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# MODEL OF POSSIBLE COOPERATION IN FINANCIAL MARKETS IN PRESENCE OF TAX ON SPECULATIVE TRANSACTIONS

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ABSTRACT. In this paper, we propose an economic and mathematical model to protect the financial markets from speculative attacks, by introducing a tax on financial speculative transactions. By using Game Theory, we focus on the interaction between two general players: a real economic subject (we call Enterprise) acting with hedging scopes and a bank (we call Financial Institute) acting with speculative purposes. We find different equilibria of our game, by considering friendly, selfless, selfish, fearful or aggressive behavior of players; we note that no equilibrium is good for both players, and each of them prevent at least one of the two economic subjects to obtain profits. So, we propose two different transferable utility solutions, in order to achieve a result satisfying both economic subjects and, at the same time, to achieve a condition promoting the stability of the financial markets in which our two players are interacting.

## 1. Introduction

In this paper, by the introduction of a tax on financial transactions, we propose a method aiming at limit the speculations of medium and big financial operators and, consequently, a way to make more stable the financial markets (see also Carfì and Musolino 2011, 2012a,b, 2013a,b,c,d,e; Musolino 2012, 2013a); our aim is attained without inhibiting the possibilities of profits. At this purpose, we present and study a game (for a complete study of a game see Carfì and Musolino 2013f) as an example and we shall propose a cooperative solution that gives to both players mutual economic advantages.

## 2. Methodologies

The normal-form game $G$, we propose to model our financial interaction, requires a dynamical construction which takes place on 3 times, which we say time 0, time 1 and time 2.

At time 0, the Enterprise can choose to buy futures contracts to hedge the market risk of the underlying asset, asset that the Enterprise itself should buy at time 1, in order to conduct its business activities.
The Financial Institute, on the other hand, acts - with speculative purposes - on the spot market - buying or short-selling the asset at time 0 - and on the futures market, with an action contrary to that on the spot market: if the Financial Institute short-sells on spot market, it purchases on the futures market, and vice versa. The Financial Institute may so

take advantage of the temporary misalignment of the spot and futures prices created as a result of the Enterprise hedging strategy.
At the time 2 the Financial Institute cashes or pays the sum determined by the behavior used in the futures market at time 1.

**Remark.** In the game, we do not introduce the uncertainty (and we do not consider extreme events in our economic world) and so we suppose that attempts of speculative profit (modifying the asset price) are successful. In fact, our interest is to show that a tax on speculative profits can limit speculation, and not to determine if or how much speculators gain. Anyway, even without uncertainty, our model remains likely, plausible and very topical because:

- in a period of crisis, behavioral finance suggests (Asch 1952; Deutsch and Gerard 1955; Milgram 1974) the vertical diffusion of a behavior, the so-called herd behavior (Ford, Kelsey, and Pang 2012; Scharfstein and Stein 1990), conforming to that adopted by the great investors;
- just the decrease (or increase) in demand influences the price of the asset (Marshall 2010).

## 3. The game and stabilizing proposal

**3.1. The description of the game.** We assume that our first player is the Enterprise that choose to buy futures contracts to hedge by an upwards change in the price of the underlying asset that the Enterprise has to buy at time 1, for the conduct of its business. Therefore, the Enterprise chooses a strategy $x \in [0,1]$ representing the percentage of the quantity $M_1$ of the underlying asset that the Enterprise purchases through futures, depending on it intends:

(1) to not hedge, $x = 0$,
(2) to hedge partially, $0 < x < 1$,
(3) to hedge totally, $x = 1$.

On the other hand, our second player is a the Financial Institute operating on the spot market of the underlying asset that the Enterprise should buy at time 1. The Financial Institute works in our game also on the futures market:

- taking advantage of possible gain opportunities - given by misalignment between spot prices and futures prices of the asset;
- or accounting for the loss obtained, because it has to close the position of short-sales opened on the spot market.

These actions determine the gain or the loss of the Financial Institute.

The Financial Institute therefore choose a strategy $y \in [-1,1]$ representing the percentage of the quantity $M_2$ of the underlying asset that it can buy (in algebraic sense) with its financial resources, depending on it intends:

(1) to purchase the underlying on the spot market, $y > 0$;
(2) to short-sell the underlying on the spot market, $y < 0$;
(3) to not intervene on the market of the underlying, $y = 0$.

In Fig.1 we illustrate graphically the bi-strategy space $E \times F$ of the game.

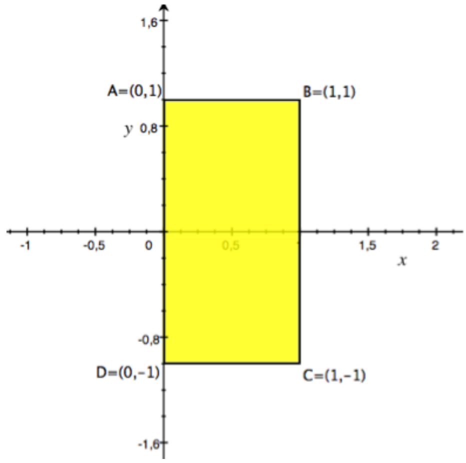


FIGURE 1. The bi-strategy space of the game

**3.2. The payoff function of Enterprise.** The payoff function of the Enterprise, say $f_1$, is the function $f_1 : E \times F \to \mathbb{R}$ representing the quantitative loss or gain of the Enterprise, referred to time 1, is given, at every pair $(x,y)$, by the gain (or loss) obtained on not hedged asset $(1-x)M_1$. The gain related with the not hedged asset, at the bi-strategy pair $(x,y)$, is given by the quantity of the not hedged asset $(1-x)M_1$, multiplied by the difference $F_0 - S_1(y)$, between the futures price at time 0 (the term $F_0$) - which the Enterprise should pay, if it decides to hedge - and the spot price $S_1(y)$ at time 1, when the Enterprise actually buys the asset that it did not hedge.

**Payoff function of the Enterprise**. The payoff function $f_1 : E \times F \to \mathbb{R}$ is defined by

$$f_1(x,y) = M_1(1-x)(F_0 - S_1(y)), \tag{1}$$

for every bi-strategy $(x,y)$ in $E \times F$, where:

- $M_1$ is the amount of asset that the Enterprise has to buy at time 1;
- $(1-x)$ is the percentage of the underlying asset that the Enterprise buys on the spot market at time 1 without hedging (and therefore, the percentage of the underlying asset exposed to the price fluctuations);
- $F_0$ is the futures price at time 0. It represents the price established at time 0 that the Enterprise has to pay at time 1 to buy the asset. According to Hull's relation (Hull 2002), assuming the absence of dividends, known income, storage costs and convenience yield to keep possession the underlying, the futures price is given by

$$F_0 = S_0(1+i)^t, \tag{2}$$

where $u^t := (1+i)^t$ is the capitalization factor with interest rate $i$ at the maturity $t$. By $i$ we mean the risk-free interest, in our case the rate charged by banks on deposits of other banks, the so-called "LIBOR" rate.

$S_0$ is, on the other hand, the spot price of the underlying asset at time 0. $S_0$ is a constant of our model because it is not influenced by our strategies $x$ and $y$.

