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STRINGS AND BRANE WORLD SCENARIOS IN FINANCIAL MARKET DATA

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ABSTRACT. In the paper, we study the projections of the real exchange rate dynamics onto the string-like topology. Our approach is inspired by the contemporary movements in the string theory. The string map of data is defined here by the boundary conditions, characteristic length, real valued and the method of redistribution of information. As a practical matter, this map represents the detrending and data standardization procedure. We introduced maps onto 1-end-point and 2-end-point open strings that satisfy the Dirichlet and Neumann boundary conditions. The questions of the choice of extra-dimensions, symmetries, duality and ways to the partial compactification are discussed. Subsequently, we pass to higher dimensional and more complex objects. The 2D-Brane was suggested which incorporated bid-ask spreads. The systematic way which allows one suggest more structured maps suitable for a simultaneous study of several currency pairs was analyzed by means of the Gâteaux generalized differential calculus. The effect of the string and brane maps on test data was studied by comparing their mean statistical characteristics. The possible utilizations of the string theory approach in financial market are slight.

1. Introduction

We are currently in the process of transfer of modern physical ideas into the neighboring field called econophysics. The physical statistical view point has proved fruitful, namely, in the description of systems where many-body effects dominate. However, standard, accepted by physicists, bottom-up approaches are cumbersome or outright impossible to follow the behavior of the complex economic systems, where autonomous models encounter the intrinsic variability.

Digital economy is founded on data. In the paper, we suggest and analyze statistical properties of heuristics based on the currency rate data which are arranged to mimic the topology of the basic variants of the physical strings and branes. Our primary motivation comes from the actual physical concepts (McMahon 2008; Zwiebach 2009); however, our realization differs from the original attempts in various significant details. The second aspect of our method is that it enables a transformation into a format which is useful for an analysis of a partial trend or relative fluctuations on the time scale window of interest.

As with most science problems, the representation of data is the key to efficient and effective solutions. The underlying link between our approach and the string theory may be seen in the switching from a local to a non-local form of the data description. This line passes from the single price to the multivalued collection of prices from the temporal neighborhood which we term here the string map. As we will see later, an important role in our considerations is played by the distance measure of the string maps. The idea of exploring the relationship between more intuitive geometric methods and financial data is not new. The discipline called the geometric data analysis includes many diverse examples of the conceptual schemes and theories grounded on the geometric representation and properties of data. Among them we can emphasize the tree network topology that exhibits usefulness in the studies of the world-trade network (He and Deem 2010) and other network structures of the market constructed by means of inter-asset correlations (M. Eryiğit and R. Eryiğit 2009; Jung *et al.* 2008). The multivariate statistical method called *cluster analysis* deals with data mapping onto representative subsets called *clusters* (Krishnaiah and Kanal 1982). Here we work on the concept that is based on projection data into higher dimensional vectors in the sense of the work by Grassberger and Procaccia (1983) and Yongwei *et al.* (2010). Also, arguments based on the metrics are consistent with our efforts but not too obvious points in common with the original objectives of the nonlinear analysis.

The string theory development over the past 25 years achieved a high degree of popularity among physicists (Green 1988; Polchinski 1998). The reason lies in its inherent ability to unify theories that come from diverse physical spheres. The prime instrument of the unification represents the concept of extra dimension. The side-product of theoretical efforts can be seen in the elimination of the ultraviolet divergences of Feynman diagrams. However, despite the considerable achievements, there is a lack of the experimental verification of the original string theory. In contrast, in the present work we exploit time-series which can build the family of the string motivated models of boundary-respecting maps. In a narrow sense, the purpose of the present data-driven study is to develop statistical techniques for the analysis of these objects.

The work is organized as follows: In Sec. **2**, we specify the data selection and data pretreatment. In Sec. **3**, we introduce the notion of the string map of data time series. The symmetry of the maps is discussed in Sec. **4**. To examine generality and specificity of these ideas, the calculations have been performed for several representative currencies and ask (buyer initiated) or bid (seller initiated) prices. In Sec. **5**, we give a generalization for a higher dimensional case which encodes the ask-bid spread difference. In this section, we also discuss the problem of partial compactification (subsection **5.1**). Efforts have been made to study inter-currency relations by means of the projections onto *rotating strings* (Sec. **6**) and via a generalization of the derivative (Sec. **7**).

