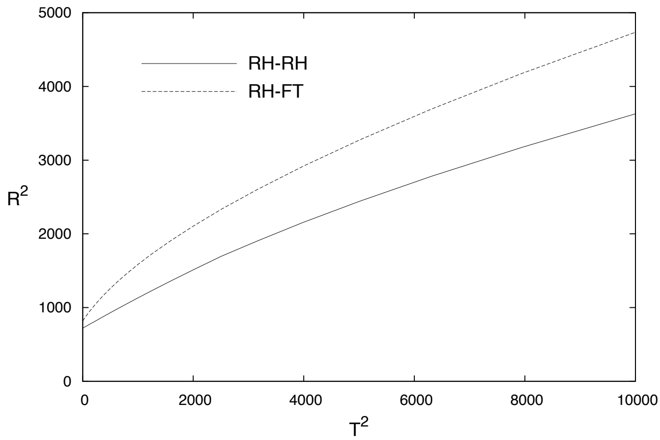

Figure 3. Dependence on Taylor number T^2 of the critical thresholds for Rayleigh numbers R^2 in the case of RH-RH boundary conditions (solid) and RH-FT boundary conditions (dashed).

3. Nonlinear stability

The stabilizing effects of rotation in the Bénard problem have been analyzed in [5] and [17, 18] with stress-free, and fixed temperature boundary conditions. In [5] the authors gave a possible Lyapunov functional and proved the stabilizing effect of rotation for the nonlinear system, they also gave a nonlinear critical threshold for the Rayleigh number that is numerically shown to be below the linear critical threshold. It was later shown in [17] the existence of a Lyapunov function which gives the coincidence of nonlinear stability and linear instability thresholds for T^2 up to $80\pi^4$ (see also [10]). Although no proof exists, we expect that the gap between nonlinear stability and linear instability is due to a poor choice of the Lyapunov function, or to non-sharp estimates in the investigation of monotonicity of its orbital derivative. Despite the fact that in many articles has been conjectured the existence of a Lyapunov function which gives coincidence, for large T such coincidence is still unproven, and for many accurately chosen Lyapunov functions it is possible to find explicit solutions of the problem, converging to zero, along which such functions are not monotonically decreasing.

A large part of the community shares the idea that coincidence can always be shown with energy methods, and even the puzzling experimental results of Rossby [20] can be interpreted in a way that excludes the existence of subcritical instabilities [8, 14].

3.1. Our results with the energy method. The Lyapunov function introduced in [5], later revisited and adapted in [17], has been reconsidered and simplified in [15, 16]. It turns out that to prove the coincidence of linear and nonlinear thresholds up to $T^2 = 80\pi^4$ (in the stress-free case) it is possible to use the function $E = E_1 + bE_2$ with

$$E_1 = \frac{1}{2}\left(\frac{R^2a^2}{R^2a^2 - T^2\pi^2}||R\zeta_z + T\pi^2\vartheta||^2 + (R^2a^2 - T^2\pi^2)||\nabla w||^2 + ||T\zeta_z + Ra^2\vartheta||^2\right),$$

$$E_2 = \frac{1}{2}\left(||\nabla\mathbf{u}||^2 + P_r||\Delta\vartheta||^2 + ||\nabla\zeta||^2 + ||\Delta w||^2 + ||\nabla\Delta^*\mathbf{u}||^2\right).$$

The nonlinearities can be estimated with techniques similar to those in [16], but in this article we simply plan to show that a numerical investigation of the stability threshold with such Lyapunov function gives, in the RH-FT case, a line that lies below the linear instability threshold for every value of the parameter, but is monotonically increasing.

Since our nonlinear analysis is restricted to the case of a single rotating fluid, whose equations are

$$\begin{cases} \Delta w_t = R\Delta^*\vartheta - T\zeta_z + \Delta^2 w + \mathcal{N}_w \\ \zeta_t = Tw_z + \Delta\zeta + \mathcal{N}_\zeta \\ P_r\vartheta_t = Rw + \Delta\vartheta + \mathcal{N}_\vartheta, \end{cases} \tag{3}$$

and since we also consider only the case $P_r = 1$, a natural choice of fields is $\phi = R\Delta^*\vartheta - T\zeta_z$, and $\psi = R\zeta - T\vartheta_z$ allows to recast system (3) as

$$\begin{cases} \Delta w_t = \phi + \Delta^2 w + \mathcal{M}_w \\ \phi_t = R^2\Delta^* w - T^2 w_{zz} + \Delta\phi + \mathcal{M}_\phi \\ \psi_t = \Delta\psi + \mathcal{M}_\psi, \end{cases} \tag{4}$$

where $\mathcal{M}_w, \mathcal{M}_\phi, \mathcal{M}_\psi$ can be expressed in terms of u, v, w, ϕ, ψ. Observe that ϕ is precisely the field associated to $s = R\vartheta\mathbf{k} + T\mathbf{u}\times\mathbf{k}$ in [16] once the double curl is applied, and is deduced from the expression of the eigenvector associated to the critical eigenvalue in the stress-free case.

Remark 1. *The third equation has a clearly stabile behaviour, and is uncoupled with the other two. It follows that the stability of the system requires the investigation of the first two equations of (4). Hence the zero solution of system (3) is stable if and only if the origin* $w = 0$, $\phi = 0$ *is stable for the system* ($4_{1,2}$).

Boundary conditions that transform into boundary conditions for the fields ϕ, ψ are either stress-free on velocity and fixed temperature (FT), which induce the conditions $\phi = 0$ and $\psi_z = 0$, or rigid boundary conditions on velocity and fixed heat flux (RH), which induce the conditions $\phi_z = 0$ and $\psi = 0$. These are the conditions that we allow at the boundaries of the fluid.