- $S_1(y)$ is the spot price of the underlying asset at time 1, after that the Financial Institute has played its strategy $y$. We assume an affine (first order) dependence of the price $S_1$ on $y$, precisely

$$S_1(y) = S_0(1+i) + n(1+i)y, \tag{3}$$

where $n > 0$ is a marginal coefficient representing the effect of the strategies $y$ of the second player on the price-function $S_1$. The value $n$ depends on the ability of the Financial Institute to influence the asset market and the behaviors of other financial agents. $S_1$ depends on $y$ because any demand change has an effect on the asset price (see Besanko and Braeutigam 2010; Ecchia and Gozzi 2002). We are assuming (in a first approximation) a linear influence $n \mapsto ny$ of $y$ on $S_1(y)$.
The values $S_0$ and $ny$ should be capitalized, because they should be "transferred" from time 0 to time 1.

**Remark.** According to our assumption, if the Financial Institute plays a strategy $y = 0$, the value of the function $S_1$ is equal to the capitalized spot price $S_0$. In fact, since the times of the model are fairly close to each other, we do not consider other variables influencing the asset price (economic crisis, inflation, war and social unrest, technological progress etc.).

***Payoff function of the Enterprise.*** Therefore, recalling the equations 3 and 2 and replacing in Eq. 1, the payoff function of the Enterprise, is defined:

$$f_1(x,y) = M_1(1-x)(-ny(1+i)), \tag{4}$$

for every $(x,y)$ in $E \times F$.
From now on, the value $n(1+i)$ will be indicated by $\nu$.

**Remark (the case of collateral deposits).** Even if the price $F_0$ is paid at time 1, the Enterprise could deposit, already at time 0, a collateral. Also in this case, however, there are no differences between the payoff function of the Enterprise without collateral deposits and the payoff function of the Enterprise with collateral deposits. In fact, the value - net of interest received - paid by the Enterprise to buy the asset via futures is given by the difference between

- the value $F_0$ - that is given as collateral at time 0 - capitalized at time 1,
- the interests $F_0 i$ cashed by the Enterprise on the deposit of collateral.

So we have

$$F_0(1+i) - F_0 i,$$

that is exactly the futures price $F_0$ paid at time 1 by the Enterprise in absence of collateral.

So, we have shown that the payoff function of the Enterprise is valid also with collateral deposits.

**3.3. The payoff function of the Financial Institute.** The payoff function $f_2 : E \times F \to \mathbb{R}$ of the Financial Institute at time 1 is defined, at any bi-strategy $(x,y)$, by the multiplication of the quantity of asset bought on the spot market, that is $yM_2$, times the difference between the futures price $F_1(x,y)$ discounted at time 1 (it is a price established at time 1 but cashed at time 2) and the purchase unit price of asset at time 0, say $S_0$, capitalized at time 1 (in other words, we are accounting for all balances at time 1).

**Stabilizing strategy of normative authority.** In order to avoid speculations of the Financial Institute on spot and futures markets, we propose the imposition of a tax by the normative authority:

- the normative authority should impose, to the Financial Institute, the payment of a tax(see also Musolino 2013b) on the sale of the futures (see Palley 1999; Wei and Kim 1999; Westerhoff and Dieci 2006), for the benefits of the taxation on financial transactions).
- this new tax is motivated firstly because the F. I., in this model, is the only one able to determine variations on the spot price (and consequently also on the futures price) of the underlying asset.
- we propose, moreover, that this tax should be fairly equal to the incidence of the strategy of the Financial Institute on the spot price; so, the price effectively cashed or paid for the futures by the Financial Institute is
$$F_1(x,y)(1+i)^{-1} - ny(1+i),$$
where $nuy$ is the tax paid by the Financial Institute, referred to time 1.
- with this new tax, the Financial Institute can't take anymore free advantage of price swings caused by its behavior.

**Remark**. We note that, if the Financial Institute gains, it acts on the futures market at time 2 to cash the gain, but also in case of loss it must necessarily act in the futures market and account for its loss because at time 2 (in the futures market) it should close the short-sale position opened on the spot market.

**Payoff function of the Financial Institute**. The function $f_2 : E \times F \to \mathbb{R}$ is defined by
$$f_2(x,y) = yM_2[F_1(x,y)(1+i)^{-1} - ny(1+i) - S_0(1+i)], \tag{5}$$
for every $(x,y)$ in $E \times F$, where:

- y is the percentage of $M_2$ quantity of the asset that the Financial Institute purchases or sells on the spot market;
- $M_2$ is the amount of asset that the Financial Institute can buy or sell on the spot market according to its disposable income;
- $S_0$ is the unit price at which the Financial Institute buy the asset at time 0. $S_0$ is a constant because our strategies $x$ and $y$ does not have impact on it.
- $n(1+i)y$ is the normative tax on the price of the futures paid at time 1 by the F.I.. We assume the tax is equal to the incidence of the strategy $y$ of the Financial Institute on the price $S_1$.
- $F_1(x,y)$ is the price of the futures market (established) at time 1, after the Enterprise has played its strategy $x$ and the Financial Institute has played the strategy $y$.

We assume that the price $F_1(x,y)$ is given by

$$F_1(x,y) = S_1(1+i) + mx(1+i), \tag{6}$$

where

- $u := (1+i)$ is the capitalization factor corresponding to the risk-free interest rate that the F.I. can obtain;
- by $m$ we intend the marginal coefficient measuring the impact of the strategy $x$ on the price $F_1(x,y)$.

The price $F_1(x,y)$ depends on $x$ because, if the Enterprise buys futures with a strategy $x \neq 0$, the price $F_1$ changes because an increase of futures demand influences the futures price (see Besanko and Braeutigam 2010; Ecchia and Gozzi 2002). The value $S_1$ should be capitalized because it respects the relationship between futures and spot prices expressed in Eq. 2. The value $mx$ is also capitalized because the strategy $x$ is played at time 0 but it has effect on the futures price at time 1.

- $u^{-1} := (1+i)^{-1}$ is the discount factor corresponding to the risk-free interest rate that the F.I. can obtain. $F_1(x,y)$ must be discounted at time 1 because the money for the sale of futures is cashed in a time 2.

**Remark (on the deterministic evolution of our asset price with respect to time).** As suggested in Schwartz (1997), the price evolution of our asset should follow a stochastic behavior with respect to time. Nonetheless, our model is not a time continuous model but a discrete one, and moreover we consider only two future times (times 1 and 2). Consequently the stochastic behavior will influence our prices only by two possible errors $\varepsilon_1$ and $\varepsilon_2$. These two unpredictable, but fixed errors, do not influence the quality of our model but only the dimensions of our payoff space and, in usual conditions on the brief time horizon we consider, the two errors could be considered qualitatively negligible.

**Remark (on the deterministic evolution of our asset price with respect to the strategies of the players).** Analogously, in our opinion, the price evolution of our asset should follow a stochastic behavior with respect to the strategies of the players. Nevertheless, we do not know literature considering a stochastic dependence of the assets prices on the actions of the investors, on the contrary there is a wide literature admitting a deterministic (although not precisely defined) dependence. In any case, we think that the stochastic behaviors could be considered as a deterministic kernel $k : (x,y) \mapsto k(x,y)$ disturbed by a stochastic perturbation $\eta$ of mean 0 and significantly smaller in module than the absolute value of the kernel $k$. Consequently the stochastic perturbation will influence our prices only by a negligible and 0 mean error $\eta(x,y)$. More important, in our opinion, is the limitation to the choice of linear dependence, we did so in order to consider a first approximate examination, more than for simple sake of simplicity.

