

## SOLVING DIFFRACTION PROBLEMS BY METHOD OF ELEMENTARY SCATTERERS

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**ABSTRACT.** The method of modeling scattering characteristics of bodies with complex geometry and structure using elementary scatterers, which together reproduce the geometry and structure of initial object, is proposed. The efficiency of this approach is shown by a simple example the diffraction problem on a strip.

### 1. Introduction

One of the Pattern Equation Method (PEM) [1] advantages is a weak dependence of the speed and accuracy of computational algorithm on the distance between scatterers. This fact prompted us to model scattering characteristics of complex geometry bodies by solving wave diffraction problem on the group of bodies with more simple geometry, which together reproduce the original complex object. Calculations have proven the efficiency of such approach [2]. Natural generalization of this idea is to substitute a scatterer with complex geometry and structure, for example a non homogeneous magneto-dielectric scatterer, by the group of simple homogeneous bodies, for example spheres (or circles in two dimensional case), which sizes are small in comparison to the wavelength. Note, that the problem of wave scattering on ensemble of spherical particles was treated earlier based on T-matrix method [3] (see also [4]). It is expedient to use PEM in this approach, because the solution of corresponding problem diffraction problem on a group of  $N$  bodies with small wave sizes could be obtained analytically [1] by reducing the problem to an algebraic system of  $N$  equations.

### 2. Formulation of the problem and its solution

Let us consider a realization of this idea using the example of diffraction on a perfectly conductive cylindrical scatterers, when electric intensity vectors of incident and scattered fields has single component  $E_Z$ , oriented along generatrix of these cylindrical bodies. In this case problem becomes two-dimensional and Dirichlet boundary condition is satisfied on the boundaries of cylindrical bodies. Lets write PEM integral-operator system for

diffraction problem on  $N$  bodies

$$\begin{aligned}
 g_j(\alpha) = & g_j^0(\alpha) + \frac{1}{\pi} \int_0^{2\pi} \int_{-\pi/2-i\infty}^{\pi/2+i\infty} \frac{k}{4} [\rho_j(\varphi_j) \cos \psi - \rho'_j(\varphi_j) \sin \psi] g_j(\varphi_j + \psi) \times \\
 & \times \exp \{-ik\rho_j(\varphi_j)[\cos \psi - \cos(\alpha - \varphi_j)]\} d\psi d\varphi_j + \\
 & + \sum_{l=1}^N (1 - \delta_{jl}) \frac{1}{\pi} \int_0^{2\pi} \exp \{ik\rho_j(\varphi_j) \cos(\alpha - \varphi_j)\} D_j \times \\
 & \times \int_{-\pi/2-i\infty}^{\pi/2+i\infty} g_l(\varphi_l + \psi) e^{-ikr_l \cos \psi} d\psi d\varphi_j, \quad j = \overline{1, N}
 \end{aligned} \tag{1}$$

In this system  $g_j(\varphi)$  is a scattering pattern of body  $j$  [1], i.e. a function, related to body  $j$  scattered field by the following asymptotic equation

$$E_{zj}(r, \varphi) = \sqrt{\frac{2}{\pi kr}} e^{-ikr+i\pi/4} g_j(\varphi) + O\left(\frac{1}{(kr)^{3/2}}\right),$$

and

$$\begin{aligned}
 g_j^0(\alpha) = & \frac{k}{4} e^{-i\vec{k}\vec{r}_{0j}} \int_0^{2\pi} [\rho_j(\varphi_j) \cos(\varphi_j - \varphi_0) + \rho'_j(\varphi_j) \sin(\varphi_j - \varphi_0)] \times \\
 & \times \exp \{ik\rho_j(\varphi_j)[\cos(\alpha - \varphi_j) - \cos(\varphi_0 - \varphi_j)]\} d\varphi_j
 \end{aligned} \tag{2}$$

$r_j, \varphi_j$  - polar coordinates of viewpoint from the center of body  $j$ ,

$r_j = \rho_j(\varphi_j)$  - equation of body  $j$  cross-section,

$\vec{k}\vec{r}_{0j} = kr_{0j} \cos(\varphi_0 - \varphi_{0j})$ , where  $r_{0j}, \varphi_{0j}$  - body  $j$  center coordinates from common origin, and  $\varphi_0$  - primary plane wave incident angle,

$D_j = \frac{i}{4} \left[ \rho_j(\varphi_j) \frac{\partial}{\partial r_j} - \frac{\rho'_j(\varphi_j)}{\rho_j(\varphi_j)} \frac{\partial}{\partial \varphi_j} \right] \Big|_{r_j=\rho_j(\varphi_j)}$  - differential operator.