2. Data analysis

First of all we need to mention some facts about data streams we analyzed. We analyze tick by tick data of EUR/USD, GBP/USD, USD/JPY, USD/CAD, USD/CHF major currency pairs from the OANDA market maker. We focussed on the three month period within three selected periods of 2009 which capture moments of the financial crisis. The streams are collected in such a way that each stream begins with Monday. More precisely, we selected periods denoted as Aug-Sep (from August 3rd. to September 7th.), Sep-Oct (Sep.7-Oct.5) and Oct-Nov (Sep.5-Nov.2). At first, the data sample has been decimated - only each 10th tick was considered. This delimits results to the scales larger than 10 ticks. The mean time corresponding to the string length l_s in ticks is given by

$$T(l_\mathrm{s}) = \langle t(\tau+l_\mathrm{s}) - t(\tau)\rangle \simeq \frac{1}{\tau_\mathrm{up}-\tau_\mathrm{dn}} \sum_{\tau=\tau_\mathrm{dn}}^{\tau_\mathrm{up}} \left[t(\tau+l_\mathrm{s}) - t(\tau)\right]. \tag{1}$$

Data for study of rotating strings and angular moments (see Sec. **6**) were preformatted in a different way. In this case, the currency information has been projected onto the grid of the regularly spaced 10 sec intervals.

3. One dimensional maps

By applying standard methodologies of detrending one may suggest to convert original series of the quotations of the mean currency exchange rate $p(\tau)$ onto a series of returns defined by

$$\frac{p(\tau+h)-p(\tau)}{p(\tau+h)}, \tag{2}$$

where h denotes a tick lag between currency quotes $p(\tau)$ and $p(\tau+h)$, τ is the index of the quote. The mean $p(\tau) = (p_\mathrm{ask}(\tau)+p_\mathrm{bid}(\tau))/2$ is calculated from $p_\mathrm{ask}(\tau)$ and $p_\mathrm{bid}(\tau)$.

In the spirit of the string theory it would be better to start with the 1-end-point open string map

$$P^{(1)}(\tau,h) = \frac{p(\tau+h)-p(\tau)}{p(\tau+h)}, \qquad h \in < 0, l_\mathrm{s} > \tag{3}$$

where superscript (1) refers to the number of endpoints.

Later, we may use the notation $P\{p\}$ which emphasizes the functional dependence upon the currency exchange rate $\{p\}$. It should also be noted that the use of P highlights the canonical formal correspondence between the *rate of return* and the internal *string momentum*.

Here the tick variable h may be interpreted as a variable which extends along the extra dimension limited by the string size l_s. A natural consequence of the transform, Eq.(3), is the fulfilment of the boundary condition

$$P^{(1)}(\tau,0) = 0, \tag{4}$$

which holds for any tick coordinate τ. Later on, we want to highlight effects of the rare events. For this purpose, we introduce a power-law q-deformed model

$$P_q^{(1)}(\tau,h) = f_q\left(\frac{p(\tau+h)-p(\tau)}{p(\tau+h)}\right), \qquad h \in < 0, l_\mathrm{s} > \tag{5}$$

by means of the function

$$f_q(x) = \mathrm{sign}(x)\,|x|^q, \qquad q > 0. \tag{6}$$

The 1-end-point string has defined the origin, but it reflects the linear trend in $p(.)$ at the scale l_s. Therefore, the 1-end-point string map $P_q^{(1)}(.)$ may be understood as a q-deformed generalization of the *currency returns*. The illustration of the 1-end-point model is given in Fig.(1). The corresponding statistical characteristics displayed in Fig.(2) have been obtained on the basis of a statistical analysis discussed in Sec. **2**.