***Payoff function of the Financial Institute.*** Replacing Eq. 6 in Eq. 5, we obtain the final expression of the Financial Institute payoff function:

$$f_2(x,y) = mM_2xy, \tag{7}$$

for every (x,y).

***Payoff function of the Game.*** Summarizing, the payoff function of the game is the function $f : E \times F \to \mathbb{R}^2$ : defined by

$$f(x,y) = (-\nu M_1(1-x)y, mM_2xy), \tag{8}$$

for every $(x,y) \in E \times F$. In the following figure 2 (in 3D) we shall give a representation of this vector function $f$.

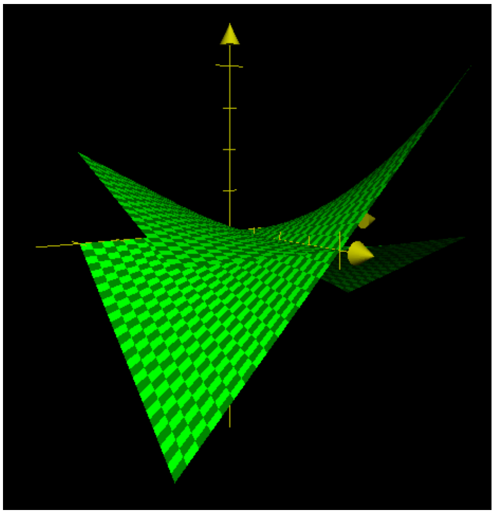

FIGURE 2. The 3D representation of the game

**Remark.** Even if the price $F_1$ is paid at time 2, the Financial Institute could deposit, already at time 1, a collateral. In this case, at the value $F_1(x,y)$ (that is paid as collateral at time 1) we must subtract the interests - discounted at time 1 - cashed at time 2 by the Financial Institute on the deposit of collateral. These interests are given by

$$F_1(x,y)i(1+i)^{-1}.$$

So, in presence of collateral, the value to put in $f_2(x,y)$ instead of $F_1(x,y)(1+i)^{-1}$ is

$$\delta := F_1(x,y) - F_1(x,y)i(1+i)^{-1}.$$

Recalling the Eq. 6, that is

$$F_1(x,y) = uS_1(y) + mux,$$

we have

$$\delta = (uS_1(y) + mux) - iu^{-1}(uS_1(y) + mux),$$

and therefore

$$\delta = S_1(y) + mx,$$

that is exactly the value $F_1(x,y)$ discounted at time 1.

**Remark.** So we have shown that the payoff function of the Financial Institute, that we have found before without considering eventual collaterals, results valid also with collateral deposits.

## 4. Critical space of the game

Since we are dealing with a non-linear game, it is necessary to study the critical zone of the game. To find the critical area of the game, we consider the Jacobian matrix of $f$ and we consider the set of bi-strategies in which its determinant equals 0.

The Jacobian matrix of $f$ at $(x,y)$ is the matrix

$$J_f(x,y) = \begin{pmatrix} \nu M_1 y & -\nu M_1(1-x) \\ M_2 m y & M_2 m x \end{pmatrix},$$

and its determinant is

$$\det J_f(x,y) = m\nu M_1 M_2 xy + m\nu M_1 M_2 (1-x)y.$$

Therefore the critical space of the game is the set

$$Z = \{(x,y) \in E \times F : xy + (1-x)y = 0\} = \{(x,y) \in E \times F : y = 0\},$$

because the coefficient $m\nu M_1 M_2$ is strictly greater than 0.
The critical area of our bi-strategy space is represented in the Fig. 3: it is the segment $[H,K]$.

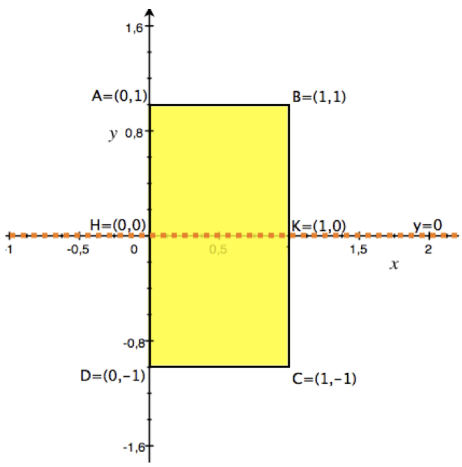


FIGURE 3. The critical space of the game

## 5. Payoff space

In order to represent graphically the payoff space $f(E \times F)$, we transform, by the function

$$f : E \times F \to \mathbb{R}^2 : (x,y) \mapsto (-\nu M_1(1-x)y, mM_2 xy),$$

all the sides of bi-strategy rectangle $E \times F$ and the critical space $Z$ of the game.

1) The segment $[A,B]$ is the set of all the bi-strategies $(x,y)$ such that $y = 1$ and $x \in [0,1]$. Calculating the image of the generic point $(x,1)$ of $[A,B]$, we have:

$$f(x,1) = (-\nu M_1(1-x), mM_2 x).$$

Therefore setting, for every $x \in [0,1]$, $X := -\nu M_1(1-x)$ and $Y = mM_2 x$, and assuming $M_1 = 1, M_2 = 2$, and $\nu = m = 1/2$, we have $X = -(1/2)(1-x)$ and $Y = x$; thus $X = -(1/2)(1-Y)$, and so the image of the segment $[A,B]$ by $f$ is the set of bi-gains $(X,Y)$ such that

$$X = -(1/2)(1-Y) = -(1/2) + (1/2)Y \text{ with } Y \in [0,1],$$

that is, it is the line segment $[A',B']$, with extremes $A' := f(A)$ and $B' := f(B)$.

Following the procedure described above for the other sides of the bi-strategy rectangle and for the critical space, the segments $[B,C]$, $[C,D][D,A]$ and $[H,K]$, we obtain the figures 4.

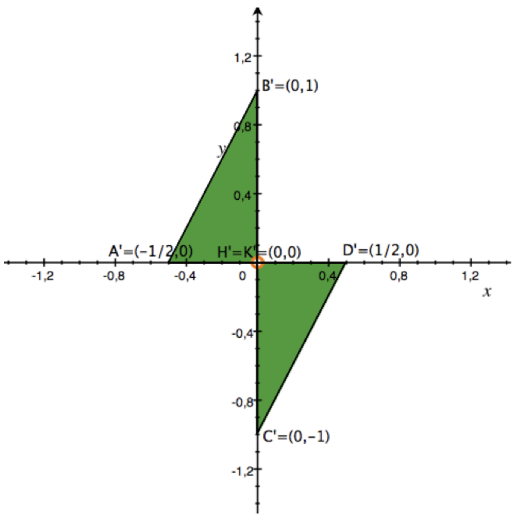


FIGURE 4. The payoff space of the game

We can see how the set of possible gaining combinations of our players take a curious butterfly shape, that promises the game particularly interesting.

