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SCATTERING BY CLOSELY SPACED INFINITE CYLINDERS IN AN ABSORBING MEDIUM

SIU-CHUN LEE*

ABSTRACT. Scattering by closely spaced parallel infinite cylinders in an absorbing medium is considered in this paper. The source wave is arbitrarily polarized and propagates in a general direction at the cylinders. The formulation utilizes the Hertz potential approach, and the scattering cross section and intensity distribution in the far-field are developed. Numerical results are presented to illustrate the influence of the absorbing medium on the scattering properties of two configurations of closely-spaced cylinders.

1. Introduction

Scattering of electromagnetic wave by small particles is a subject of extensive research due to its relevance to problems in atmospheric scattering, remote sensing, and radiative transfer. Scattering by an infinite cylinder is generally applied to radiative transfer analyses of fiber materials [1]. Many fiber composites contain densely packed parallel fibers embedded in a matrix that vary from a dielectric to absorbing at different wavelengths.

The scattering formalisms for a single infinite cylinder or closely spaced infinite cylinders in a non-absorbing medium have been well established [2, 3, 4, 5, 6]. The corresponding problem in an absorbing medium has been studied for normal [7, 8, 9] and oblique incidence [10]. The purpose of this paper is to examine scattering by closely spaced infinite cylinders in an absorbing medium. The influence of absorption by the host medium on the scattering properties is illustrated by numerical examples.

2. Theory

Figure 1 shows an arbitrary configuration of cylinders located in an absorbing medium with complex refractive index ($\tilde{m}_1$) and magnetic permeability (m_1). Each cylinder is prescribed by its radius (r_j), refractive index ($\tilde{m}_j$), and permeability (μ_j), where $\tilde{m}_j$ and μ_j are in general complex. The incident wave propagates in a general direction prescribed by polar angle f_1 from the XY plane and azimuth angle q_1 in the XY plane. The cylinders are aligned parallel to the Z-axis. The location of cylinder j is designated by the radial distance R_j and azimuth angle g_j with respect to OXYZ.

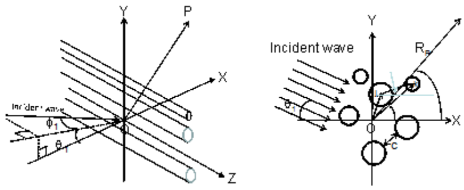

Figure 1. Schematic diagrams showing an EM wave incident on a collection of cylinders.

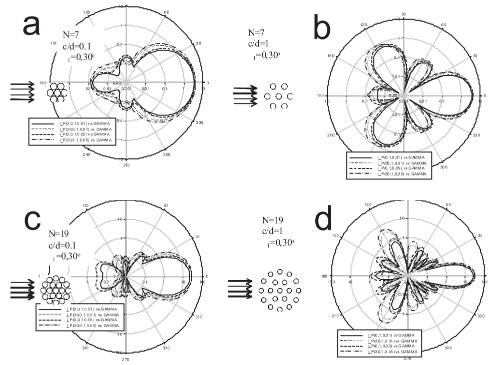

Figure 2. Scattered intensity distribution for (a) 7 SiC cylinders with c/d=0.1, (b) 7 SiC cylinders with c/d=1.0, (c) 19 SiC cylinders for c/d=0.1 and (d) 19 SiC cylinders for c/d=1.0.

The Hertz potentials external to the cylinder consist of contributions from the incident, scattered, and secondary incident waves due to scattered waves from other cylinders:

$$\psi_j\left(\vec{R}_P\right) = \psi_j^{inc}\left(\vec{R}_{jP}\right) + \psi_j^{sca}\left(\vec{R}_{jP}\right) + \sum_{k\neq j}^{N} \psi_{kj}^{sca}\left(\vec{R}_{kP}\right), \qquad R_{jP} > r_{j,Q_j} \tag{1}$$

where $\psi \in (u,v)$ are the Hertz potentials, and the superscripts *inc* and *sca* denote the incident and scattered waves, respectively. The Hertz potentials for the scattered waves are given by

