

## THE DIFFICULTY OF MEASURING ORBITAL ANGULAR MOMENTUM

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ABSTRACT. Light can carry angular momentum as well as energy and momentum; the transfer of this angular momentum to an object results in an optical torque. The development of a rotational analogue to the force measurement capability of optical tweezers is hampered by the difficulty of optical measurement of orbital angular momentum. We present an experiment with encouraging results, but emphasise the difficulty of the task.

### 1. Introduction

It is well established that a light beam can carry angular momentum. Thus, when using optical tweezers it is possible to exert torques, produced by the transfer of spin and orbital angular momentum. For example, spin torque can be exerted on birefringent particles by a Gaussian circularly polarized beam. Since the spin angular momentum depends on the Stokes parameters of the light, the change in spin, and hence the torque, can be readily measured optically. On the other hand, it is much more challenging to measure orbital angular momentum and torque.

One promising method is Laguerre–Gauss mode decomposition, as used for orbital angular momentum encoding for communication, and has been demonstrated as a macroscopic table-top experiment [1]. However, the situation becomes more complicated when a measurement is done on micro-scale, especially with highly focused laser beams and the high numerical aperture optical system used for optical tweezers. We demonstrate a promising experimental result for the measurement of orbital angular momentum in a high numerical aperture system, and consider the difficulties and challenges faced.

### 2. Spin and orbital angular momentum

The spin angular momentum density of an electromagnetic field is the *intrinsic* angular momentum density, and is, by definition, independent of the choice of origin about which moments are taken. The orbital angular momentum is *extrinsic*, and does depend on the choice of origin. The spin angular momentum of a beam is associated with its polarization and for circularly polarized light, depending on the handedness, the beam carries  $\pm\hbar$  per photon. Linearly polarized light, which can be described as a coherent superposition

of two circularly polarized beams with opposite handedness and equal intensities, carries no spin angular momentum and elliptical polarized light carries between 0 and  $\pm\hbar$  per photon depending on the intensity ratio of the right and left-handed circularly polarized components.

While it is straightforward to measure the spin angular momentum, it is much more difficult to measure the orbital angular momentum. Since in the general case, both spin and orbital angular momenta will contribute to the total torque, it is desirable to be able to measure both. A number of methods have been used successfully for the measurement of orbital angular momentum of light beams, and the key issue is whether these methods can be practically implemented in optical tweezers.

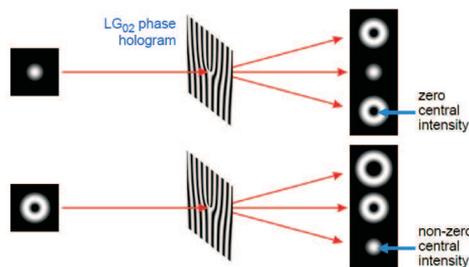
In the paraxial limit, a beam can be represented as a superposition of paraxial beam modes, such as Hermite–Gauss or Laguerre–Gauss modes, with the beam amplitude written as  $A = \sum_{p,\ell} a_{p\ell} \psi_{p\ell}$  where  $\psi_{p\ell}$  are the mode functions, or basis functions, and  $a_{p\ell}$  are the amplitudes of the modes. If we choose the Laguerre–Gauss modes as a basis, we note that each mode carries  $\ell\hbar$  orbital angular momentum per photon, and the total orbital angular momentum of the beam about the beam axis is

$$L_z = \sum_{p,\ell} \ell |a_{p\ell}|^2. \quad (1)$$

Therefore, if the power in the individual modes can be found, i.e., the individual  $|a_{p\ell}|^2$ , the total orbital angular momentum will be known. If the method used is not sensitive to the radial mode index  $p$ , then instead of measuring the power in the individual modes, it is possible to measure the power for all  $p$  and a single value of  $\ell$  simultaneously, i.e., to measure

$$P_\ell = \sum_p |a_{p\ell}|^2. \quad (2)$$

This greatly reduces the number of mode powers that need to be measured.



**Figure 1.** Measuring the orbital angular momentum of a beam using an analyzing hologram. A fork hologram can be used to generate beams carrying orbital angular momentum from an incident Gaussian beam (top). The transmitted beams carrying non-zero orbital angular momentum have a dark center. If, instead of a Gaussian beam, the incident beam is carrying  $\ell\hbar$  orbital angular momentum per photon, a different transmitted beam will have zero orbital angular momentum, and the formerly dark core of the beam will be bright.