## 6. Pareto analysis

The supremum of the game, that is the bi-gain $\alpha = (1/2,1)$, is a shadow maximum because it does not belong to the payoff space:

$$\alpha = (1/2,1) \notin f(E \times F).$$

The infimum of the game, that is the bi-gain $\beta = (-1/2, -1)$, is a shadow minimum because it does not belong to the payoff space:

$$\beta = (-1/2, -1) \notin f(E \times F).$$

The weak Pareto maximal boundary of the payoff space is

$$[B', K'] \cup [H', D'].$$

The proper Pareto maximal boundary of the payoff space is

$$\partial^* f := \partial^* f(E \times F) = \{B', D'\}.$$

The weak minimal Pareto boundary of the payoff space is

$$[A', H'] \cup [K', C'].$$

The proper Pareto minimal boundary of the payoff space is

$$\partial_* f := \partial_* f(E \times F) = \{A', C'\}.$$

**Definition of Pareto control**. We say that the first player can determine a Pareto maximal solution, or, equivalently, we say that the first player has the control of the Pareto maximal boundary if there exists a strategy $x_0 \in E$ such that for every strategy $y$ of the Financial Institute the pair $(x_0, y)$ is a Pareto maximal bi-strategy (that is, its image by $f$ belongs to $\partial^* f$), in this case, we say that the strategy $x_0$ is a maximal control-strategy of the first player. Analogous definitions could be given for the second player and for the minimal boundary.

**Control and accessibility of Pareto boundaries.** In our game there are no maximal controls, nor minimal. So no player can decide to go on the Pareto boundary without cooperating with the other one. Moreover, the game seems to be quite complex to solve in a satisfactory way for both players, because of the geometry of the Pareto maximal boundary.

## 7. Nash equilibria

If any player wants to think only on its own, they would choose the strategy that makes maximum their gain regardless of the other player's payoff. In this case we talk about multifunction of best reply and of Nash equilibria. Classically, we should maximize, for instance in the case of the first player, the partial payoff function $f_1(., y)$, for every possible strategy $y$ of the other player.
In formula, the best reply multifunction of the Enterprise is:

$$B_1 : F \to E : y \mapsto \max_{f_1(\cdot, y)} E$$

($\max_{f_1(\cdot, y)} E$ is the set of strategies in $E$ maximizing the section $f_1(\cdot, y)$).

Symmetrically, the multifunction of best reply of the Financial Institute is:

$$B_2 : E \to F : x \mapsto \max_{f_2(x, \cdot)} F$$

($\max_{f_2(x, \cdot)} F$ is the set of strategies in $F$ maximizing the section $f_2(x, \cdot)$).

Recalling Eq. 4 and the values $M_1 = 1$, $\nu = 1/2$, $M_2 = 2$, $m = 1/2$ (positive numbers), we have

$$\partial_1 f_1(x,y) = \nu M_1 y,$$

this derivative is positive if $y > 0$, and so:

$$B_1(y) = \begin{cases} \{1\} & \text{if} \quad y > 0 \\ E & \text{if} \quad y = 0 \\ \{0\} & \text{if} \quad y < 0 \end{cases}.$$

Recalling also Eq. 7, we have

$$\partial_2 f_2(x,y) = mM_2 x$$

this derivative is positive if $x > 0$, and so:

$$B_2(x) = \begin{cases} \{1\} & \text{if} \quad x > 0 \\ F & \text{if} \quad x = 0 \end{cases}.$$

In Fig. 5 we represent the graph of $B_1$ (in red) and that of $B_2$ (in blue).

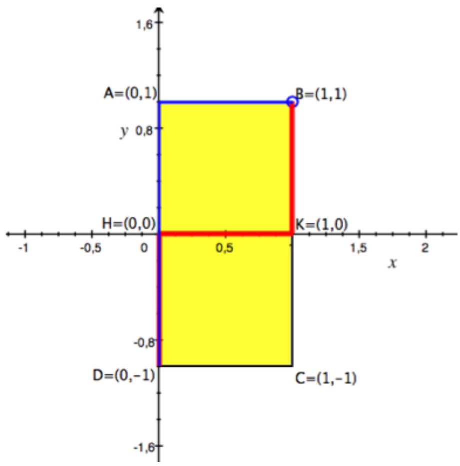


FIGURE 5. Nash equiliria

**Set of Nash equilibria**. It is the intersection of the two best reply graphs and thus we have

$$\mathrm{Eq}(B_1, B_2) = \{(1,1)\} \cup [H,D].$$

The Nash equilibria could appear not so bad, at a first sight, because they belong to the weak maximal boundary. But it is not the case, since it is impossible to reach non-cooperatively anyone of the equilibria: the first player wants to go on $D$ and so it plays 0; the second player wants to go on $B$ and so it plays 1; the result is $A = (0,1)$ a very bad non-equilibrium state in which the game gains $(-1/2, 0)$. The selfishness, in this case, does not pay well. The following analysis explains other possible aspects of the problem.

**Analysis of Nash equilibria**.
***Enterprise.*** The Enterprise has two Nash possible alternatives (two Nash-equilibrium strategies): not to hedge, by playing 0, or to hedge totally, by playing 1. With 0, E. could gain or lose in $[-1/2, 1/2]$, depending on the strategy employed by the Financial Institute; with strategy 1, the Enterprise does not obtain any loss or gain.
***Financial Institute.*** The F. equilibrium strategy set is the union $[-1,0] \cup \{1\}$. If the Enterprise adopts a strategy $x \neq 0$, the Financial Institute plays the strategy 1 as best reply (winning something >0), and if the Enterprise plays 0, the Financial Institute can play all its strategy set $F$ as best replies, without obtaining any gain or loss. So, the Financial Institute has a **dominant strategy**: it will play 1, as dominant action (gaining in [0,1]) and the Enterprise, which presume the dominant choice of the Financial Institute, will hedge by the defensive strategy 1.

**Nash solution.** So, despite the Nash equilibria are infinite, very likely the two players will arrive in the defensive-dominant equilibrium $B = (1,1)$, which belongs to the maximal Pareto boundary. The Nash approach gives here a viable, feasible and satisfactory solution for one of two players: the Financial Institute.

## 8. Offensive equilibria

If any player wants only to ruin the other one, it chooses the strategies determining the maximum the loss of the other one. In this case, we talk about worst offense multifunctions.

Any player should minimize with respect to its own strategy set, the payoff function of the other player, fixed its every possible strategy. In formula, the multifunction of worst offense of the Financial Institute against the Enterprise is the following:

$$O_2 : E \to F : x \mapsto \min_{f_1(x,\cdot)} F$$

($\min_{f_1(x,\cdot)} F$ is the set of strategies in $F$ that minimize the section $f_1(x,\cdot)$).
On the other hand, the multifunction of worst offense of the Enterprise against the Financial Institute is:

$$O_1 : F \to E : y \mapsto \min_{f_2(\cdot,y)} E$$

($\min_{f_2(\cdot,y)} E$ is the set of the strategies in $E$ that minimize the section$f_2(\cdot,y)$).
Recalling Eq. 4, we have

$$O_2(x) = \begin{cases} \{1\} & \text{if} \quad 0 \le x < 1 \\ F & \text{if} \quad x = 1 \end{cases} .$$

Recalling also Eq. 7, we have

$$O_1(y) = \begin{cases} \{0\} & \text{if} \quad y > 0 \\ E & \text{if} \quad y = 0 \\ \{1\} & \text{if} \quad y < 0 \end{cases} .$$

Note that F. has an offensive-dominant strategy, 1, and that E. has not. We observe in Fig. 6 the graphs of $O_2$ (in blue) and of $O_1$ (in red).