Using Fourier series expansion on Eq. 1,

$$g_j(\alpha) = \sum_{n=-\infty}^{\infty} c_{jn} e^{in\alpha}, \quad g_j^0(\alpha) = \sum_{n=-\infty}^{\infty} c_{jn}^0 e^{in\alpha}, \quad j = \overline{1, N} \tag{3}$$

we obtain the following algebraic system relatively to coefficients  $c_{jn}$

$$c_{jn} = c_{jn}^0 + \sum_{l=1}^N \sum_{m=-\infty}^{\infty} G_{nm}^{jl} c_{lm}, \quad j = \overline{1, N} \tag{4}$$

Equations system 4 is solvable by reduction method, if scatterers fall into weakly non-convex body class and their boundaries are not intersected [1]. In this system

$$G_{nm}^{jj} = \frac{i^{n-m+1}}{4} \int_0^{2\pi} J_n(k\rho_j) \left[ k\rho_j(\varphi_j) H_m^{(2)'}(k\rho_j) + \right.$$

$$+im \frac{\rho'_j(\varphi_j)}{\rho_j(\varphi_j)} H_m^{(2)}(k\rho_j) \Big] e^{i(m-n)\varphi_j} d\varphi_j, \tag{5}$$

$$G_{nm}^{jl} = \frac{i^{n-m+1}}{4} \sum_{p=-\infty}^{\infty} H_{m-p}^{(2)}(kr_{lj}) e^{i(m-p)\varphi_{lj}} \int_0^{2\pi} J_n(k\rho_j) [k\rho_j(\varphi_j) J'_p(k\rho_j) -$$

$$- ip \frac{\rho'_j(\varphi_j)}{\rho_j(\varphi_j)} J_p(k\rho_j)] e^{i(p-n)\varphi_j} d\varphi_j, \quad j \neq l \tag{6}$$

$$c_{jn}^0(\varphi_0) = e^{-i\vec{k}\vec{r}_{0j}} \frac{i^{n+1}}{4} \sum_{p=-\infty}^{\infty} (-i)^p e^{-ip\varphi_0} \int_0^{2\pi} J_n(k\rho_j) [k\rho_j(\varphi_j) J'_p(k\rho_j) -$$

$$- ip \frac{\rho'_j(\varphi_j)}{\rho_j(\varphi_j)} J_p(k\rho_j)] e^{i(p-n)\varphi_j} d\varphi_j \equiv e^{-i\vec{k}\vec{r}_{0j}} c_{j0n}^0(\varphi_0) \tag{7}$$

Note that in [5] the problem of wave scattering by a group of parallel cylinders was reduced to an algebraic equation system, which looks similar to system 4. However, unlike system 4, the system matrix in [5] could not be written explicitly in general case (apart from circular cylinder case). At  $kr_{lj} \gg 1$  equation system 1 allows the following solution [1]

$$g_j(\varphi; \varphi_0) \cong e^{-i\vec{k}\vec{r}_{0j}} g_j^\infty(\varphi; \varphi_0) + \sum_{l=1}^N (1 - \delta_{jl}) Q_{jl} g_l(\varphi_{lj}; \varphi_0) g_j^\infty(\varphi; \varphi_{lj}), \tag{8}$$

where

$$Q_{jl} = \sqrt{\frac{2}{\pi k r_{lj}}} e^{-ikr_{lj} + i\pi/4}.$$

System 8 allows to find unknown values  $g_j(\varphi; \varphi_0)$  with quite acceptable accuracy in the range of distances between scatterers down to 0 (touching), if scatterer sizes are small enough [1]. In case of circular cylinders the relations 5 to 7 take the following form respectively

$$G_{nm}^{jj} = \frac{i\pi k a_j}{2} J_n(k a_j) H_n^{(2)'}(k a_j) \delta_{mn}, \tag{9}$$

$$G_{nm}^{jl} = \frac{i\pi k a_j}{2} J_n(k a_j) J'_n(k a_j) H_{m-n}^{(2)}(kr_{lj}) e^{i(m-n)(\varphi_{lj} - \pi/2)}, \quad j \neq l, \tag{10}$$

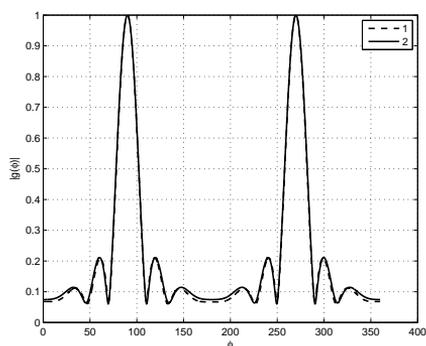
$$c_{jn}^0(\varphi_0) = e^{-i\vec{k}\vec{r}_{0j}} \frac{i\pi k a_j}{2} J_n(k a_j) J'_n(k a_j) e^{-in\varphi_0}, \tag{11}$$

where  $a_j$  - cylinder radius. If  $ka_j \ll 1$ , then algebraic system 4, as follows from Eqs. 9-11, takes the following form (one-mode approximation)

$$c_{j0} H_0^{(2)}(ka_j) + J_0(ka_j) \sum_{l=1}^N (1 - \delta_{jl}) H_0^{(2)}(kr_{lj}) c_{l0} = -e^{-i\vec{k}\vec{r}_{0j}} J_0(ka_j), \quad j = \overline{1, N}. \tag{12}$$

### 3. Numerical example

As an example let us consider a plane wave diffraction on a strip, which has a width  $kL = 2\sqrt{80}$ . Let's substitute this strip by the array of 80 circular cylinders with radius  $ka = 0,1$ , positioned next to each other, and solve the resulting problem in one-mode approximation. Figure 1 shows the scattering patterns of the strip (curve 1), which could be found in [6], and circular cylinder array (curve 2). A good coincidence of these two patterns can be seen, although we substituted infinitely thin strip by the scatterer with finite width  $\delta$  ( $k\delta = 0,2$ ). Based on these calculations we can expect even more accurate results by using similar approximation for scatterers, which are not infinitely thin.



**Figure 1.** Scattering patterns of strip (curve 1) and circular cylinder array (curve 2).

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