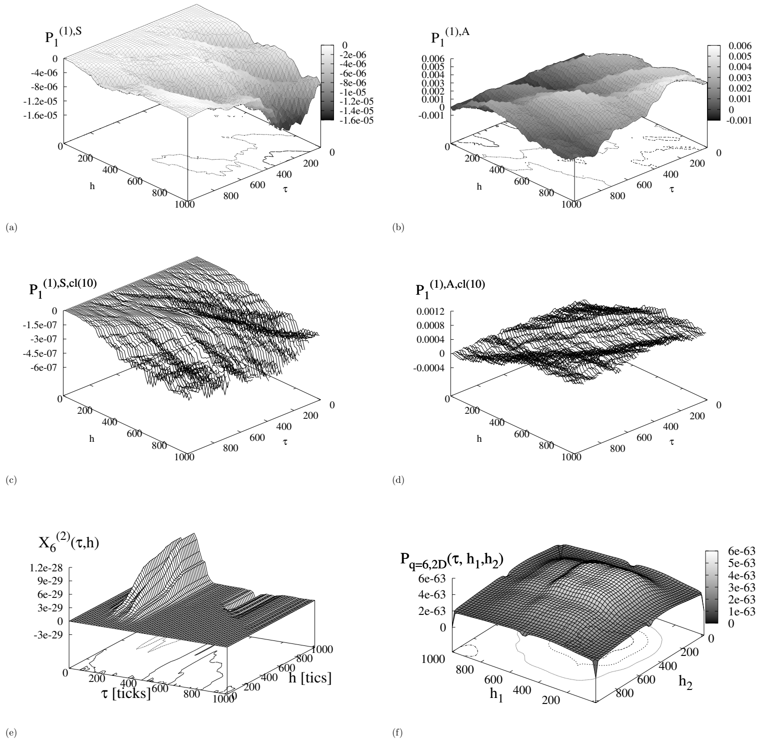

FIGURE 1. The illustrative examples of the currency data map for GBP/USD. The parts (a)-(d) constructed for date Fri, 31 Jul 2009 time interval 15:06:37 - 15:43:09 GMT. Time evolution of symmetric ($P_{q=1}^{(1),\mathrm{S}}$) and anti-symmetric ($P^{(1),\mathrm{A}}$) component of the 1-end-point string of size $l_{\mathrm{s}} = 1000$ calculated for $q = 1$ (by means of Eq.(18)). In (c),(d) we see the same data mapped by means of the partially closed 1-end-point string ($q = 1$) for $N_{\mathrm{m}} = 10$, according to Eq.(26)). (e) The calculation carried out for the 2-end-point string for $l_{\mathrm{s}} = 1000$, $q = 6$ at some instant. We see that conjugate variable $X_{q=6}^{(2)}(\tau,h)$ satisfies the Neumann-type boundary conditions; (f) The instantaneous 2D-Brane state (date Fri, 31 Jul 2009 15:11:47 GMT) is computed according to Eq.(23).

Clearly, the situation with a long-term trend is partially corrected by fixing $P_q^{(2)}(\tau,h)$ at $h = l_{\mathrm{s}}$. The open string with two end points is introduced via the nonlinear map which combines information about trends of p at two sequential segments

$$P_q^{(2)}(\tau,h) = f_q\left(\left(\frac{p(\tau+h)-p(\tau)}{p(\tau+h)}\right)\left(\frac{p(\tau+l_{\mathrm{s}})-p(\tau+h)}{p(\tau+l_{\mathrm{s}})}\right)\right), \qquad h \in < 0, l_{\mathrm{s}} > . \quad (7)$$