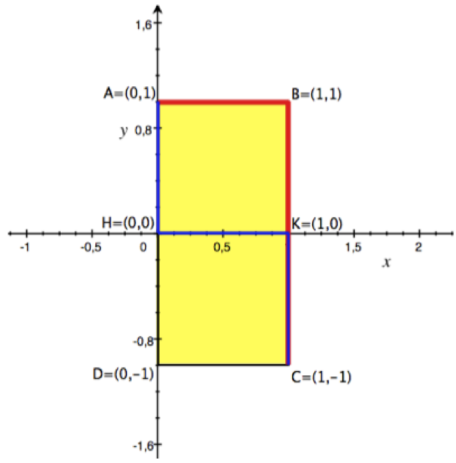


FIGURE 6. Offensive equilibria

**Set of offensive equilibria**. It is the intersection of two graphs of worst offense, and thus

$$\mathrm{Eq}(O_1, O_2) = \{(0,1)\} \cup [K, C].$$

The offensive equilibria may be considered collectively very bad, because they lie on the Pareto weakly minimal boundary.

**Analysis of offensive equilibria**. Very likely, the Financial Institute will play the offensive-dominant strategy 1, it is the only one able to maximize the damage of the Enterprise when E. chooses a strategy $\neq 1$; note that, if the Enterprise chooses the strategy 1, the Financial Institute is indifferent while choosing a strategy, since the damage caused to the Enterprise is constant.

On the other hand, if E. is conscious of the bad intentions of F., E. presumes that the Financial Institute chooses the strategy 1 to hurt, and the Enterprise most likely chooses 0, following the logic of $O_1$, to be sure that the Financial Institute gets the minimum possible gain (which, in this case, is equal to 0).

So, despite the offensive equilibria are infinite, the two players most likely arrive in $A = (0,1)$, which is on the proper minimal Pareto boundary: the offensive strategies of both players can be considered a credible threat. We want to highlight that, very likely, even if the Enterprise plays its offensive strategies, however, the Financial Institute does not lose.

## 9. Defensive phase

When a players is not aware of the will of the other, or when it is, by its nature, cautious or risk averse, then it chooses strategies minimizing its own loss: the defensive strategies.

**Conservative value of a player**. It is defined as the supremum of its worst gain function $f_i^\sharp$. Therefore, the conservative value of the Enterprise is

$$v_1^\sharp := \sup_{x\in E} f_1^\sharp(x),$$

where $f_1^\sharp : E \to \mathbb{R}$ is the Enterprise worst gain function, and it is defined by

$$f_1^\sharp(x) = \inf_{y\in F} f_1(x,y).$$

Recalling the E.'s payoff function (Eq. 4), we have

$$f_1^\sharp(x) = \inf_{y\in F} -\nu M_1(1-x)y.$$

Since the offensive multifunction of the Financial Institute is

$$O_2(x) = \begin{cases} \{1\} & \text{if} \quad 0 \le x < 1 \\ F & \text{if} \quad x = 1 \end{cases},$$

we obtain:

$$f_1^\sharp(x) = \begin{cases} -\nu M_1(1-x) & \text{if} \quad 0 \le x < 1 \\ 0 & \text{if} \quad x = 1 \end{cases}.$$

In Fig. 7 appears $f_1^\sharp$.

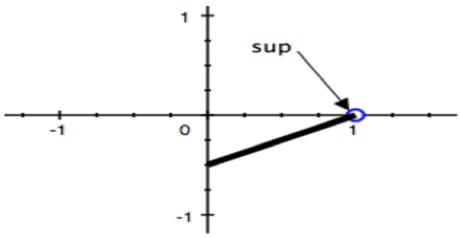


FIGURE 7. Representation of $f_1^\sharp$, the worst gain function of the Enterprise.

So the unique defense (or conservative) strategy of the Enterprise is $x^\sharp = 1$ and the conservative value of the Enterprise is

$$v_1^\sharp = \sup_{x\in E} \inf_{y\in F} -\nu M_1(1-x)y = 0. \tag{9}$$

On the other hand, the conservative value of the Financial Institute is given by

$$v_2^\sharp = \sup_{y\in F} f_2^\sharp(y)$$

where $f_2^\sharp : F \to \mathbb{R}$ is the Financial Institute worst gain function. It is given by

$$f_2^\sharp(y) = \inf_{x\in E} f_2(x,y).$$

Recalling the F.'s payoff function (Eq. 7), we have:

$$f_2^\sharp(y) = \inf_{x\in E} mM_2xy.$$

Therefore since the offensive correspondence of the Enterprise is

$$O_1(y) = \begin{cases} \{0\} & \text{if} \quad y > 0 \\ E & \text{if} \quad y = 0 \\ \{1\} & \text{if} \quad y < 0 \end{cases},$$

we obtain:

$$f_2^\sharp(y) = \begin{cases} 0 & \text{if} \quad y \geq 0 \\ mM_2 y & \text{if} \quad y < 0 \end{cases}.$$

In Fig. 8 appears $f_2^\sharp(y)$.

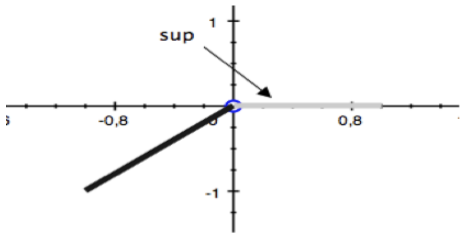


FIGURE 8. Representation $f_2^\sharp$, the worst gain function of the Financial Institute.

So the defensive (or conservative) strategy set of the Financial Institute is given by

$$F^\sharp = F$$

and the conservative value of the Financial Institute is

$$v_2^\sharp = \sup_{y\in F} \inf_{x\in E} mM_2 xy = 0. \tag{10}$$

Therefore the conservative bi-value is

$$v_f^\sharp = (v_1^\sharp, v_2^\sharp) = (0,0).$$

**9.1. Conservative crosses.** They are the bi-strategies $(x_\sharp, y_\sharp)$, with $y_\sharp \in F_\sharp$. So, the conservative crosses forms the whole of the segment $[B,K]$. If the Enterprise and the Financial Institute decide to defend themselves against any opponent's offensive strategy, they arrive on the payoffs subset $[B', K']$, which is part of the weak maximal Pareto boundary.

**Analysis of conservative crosses**. In this simplified model, the Financial Institute presumably will choose the defensive strategy $y_\sharp = 1$, because it's the only one that allows it to obtain the maximum possible profit (without incurring losses in any cases). Therefore, the players arrive in $B'$, the optimal solution for the Financial Institute.

From an economic point of view, this happens because the Enterprise was unable, by adopting its strategies $x \in [0,1]$, to determine a decreasing of the futures price.

**Remark.** In real economic world, however, in addiction to the Enterprise there are other traders, which could also cause a fall in futures price and then, if the Financial Institute would choose a defensive strategy, presumably it would decide to not act in the market, employing the null-strategy 0. In this case, the conservative cross reached by the players would be $K = (1,0)$ leading to the payoff $(0,0)$.