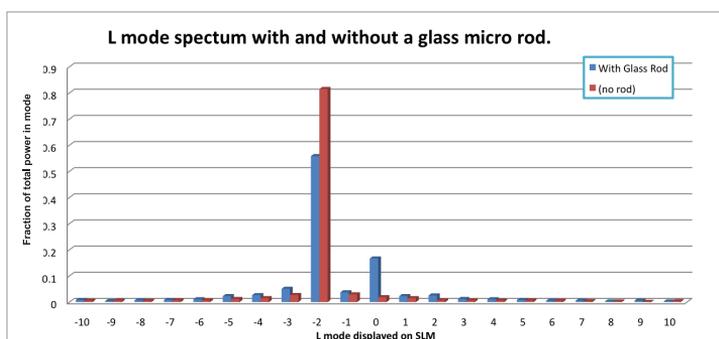
### 3. Experimental measurement

An  $LG_{02}$  Laguerre–Gauss beam was generated by a conventional fork hologram [2]. The first diffracted order from the hologram is selected via an aperture and directed to the microscope objective to form the optical trap. The transmitted beam is collected by a second object, and recollimated and passed onto an SLM which acts as a mode analyzer hologram for the laser beam that is incident on the hologram. Holograms corresponding to different  $\ell$  modes can be generated on the SLM in succession. For example, an  $LG_{0,-2}$  hologram will convert the original incident  $LG_{02}$  beam back to a simple Gaussian mode. The power in the mode is found by measuring the on-axis intensity of the first diffracted order using a CCD camera. A mode spectrum is obtained by cycling through holograms for the different values of  $\ell$ . In this experiment, the analyzer hologram was cycled from  $LG_{0,-10}$  to  $LG_{0,10}$ .

The object that the torque was exerted on was a glass rod of length  $15\ \mu\text{m}$  and diameter  $1\ \mu\text{m}$ . The rod was stuck to the cover slip so that it did not align vertically along the beam axis.

An IPG fiber laser (model YLD-5), with wavelength  $1064\ \text{nm}$ , with a plane polarized beam, was used to illuminate the fork hologram. The resulting  $LG_{02}$  beam was then passed into the back aperture of a  $1.3\ \text{N.A.}$  microscope objective. A second objective was used to then recollimate the focused beam. The collimated beam is directed onto a liquid crystal spatial light modulator (SLM) (Holoeye HEO 1080 P). A camera was used to record and view images of the focused beams.

In order to calibrate the experiment and estimate the effect of the objectives on the beam, the microscope objectives were first removed from the experiment and the beam was allowed to go through the apparatus without becoming highly focused at any point.



**Figure 2.** Mode spectra produced with and without a glass microrod in the system.

When a glass rod was placed at the beam focus, changes to the mode spectrum could be seen. A clear change in the angular momentum was seen, with a large transfer of power from the  $LG_{0,-2}$  mode to the  $LG_{0,0}$ , as expected from the second-order rotational symmetry of the rod [3]. With no rod in the beam, the transmitted light carries  $1.8 \pm 0.2\hbar$  orbital angular momentum per photon. With the rod in the beam, the transmitted light carries  $1.3 \pm 0.1\hbar$ . Thus, the angular momentum transfer per photon, or the torque efficiency, is  $0.5 \pm 0.2\hbar$  per photon. For glass rods of the sizes used in our experiment, the

torque efficiency calculated using T-matrix simulation of the optical forces and torques [4] varies from  $0.3\hbar$  to  $0.4\hbar$ , depending on the radius of the rods.

#### 4. Conclusion

Measurement of the orbital angular momentum by mode decomposition appears to work, and is in agreement with expected torque efficiencies. However, this method is difficult to implement, and is sensitive to alignment. In particular, if the beam has non-zero transverse momentum, the total axial component of the orbital angular momentum depends on the choice of origin. Such transverse momentum can result from angular misalignment of the trapping beam, or from transverse forces exerted within the trap. As a result, the measurement is sensitive to the positioning of the analysing hologram. One advantage of using an SLM as the analyser is the ease of moving the pattern on the SLM.

Beyond this, it is very likely that the different radial modes, i.e., the Laguerre–Gauss modes of the same  $\ell$  but differing  $p$  require different calibrations. In this case, it is not sufficient to measure all modes of a given  $\ell$  simultaneously; measurements for the individual combinations of  $\ell$  and  $p$  would be required.

The measurement of orbital angular momentum in optical tweezers is not an impossible task, but it is a difficult task to perform accurately and reliably, and much work remains to be done in the improvement of existing methods and the development of new methods.

Finally, to measure the torque, it is necessary to measure both the spin and orbital components. Their relative contributions will vary, depending on the object (for the glass rod here, the spin torque will be about 1/10 of the orbital torque [5]). Furthermore, focussing and collimating the beam can convert spin to orbital angular momentum, and vice versa [6].

#### References

- [1] S. J. Parkin, T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop, *Phys. Rev. A* **70**, p. 023816 (2004).
- [2] N. R. Heckenberg, R. McDuff, C. P. Smith, and A. G. White, *Opt. Lett.* **17**, pp. 221–223 (1992).
- [3] T. A. Nieminen, T. Asavei, V. L. Y. Loke, N. R. Heckenberg, and H. Rubinsztein-Dunlop, *J. Quant. Spectr. Radiative Transfer* **110**, pp. 1472–1482 (2009).
- [4] T. A. Nieminen, V. L. Y. Loke, A. B. Stilgoe, G. Knöner, A. M. Brańczyk, N. R. Heckenberg, and H. Rubinsztein-Dunlop, *J. Opt. A* **9**, pp. S196–S203 (2007).
- [5] A. I. Bishop, T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop, *Phys. Rev. A* **68**, pp. 033802 (2003).
- [6] T. A. Nieminen, A. B. Stilgoe, N. R. Heckenberg, and H. Rubinsztein-Dunlop, *J. Opt. A* **10**, pp. 115005 (2008).

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Paper presented at the ELS XIII Conference (Taormina, Italy, 2011), held under the APP patronage; published online 15 September 2011.

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