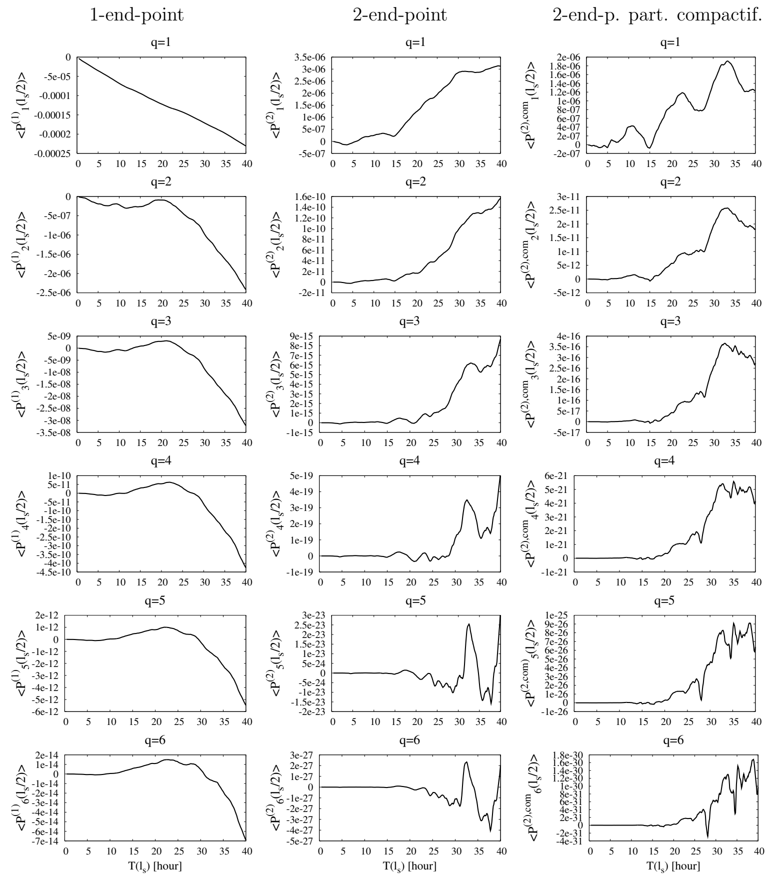

FIGURE 2. The variability in statistical characteristics caused by differences in topology and q. Calculated for the period Aug-Sep, GBP/USD currency. The model with $q = 1$ has ability to reveal the currency long trend, on the other hand, the rare events are better visible for the 2-end-point string. The effect of the partial compactification with $N_{\rm m} = 4$ [see Eq.(26)] is demonstrated in the third column (again for the 2-end-point string).

The map is suggested to include boundary conditions of *Dirichlet type*

$$P_q^{(2)}(\tau,0) = P_q(\tau,l_{\rm s}) = 0\,, \qquad \text{at all ticks } \tau\,. \tag{8}$$

In particular, the sign of $P_q^{(2)}(\tau,h)$ comprises information about the behavior differences of $p(.)$ at three quotes $(\tau,\, \tau+h,\, \tau+l_{\rm s})$. The $P_q^{(2)}(\tau,h) < 0$ occurs for trends of the different sign, whereas $P_q^{(2)}(\tau,h) > 0$ indicates the match of the signs.

In addition to the variable $P_q^{(2)}(\tau,h)$ we introduced the conjugate variable $X_q^{(2)}(\tau)$ via the recurrent summation

$$X_q^{(2)}(\tau,h+1) = X_q^{(2)}(\tau,h) + P_q^{(2)}(\tau,h-1)\left[t(\tau+h) - t(\tau+h-1)\right] \tag{9}$$

(here $t(.)$ stands for a time-stamp corresponding to the quotation index τ in the argument). The above discrete form is suggested on the basis of the time-continuous Newton second law of motion $\dot{X}_q^{(2)}(t,h) = P_q^{(2)}(t,h)$ (written here for a unit mass). The form is equivalent to the imposing of the quadratic kinetic energy term $\frac{1}{2}(P_q^{(2)})^2$. Thus, the Hamiltonian picture (Green 1988) can be reconstructed in the following way;

$$\mathscr{H} = \frac{1}{2}\sum_{h=0}^{l_s}\left[(P_q^{(2)}(\tau,h))^2 - [\phi_{\mathrm{ext}}(\tau,h+1) - \phi_{\mathrm{ext}}(\tau,h)]\, X_q^{(2)}(\tau,h)\right], \tag{10}$$

where $\phi_{\mathrm{ext}}(\tau,h)$ is the external field term which depends on the transform of the currency rate [see e.g. Eq.(7)]. We pass from the continuum to discrete theory by means of the functional form

$$\dot{P}_q^{(2)} = -\frac{\delta\mathscr{H}}{\delta X_q^{(2)}(h)} = \phi_{\mathrm{ext}}(\tau,h+1) - \phi_{\mathrm{ext}}(\tau,h) = P_q^{(2)}(\tau,h+1) - P_q^{(2)}(\tau,h), \tag{11}$$

where $P_q^{(2)}(\tau,h)$ can be calibrated equal to $\phi_{\mathrm{ext}}(\tau,h)$.