**Remark.** At this level, it seems that F. has no reasonable motivations to find a bargaining solution with E.; indeed. from the Nash analysis and from the conservative analysis we obtain the bi-strategy solution $B = (1,1)$ and the consequent vector-payoff $(0,1)$. Nonetheless, E. could menace to play 0, leading the game to the state $A = (0,1)$ and consequently to the payoff $A' = (-1/2,0)$. Why this threat should be credible? The point lies in the utility bi-value given to $A'$ by the players. The evaluation of E. on its own losses (in the purchase of the commodity) is realistically less severe then that associated by F. with the absence of profits in its financial operation, this because the primary activity of F. is to speculate and gain from its financial operations; on the contrary the primary activity of E. takes place in the real economy. So, E. could recognize its own importance for the financial affairs of F. and could ask a kind of payment for its easygoing participation to the financial interaction. This could be one reason for F. to find a bargaining solution with the Enterprise.

**9.2. Core of the payoff space.** The payoff-core of the game is the part of the maximal boundary contained in the upper cone of the cnsrvative payoff $v_f^\sharp = (0,0)$. Therefore we have

$$\mathrm{core}'(G) = [B',K'] \cup [H',D'],$$

whose reciprocal image is the (strategy)-core of the game

$$\mathrm{core}(G) = [B,K] \cup [H,D] \cup [H,K].$$

In Fig. 9 we can see, in red, the part of the payoff space where the Enterprise has a gain greater than its conservative value $v_1^\sharp = 0$ (abscissa-axis in pink) and in blue it is shown the part of the payoff space where the Financial Institute obtains a gain higher than its conservative value $v_2^\sharp = 0$ ($y$-axis in lighter blue).

We note that if both players choose their conservative strategies $x_\sharp = 1$ and any $y \in F_\sharp = F$ (very likely 1, that is the dominant strategy of F.), the Enterprise avoids to gain less of its conservative value $v_1^\sharp = 0$, but E. is automatically unable to get also higher gains. The same remark does not apply to Financial Institute which may arrive on the segment $[B',K']$ and very likely in $B'$. The game is blocked for the Enterprise, that is clearly disadvantaged in respect of the Financial Institute, in this financial interaction.

**Remark.** Recalling the previous remark, the game would be blocked for both players, because also the Financial Institute would be unable to get a gain higher than its conservative value $v_2^\sharp = 0$, if it decides to play its defensive strategy $y_\sharp = 0$.

**Conservative part of the game on the bi-strategy space.** It is given by

$$(E \times F)^\sharp = (E \times F)_1^\sharp \cap (E \times F)_2^\sharp,$$

where $(E\times F)^{\sharp}_{1}$ and $(E\times F)^{\sharp}_{2}$ are respectively the conservative part of the Enterprise and of the Financial Institute on the bi-strategy space. By definition, we have

$$(E\times F)^{\sharp}_{1} = \{(x,y)\in E\times F : f_1(x,y)\geq v^{\sharp}_{1}\}$$

and

$$(E\times F)^{\sharp}_{2} = \{(x,y)\in E\times F : f_2(x,y)\geq v^{\sharp}_{2}\}.$$

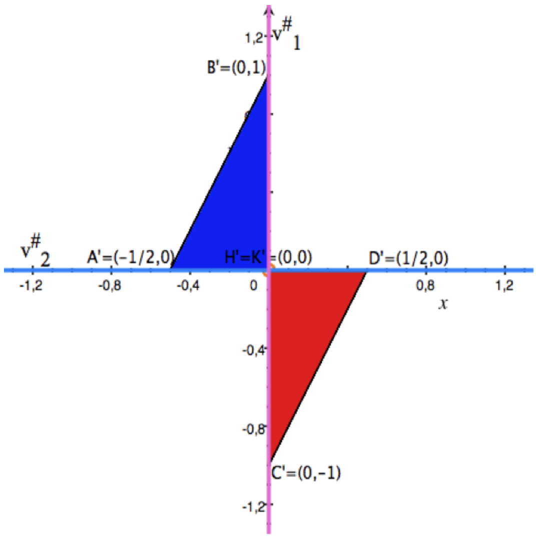


FIGURE 9. Core and conservative parts on the payoff space.

Recalling equations 4 and 9, the conservative part of the Enterprise on the bi-strategy space is given by

$$(E\times F)^{\sharp}_{1} = \{(x,y)\in E\times F : -\nu M_1(1-x)y\geq 0\},$$

which becomes

$$-\nu M_1 y\leq 0 \ \text{ et } \ 1\leq x \qquad \text{or} \qquad -\nu M_1 y\geq 0 \ \text{ et } \ 1\geq x.$$

Recalling that $M_1 = 1$ and $\nu = 0.5$ are always positive numbers (strictly greater than 0), we obtain Fig. 10.

Now talk about the Financial Institute. Recalling equations 7 and 10, the conservative part of the Financial Institute on the bi-strategy space is given by

$$(E\times F)^{\sharp}_{2} = \{(x,y)\in E\times F : M_2 mxy\geq 0\}.$$

Recalling that $M_2 = 2$ and $m = 0.5$ are always positive numbers (strictly greater than 0), we obtain Fig. 11.

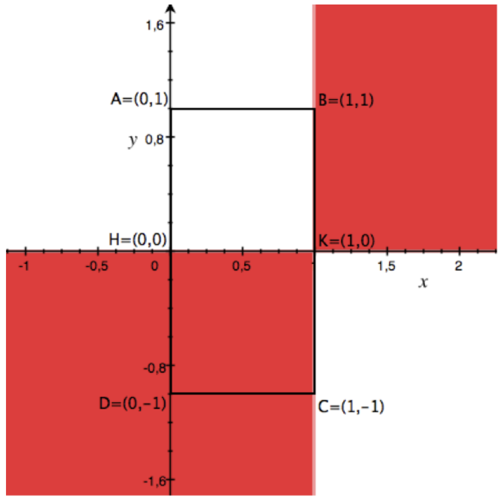


FIGURE 10. Conservative part of the Enterprise (in red) on the bi-strategy space.

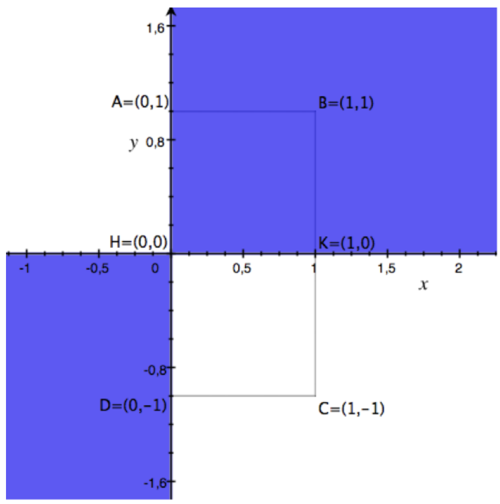


FIGURE 11. Conservative part of the Financial Institute (in light blue) on the bi-strategy space.