$$\begin{pmatrix} u_j^{sca} \\ v_j^{sca} \end{pmatrix} = -\exp(-ih_1 z) \sum_{n=-\infty}^{\infty} (-i)^n \exp(in\gamma_{jP}) H_n(\ell_1 R_{jP}) \begin{pmatrix} b_{jn} \\ a_{jn} \end{pmatrix} \tag{2}$$

where $h = k_1 \sin\phi_1,\ \ell_1 = k_1 \cos\phi_1,\ k_1 = \tilde{m}_1 k_o$ is the propagation constant in the medium, k_o is the free space propagation constant, H_n is Hankel function of the second kind, and (a_{jn}, b_{jn}) are unknown partial wave coefficients. The properties of the medium and cylinders are given by

$$(\tilde{m}, \mu, k) = \begin{cases} (\tilde{m}_1, \mu_1, k_1), & R_{jP} > r_j \\ (\tilde{m}_j, \mu_j, k_j = \tilde{m}_j k_o), & R_{jP} < r_j \end{cases} \tag{3}$$

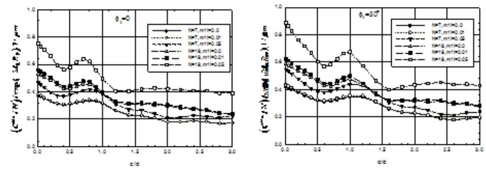

Figure 3. Scattering cross section of cluster of cylinders, (a) f$_1$=0 and (b) f$_1$=30^o.

Continuity of the tangential components of the E and H field vectors across the interface of each cylinder yields a set of equations that govern the partial wave coefficients. The unknown partial wave coefficients of each cylinder can be expressed in terms of the known coefficients and the location of the cylinders as

$$\begin{bmatrix} b_{ks} \\ a_{ks} \end{bmatrix} = \varepsilon_j \exp\left(in\theta_1\right) \times \begin{bmatrix} \delta_{jk}\delta_{ns} + \left(1-\delta_{jk}\right) G_{ks}^{jn} b_{jn}^{o,I} & \left(1-\delta_{jk}\right) G_{ks}^{jn} b_{jn}^{o,II} \\ \left(1-\delta_{jk}\right) G_{ks}^{jn} a_{jn}^{o,I} & \delta_{jk}\delta_{ns} + \left(1-\delta_{jk}\right) G_{ks}^{jn} a_{jn}^{o,II} \end{bmatrix}^{-1} \begin{bmatrix} b_{jn}^{o} \\ a_{jn}^{o} \end{bmatrix} \tag{4}$$

where δ_{jk} and δ_{ns} are Kronecker delta functions, e$_j$ is the phase shift of cylinder j relative to OXYZ,

$$G_{ks}^{jn} = \left(-i\right)^{s-n} \exp\left[i\left(s-n\right)\gamma_{kj}\right] H_{s-n}\left(\ell_1 R_{jk}\right) \tag{5}$$

γ_{kj} is the azimuth angle of cylinder k relative to j, and $\left(a_{jn}^{o,I}, b_{jn}^{o,I}\right)$ and $\left(a_{jn}^{o,II}, b_{jn}^{o,II}\right)$ are the TM and TE mode partial wave coefficients for a single cylinder in an absorbing medium [10]. In addition, the partial wave coefficients on the right hand side of Eq. (4) is given by

$$\begin{pmatrix} b_{jn}^{o} \\ a_{jn}^{o} \end{pmatrix} = \alpha_u \begin{pmatrix} b_{jn}^{o,I} \\ a_{jn}^{o,I} \end{pmatrix} + \alpha_v \begin{pmatrix} b_{jn}^{o,II} \\ a_{jn}^{o,II} \end{pmatrix} \tag{6}$$

where (α_u, α_v) are the complex amplitudes that prescribe the polarization of the source wave.