The discrete conjugate variable meets the *Neumann type* boundary conditions

$$X_q^{(2)}(\tau,0) = X_q^{(2)}(\tau,1), \qquad X_q^{(2)}(\tau,l_s-1) = X_q^{(2)}(\tau,l_s), \tag{12}$$

which is illustrated in Fig.(1)(d).

A more systematic way to obtain the 2-end-point string map represents the method of undetermined coefficients. The numerator of $q=1$ can be chosen in the functional polynomial form of degree 2 with coefficients $\beta_0,\dots,\beta_5$ as follows:

$$P_{q=1,\mathrm{Num}}^{(2)}(\tau,h) = \beta_0 p^2(\tau+h) + \beta_1 p^2(\tau) + \beta_2 p^2(\tau+l_s) \tag{13}$$

$$+ \beta_3 p(\tau)p(\tau+h) + \beta_4 p(\tau)p(\tau+l_s) + \beta_5 p(\tau+h)p(\tau+l_s). \tag{14}$$

Again, the Dirichlet conditions $P_{q=1,\mathrm{Num}}^{(2)}(\tau,0) = P_{q=1,\mathrm{Num}}(\tau,l_s) = 0$ yield $P_{q=1,\mathrm{Num}}^{(2)} = \beta_0(p(\tau)-p(\tau+h))(p(\tau+l_s)-p(\tau+h))$ with arbitrary β_0. The overlooked denominator part of fraction $P_{q=1}^{(2)}$ then servers as a normalization factor.

Another interesting issue is the generalizing 1-end-point string to include the effect of many length scales

$$P_q^{(N_{l_s})}(\tau,h;\{l\}) = \prod_{i=1}^{N_{l_s}} f_q\left(\frac{p(\tau+l_i)-p(\tau+h)}{p(\tau+h)}\right), \tag{15}$$

which relies on the sequence $\{l\} \equiv \{l_i, i=1,\dots,N_{l_s}\}$, including the end points ($\min_{i=1,\dots,N_{l_s}} l_i$ and $\max_{i=1,\dots,N_{l_s}} l_i$) as well as the $N_{l_s}-2$ interior node points that divide the string map into the sequence of unfixed segments of the non-uniform length (in general).

4. Symmetry with respect to $p(.) \to 1/p(.)$ transform

The currency pairs can be separated into *direct* and *indirect* type. In a direct quote the *domestic* currency is the base currency, while the *foreign* currency is the quote currency. An *indirect* quote is just the opposite. Therefore, it would be interesting to take this symmetry into account. Hence, one can say that this two-fold division of the market network admits *duality symmetry*. Duality symmetries are some of the most interesting symmetries in physics. The term *duality* is used to refer to the relationship between two systems that have different descriptions but identical physics (identical trading operations).

Let us analyze the 1-end-point elementary string map when the currency changes from direct to indirect. The change can be formalized by means of the transformation

$$\hat{\mathscr{T}}_{\mathrm{id}} : P\{p(.)\} \to \overline{P}\{p(.)\} \equiv P\{1/p(.)\}\,, \tag{16}$$

For the 1-end-point map model of the string, Eq.(5), we obtained

$$\hat{\mathscr{T}}_{\mathrm{id}} P_q^{(1)}(\tau,h) = \overline{P}_q^{(1)}(\tau,h) = f_q\left(\frac{p(\tau)-p(\tau+h)}{p(\tau)}\right). \tag{17}$$

Let us consider two-member space of maps $V_P^{(1)} = \{P_q^{(1)}, \overline{P}_q^{(1)}\}$. Important, we see that $\hat{\mathscr{T}}_{\mathrm{id}}$ preserves the Dirichlet boundary conditions, in addition, the identity operator $\hat{\mathscr{T}}_{\mathrm{id}}^2$ leaves the elements of $V_P^{(1)}$ unchanged. The space $V_P^{(1)}$ is closed under the left action of $\hat{\mathscr{T}}_{\mathrm{id}}$. These ideas are straightforward transferable to the 2-end-point string points.