Then intersecting the graph of the conservative part (we are talking about the bi-strategy space) of the Enterprise (player 1) and the conservative part of the Financial Institute (player 2), we have the conservative part of the game in the bi-strategy space.
It is given by the intersection

$$(E \times F)^\sharp = \{(x,y) \in E \times F : -\nu M_1(1-x)y \geq 0 \ \text{ et } \ M_2 mxy \geq 0\}.$$

We observe the graphical result in Fig. 12, where the conservative part is easily seen to be a union of three line segments (shown in yellow); this situation was, in any case, quite evident also from the analysis of the figure 9 (representing the transformation of the core of the game and the conservative parts in the payoff space).

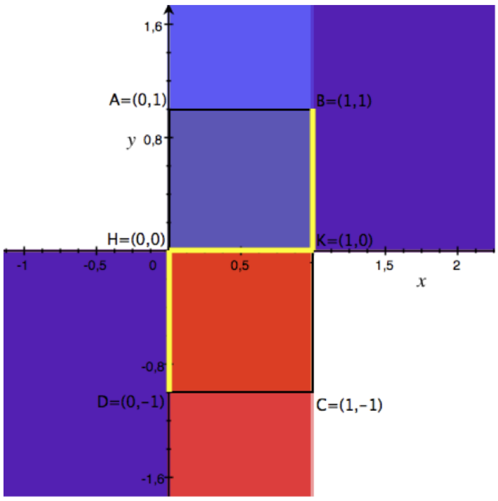


FIGURE 12. Conservative part of the game (in yellow) on the bi-strategy space.

We remark, moreover, that this conservative part coincides with the weak Pareto boundary of the game, that is the set of all bi-strategies which are not strongly dominated by other bi-strategies of the game:

$$\partial_w^* G = \{(x,y) : \text{does not exist } (u,v) \text{ in } E \times F \text{ such that } f(x,y) \ll f(u,v)\},$$

where $k \ll k'$ means that both components of $k$ are strictly less than the corresponding components of $k'$.

So, we see easily that the conservative part of the game, on the bi-strategy space, is given by

$$(E \times F)^\sharp = [B,K] \cup [K,H] \cup [H,D].$$

**Conservative knots of the game.** They are, by definition, the strategy pairs $(x,y)$ such that

$$f_1(x,y) = v_1^\sharp \quad \text{and} \quad f_2(x,y) = v_2^\sharp,$$

that are those bi-strategies whose images coincide with the conservative bi-value.
And therefore, recalling equation 4 and 9, any conservative knot verifies the equation:

$$-\nu M_1(1-x)y = 0.$$

Solving the equation, we obtain $M_1 \nu y = 0$ and $1 - x = 0$.
Recalling that $M_1$ and $\nu$ are always positive numbers (strictly greater than 0), we have:

$$y = 0 \quad \text{or} \quad x = 1.$$

Recalling also equation 7 and 10, we have:

$$M_2 mxy = 0.$$

Recalling that $M_2$ and $m$ are always positive numbers (strictly greater than 0), we have:

$$x = 0 \ \text{ or } \ y = 0.$$

Therefore, as we can see in Fig. 13, every point $(x,0)$ of the bi-strategy space, i.e. the segment $[H,K]$, is a conservative knot.

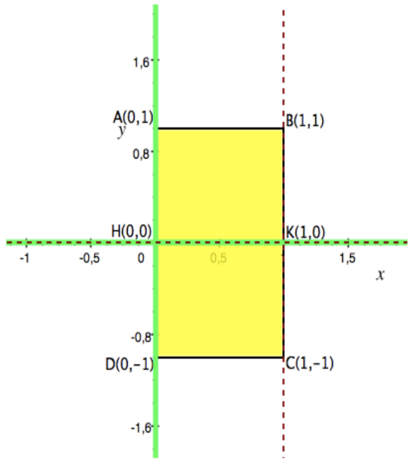


FIGURE 13. Conservative knots

## 10. Equilibria of devotion

In the event that the two players wanted to "do good" to the other one, they would choose their strategy maximizing the payoff of the other one. In this case is necessary to talk about multifunction of devotion.
We have to maximize the payoff function of the other player considering every its possible strategy. In mathematical language for the Enterprise the multifunction of devotion is:

$$L_1 : F \to E : y \mapsto \max_{f_2(\cdot,y)} E,$$

(i.e. the set of strategies of the Enterprise that maximize the section $f_2(\cdot,y)$).

For the Financial Institute the multifunction of devotion is:

$$L_2 : E \to F : x \mapsto \max_{f_1(x,\cdot)} F$$

(i.e. the set of strategies of the Financial Institute that maximize the section $f_1(x,\cdot)$).

In practice, in order to find $L_1$ we search the value of $x$ maximizing $f_2$; in order to find $L_2$ we search the $y$ maximizing $f_1$.

Recalling that $M_1 = 1$ and $\nu = 0.5$ are always positive numbers (strictly greater than 0) and recalling Eq. 4, we have:

$$L_2(x) = \begin{cases} \{-1\} & \text{if} \quad 0 \leq x < 1 \\ F & \text{if} \quad x = 1 \end{cases},$$

Recalling also Eq. 7, we have

$$L_1(y) = \begin{cases} \{1\} & \text{if} \quad y > 0 \\ E & \text{if} \quad y = 0 \\ \{0\} & \text{if} \quad y < 0 \end{cases},$$

In Fig. 14 we illustrate the graphs of $L_1(y)$ (in red) and of $L_2(x)$ (in blue).

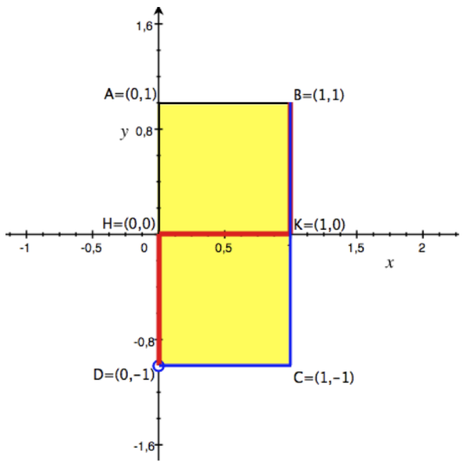


FIGURE 14. Equilibria of devotion

**Set of equilibria of devotion**. It is

$$\mathrm{Eq}(L_1, L_2) = \{(0,-1)\} \cup [B, K].$$

The equilibria of devotion can be considered good because they are on the weak maximal Pareto boundary.

**Analysis of devote equilibria**. The Financial Institute probably plays the strategy $y = -1$ because it is the only one able to maximize the gain of the Enterprise if it plays $x \neq 1$, while if the Enterprise chooses the strategy $x = 1$, the choice of strategy of the Financial Institute is indifferent about the gain (equal to 0) of the Enterprise.
On the other hand, the Enterprise, knowing that the Financial Institute chooses the strategy $y = -1$ in order to help it, most likely chooses $x = 0$. So the Financial Institute gets the highest possible gain, which in this case is equal to 0. We can see that although the equilibria of devotion are infinite, the two players most likely arrive in $D = (0, -1)$, which is on the proper maximal Pareto boundary.

In case of devote strategies adopted by the Financial Institute, most likely the Enterprise manages to gain the maximum possible sum, while it is not the same for the Financial Institute.