The Lyapunov function has a much simpler expression in this new set of fields. In fact, disregarding the role of ψ that is clearly stabilizing, the Lyapunov function E_1 has the form

$$E = \frac{1}{2}\left(\lambda||\nabla w||^2 + ||\phi||^2\right). \tag{5}$$

When doing a numerical nonlinear investigation, we will use this last function. The *complementary energy* E_2 [12] does not play a role in this numerical analysis, since its role is exclusively that of estimating the nonlinear terms (see [5, 6, 17, 21]).

The coefficients of the quadratic form E_1 have been obtained in [16] using the *method of canonical reduction* (see [12] and references therein). This method gives the best coefficients, and hence the λ that we numerically obtain by maximizing the critical Rayleigh number must, in the stress-free case, coincide with $R^2a^2 - T^2\pi^2$. This is shown in Figure 4 left. In this article, we investigate the case of RH-FT boundary conditions. The best coupling parameter λ determined numerically in this case differs from the parameter determined by Mulone [16] with the method of canonical reduction, as can be seen in Figure 4 right.

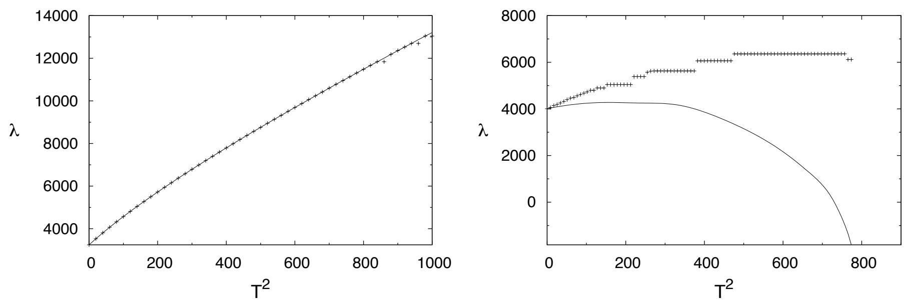

Figure 4. Plot of $R_c^2(T)a_c^2(T) - T^2\pi^2$ (solid) and, represented as a train of data, the numerically determined values of the coefficient λ in the Lyapunov function in equation (5). The left pane represents the plots for the FT-FT case, the right pane for the RH-FT cases.

The stabilizing effect of rotation in the rigid case, with the admissible boundary conditions of fixed heat flux, cannot be treated analytically and, to our knowledge, has not yet been analyzed numerically. In this short note we prove, using the a Chebyshev-tau algoritm, that the Lyapunov function (5) gives a threshold of nonlinear stability increasing with T. The orbital derivative of E is

$$\dot{E} = I - D$$

where

$$I = R^2(\Delta^* w, \phi) - T^2(w_{zz}, \phi) - \lambda(\phi, w), \qquad D = ||\nabla\phi||^2 + \lambda||\Delta w||^2. \tag{6}$$

Considering

$$\dot{E} = D(I/D - 1)\,,$$

the orbital derivative is negative-definite as long as

$$\max I/D < 1\,.$$

The Euler-Lagrange equations associated to this maximum problem are

$$\begin{cases} R^2\Delta^* w - T^2 w_{zz} - \lambda w + 2m\Delta\phi = 0 \\ R^2\Delta^*\phi - T^2\phi_{zz} - \lambda\phi - 2m\lambda\Delta\Delta w = 0. \end{cases} \tag{7}$$

This system (posing $m = 1$) can be thought as an eigenvalue problem in R, which can be solved numerically by a Chebyschev-tau method [21]. The nonlinear critical threshold can be compared with the linear one in Figure 5.

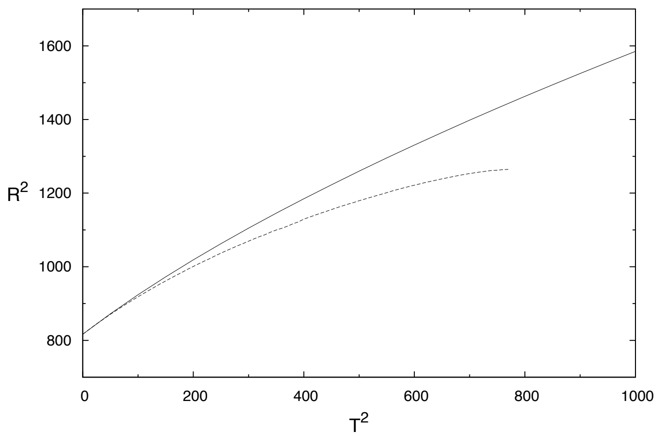

Figure 5. Linear and nonlinear critical threshold for the RH-FT boundary conditions (solid and dashed respectively).

4. Conclusions

There are many directions along which this kind of investigations can evolve. The first question to address is the existence, even for the stress-free case, of an optimal Lyapunov function or of a family of Lyapunov functions that prove coincidence.

With more general boundary conditions, the basic motion could have non-zero velocity. In such case the nondimensional parameters will depend on space, and many open questions exist already at the level of linear analysis. When the boundary conditions are non-stationary, or when the forces acting on the system explicitly depend on time, the basic solution is non-stationary, and the stability of such a solution requires the investigation of non-autonomous differential equations.

The non-autonomous case has many relevant applications, for example the sun heating a layer of water. A numerical investigation can be performed also in such cases, while any theoretical approach requires careful validation [4].

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