The Poynting vector for the total radiative energy flow is formulated by utilizing the total E and H field vectors. Of particular interests are far field radiative properties that include the scattered intensity distribution, scattering cross section, and extinction cross section. By utilizing the far-field approximations of $\gamma_{jP} \sim \gamma_{kP} \sim \gamma_P$ and $1/R_{jP}, 1/R_{kP} \sim 1/R_P$, where γ_P and R_P are defined with respect to OXYZ, we obtain the scattered intensity distribution as

$$\vec{I}^{sca}\left(\phi_1, \gamma_P\right) = \left(\left|T_u\left(\phi_1, \gamma_P\right)\right|^2 + \left|T_v\left(\phi_1, \gamma_P\right)\right|^2\right) \exp\left(-2\ell_{1i}R_P - 2h_{1i}z\right)\vec{e}_s \tag{7}$$

where $\vec{e}_s = \cos\phi_1 \vec{e}_R + \sin\phi_1 \vec{e}_z$, $h_{1i} = -Im\left(h_1\right)$, $\ell_{1i} = -Im\left(\ell_1\right)$, Im denotes the imaginary part, and

$$\begin{bmatrix} T_u\left(\phi_1, \gamma_P\right) \\ T_v\left(\phi_1, \gamma_P\right) \end{bmatrix} = \sum_{j,n} \begin{bmatrix} b_{jn} \\ a_{jn} \end{bmatrix} \exp\left[in\gamma_P + i\ell_1 R_j \cos\left(\gamma_P - \gamma_j\right)\right] \tag{8}$$

are the scattering amplitudes. Integrating the scattered intensity over a large cylindrical surface surrounding the cylinders and normalizing by the magnitude of the Poynting vector of the incident wave yields the apparent scattering cross section per unit length as:

$$C^{sca} = \frac{4}{|k_1|} \exp\left(-2\ell_{1i} R_P\right) \sum_{j,k=1}^{N} \sum_{n,s=-\infty}^{\infty} \exp\left[i\left(n-s\right)\left(\Gamma_{jk} - \pi/2\right)\right] \times J_{s-n}\left(\ell_1 \bar{R}_{jk}\right) \frac{\left(b_{jn} b_{ks}^* + a_{jn} a_{ks}^*\right)}{|\alpha_u|^2 + |\alpha_v|^2} \tag{9}$$

where (α_u, α_v) are the complex amplitudes that prescribe the polarization of the incident wave,

$$\exp\left(i\Gamma_{jk}\right) = \left[\ell_1 R_j \exp\left(i\gamma_j\right) - \ell_1^* R_k \exp\left(i\gamma_k\right)\right] / \left(\ell_1 \bar{R}_{jk}\right) \tag{10}$$

$$\ell_1 \bar{R}_{jk} = \sqrt{\left(\ell_1 R_j\right)^2 - 2\left|\ell_1\right|^2 R_j R_k \cos\left(\gamma_j - \gamma_k\right) + \left(\ell_1^* R_k\right)^2} \tag{11}$$

and the superscript * refers to the complex conjugate.

3. Results

Numerical results are shown for 2 configurations of silicon carbide (SiC) cylinders arranged in a hexagonal pattern. The source radiation propagates at normal (f_1=0) and oblique (f_1=30^o) incidence on the cylinders along the line of symmetry relative to the location of the cylinders. The free-space wavelength of the source radiation is 10 mm, at which the refractive index of SiC is $\tilde{m}(SiC) = 1.051 - i0.0255$, and unity size parameter is assumed for all the cylinders. The refractive index of the host medium is $\tilde{m}_1 = 1.2 - im_{1i}$. Figures 2a-b show the normalized scattered intensity distribution for a cluster of 7 cylinders at normal (f_1=0) and oblique (f_1=30^o) incidence in an absorbing medium (m_{1i}=01,0.05) for the spacing-to-diameter ratio (c/d) of 0.1 and 1. The corresponding results for a cluster of 19 cylinders are shown in Figs. 3a-b. The variation of the scaled scattering cross section with c/d is shown in Figs. 4a-b for the two configuration of cylinders.

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* Applied Sciences Laboratory, Inc.,
Baldwin Park, CA 91706, USA

Email: sclee@appliedscienceslab.com

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