## 11. Cooperative solutions

As already observed, E. could recognize its own importance for the financial affairs of F. and could ask a kind of payment for its easygoing participation to the financial interaction. It seems that F. has no reasonable motivations to pay E. for its participation in the Game; indeed, from the Nash analysis and from the conservative analysis we obtain the bi-strategy solution $B = (1,1)$ and the consequent vector-payoff $(0,1)$. Nonetheless, E. could menace credibly to play 0, leading the game to the state $A = (0,1)$ and consequently to the payoff $A' = (-1/2, 0)$. This threat could be credible because the utility bi-value given to $A'$ is determine by the players in two deeply different ways, the evaluation of E. on its own losses (in the purchase of the commodity) is realistically less severe than that associated by F. with the absence of profits in its financial operation, this because:

- the primary activity of F. is to speculate and gain from its financial operations;
- on the contrary, the primary activity of E. takes place in the real economy.

So, the above could be one motivation for F. to find a bargaining solution with the Enterprise. Moreover, since the evaluation of E. on its own position depends strongly on the general situation of E., as well as that of F., we propose two different solutions in which the balance of power between E. and F. are different.
So, the best way for the two players to get both a gain is to find a cooperative solution. One way would be to divide the maximum collective profit, determined by the maximum collective gain functional $g$, defined by

$$g(X,Y) = X + Y$$

on the payoffs space of the game $G$, i.e the profit $w = \max_{f(E\times F)} g$.

The maximum collective profit $w$ of the game is attained (evidently) at the point $B'$, which is the only bi-gain belonging to the straight line with equation $X + Y = 1$ and to the payoff space $f(E \times F)$.
So the Enterprise and the Financial Institute play $x = 1$ and $y = 1$, in order to arrive at the payoff $B'$. Then, they split the obtained bi-gain $B'$ by contract.

**Financial point of view.** the Enterprise buys futures to create artificially a misalignment between futures and spot prices, misalignment that is exploited by the Financial Institute, which gets the maximum gain $w = 1$.

**First possible division.** For a possible **fair division** of this gain $w = 1$, we employ a transferable utility solution: finding on the transferable utility Pareto boundary of the payoff space a *non-standard Kalai-Smorodinsky solution* (it is a non-standard K-S solution because we do not consider the whole game, but only its maximal Pareto boundary). We find the supremum of the maximal boundary,

$$\sup \partial^* f(E \times F),$$

which is the point $\alpha = (1/2, 1)$, and we join it with the infimum of the maximal boundary,

$$\inf \partial^* f(E \times F),$$

which is the origin $(0,0)$.
The coordinates of the intersection point $P$ (see fig. 15), between the straight line of maximum collective gain (i.e. $X+Y=1$) and the straight line joining the supremum of the maximal Pareto boundary with the infimum (i.e., the line $Y = 2X$) gives us the desirable division of the maximum collective gain $w = 1$ between the two players.

**Second possible division.** For another possible **fair division** of the gain $w = 1$, we propose a *transferable utility Kalai-Smorodinsky method*. The bargaining problem we face is the pair $(\Gamma, v^\sharp)$, where:

(1) our decision constraint $\Gamma$ is the transferable utility Pareto boundary of the game (straight line $X+Y=1$);
(2) we take the payoff $A' = (-1/2, 0)$ of the most likely offensive equilibrium as the threat point of our bargaining problem.

**Solution.** For what concerns the solution: we join $A' = (-1/2, 0)$ with the supremum

$$\sup(\Gamma \cap [A', \rightarrow [),$$

according to the classic Kalai-Smorodinsky method, supremum given by $(1, 3/2)$.

The coordinates of the intersection point $P'$ (see fig. 15) - between the straight line of maximum collective gain (i.e. $X+Y=1$) and the segment joining $A'$ and the considered supremum (the segment is part of the line $A' + \mathbb{R}(1,1)$) - give us the desirable division of the maximum collective gain $w = 1$, between the two players.

Thus the solution points $P = (1/3, 2/3)$ and $P' = (1/4, 3/4)$ suggest that the Enterprise has to receive respectively $1/3$ or $1/4$ (by contract) from the Financial Institute, while at the Financial Institute remains the gain $2/3$ or $3/4$.

**Why are there differences between the two possible divisions of the collective profit?**
The difference between the points $P$ and $P'$ are due to the different solution methods.
About the point $P$, we consider as threat and utopia point respectively the inf and the sup of the maximal boundary. Therefore, the division is more profitable for the Financial Institute because it can obtain a higher maximum profit (that is 2/3) than the Enterprise (that can obtain 1/3).
About the point $P'$, we consider as threat point the image $A' = (-1/2, 0)$ of the most likely offensive equilibrium and its supremum $(1, 3/2)$, according to the classic Kalai-Smorodinsky method. Therefore, the division is even more profitable for the Financial Institute because, if the two players decide to clash, the Enterprise obtains its most high possible loss while the Financial Institute manages to lose nothing.
In both cases, the gain of E. is the payment for its easygoing participation to the financial interaction, a payment determined by the balance of power between E. and F.

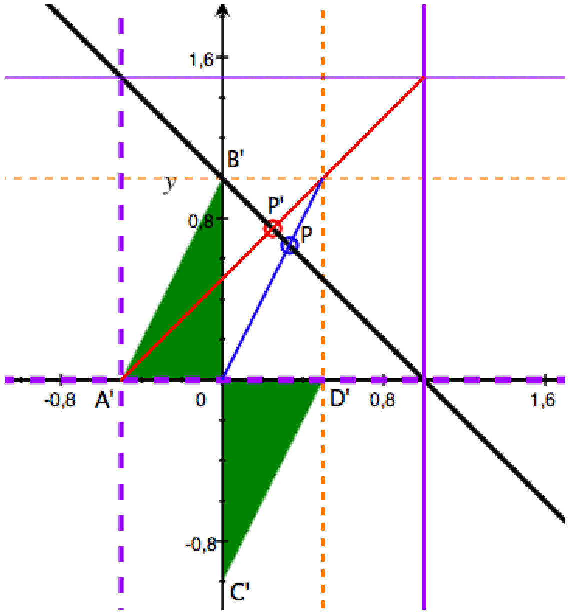


FIGURE 15. Transferable utility solution: cooperative solutions

## 12. Conclusions

In this paper we model, in a new way, a possible interaction about a not specified asset (oil, currencies or securities for instance) between a generic real economic subject, with hedging aims, and a financial subject, that try to profit by speculation.

In fact, by introducing a tax on speculative profits, the financial player is forced to an interaction with another subject to obtain a profit and this favors a stabilization of the financial market, because of the absence of uncontrolled and unpredictable speculations. Moreover, we have noted that this possible profit depends on the behavior of the real economic subject, which could prevent the gains of the financial institute.

For this reason, we suppose an agreement between the two player in order to divide the maximum collective profit: the real economic subject causes a misalignment between futures and spot prices of the asset, misalignment that is exploited by the financial institute to gain the maximum possible gain.

After, by mean of contract, the financial institute gives the enterprise part of this profit (at this purpose we propose two possible transferable utility solutions). We can consider this part of profit that is transferred to the real economic subject under a threefold perspective:

(1) it is the *fair cost* of the uncertainty to achieve the most likely Nash equilibrium (favorable to the financial institute) without cooperation;

(2) it allows a *fair redistribution* of the wealth generated by financial transactions;
(3) the right payment to E., for its easygoing participation to the financial interaction.

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