Now we omit the notation details and proceed according to Eq.(16). The map $P(.)$ is decomposable into a sum of symmetric and antisymmetric parts

$$P^{\mathrm{S}} = \frac{1}{2}(P+\overline{P})\,, \qquad P^{\mathrm{A}} = \frac{1}{2}(P-\overline{P}), \tag{18}$$

respectively. Due to of normalization by $1/2$, we get the projection properties

$$\hat{\mathscr{T}}_{\mathrm{id}} P^{\mathrm{S}} = P^{\mathrm{S}}\,, \qquad \hat{\mathscr{T}}_{\mathrm{id}} P^{\mathrm{A}} = -P^{\mathrm{A}}\,. \tag{19}$$

To be more concrete, we choose $q=1$ and obtain

$$P_{q=1}^{(1),\mathrm{S}} = 1 - \frac{1}{2}\left[\frac{p(\tau)}{p(\tau+h)} + \frac{p(\tau+h)}{p(\tau)}\right], \qquad P_{q=1}^{(1),\mathrm{A}} = \frac{1}{2}\left[\frac{p(\tau)}{p(\tau+h)} - \frac{p(\tau+h)}{p(\tau)}\right]. \tag{20}$$

and

$$P_1^{(2),\mathrm{A}} = \frac{1}{2}\left[\frac{p(\tau)}{p(\tau+l_{\mathrm{s}})} - \frac{p(\tau+l_{\mathrm{s}})}{p(\tau)} + \frac{p(\tau+h)}{p(\tau)} - \frac{p(\tau)}{p(\tau+h)} + \frac{p(\tau+l_{\mathrm{s}})}{p(\tau+h)} - \frac{p(\tau+h)}{p(\tau+l_{\mathrm{s}})}\right] \tag{21}$$

$$P_1^{(2),\mathrm{S}} = 1 + \frac{1}{2}\left[\frac{p(\tau+l_{\mathrm{s}})}{p(\tau)} + \frac{p(\tau)}{p(\tau+l_{\mathrm{s}})} - \frac{p(\tau)}{p(\tau+h)} - \frac{p(\tau+h)}{p(\tau)} - \frac{p(\tau+h)}{p(\tau+l_{\mathrm{s}})} - \frac{p(\tau+l_{\mathrm{s}})}{p(\tau+h)}\right].$$

We see that the $P_{q=1}^{(1),\mathrm{S}}$ and $P_{q=1}^{(2),\mathrm{S}}$ maps acquire formal signs of the systems with $T-$*dual symmetry* (Zwiebach 2009). When the world described by the closed string of the radius R is indistinguishable from the world of the radius $\propto 1/R$ for any R, the symmetry manifests itself by $(R \pm \mathrm{const.}/R)$ terms of the mass squared operator. The correspondence with our model becomes apparent one assumes that R corresponds to the ratio $p(\tau)/p(\tau+h)$ in Eq.(20). However, we must also refer a reader to an apparently serious difference that in our model we do not consider for the moment the compact dimension. One can also find in the option price dynamics some real example of duality symmetry (José Santiago Fajardo 2006). Concretely put-call duality which means "A call to buy foreign with domestic is equal to a put to sell domestic for foreign." Also most questions will not spell out what is domestic or foreign but let you decide what is the underlying asset and which is the strike asset.

4.1. $\mathscr{T}_{\rm id}$ transform under the conditions of bid-ask spreads. Simply, the generalization can also be made with allowing for currency variables which appear as a consequence of the transaction costs (Wagner and Edwards 1993). The occurrence of ask-bid spread complicates the analysis in several ways. Instead of one price for each currency, the task requires the availability to two prices. The impact of ask-bid spread on the time-series properties has been studied within the elementary model (Roll 1984).

Thus, for the purpose of a thorough and more realistic analysis of the market information, it seems straightforward to introduce generalized transform

$$\hat{\mathscr{T}}_{\rm id}^{\rm ab} P\{\, p_{\rm ask}(.), p_{\rm bid}(.)\,\} = \overline{P}\{\, 1/p_{\rm bid}(.), 1/p_{\rm ask}(.)\,\}\,, \tag{22}$$

which converts to Eq.(17) in the limit of vanishing spread.

5. Mapping to the model of 2D Brane

Clearly, there is a possibility to go beyond a string model towards more complex maps including alternative spread-adjusted currency returns. This is extension of string theory into the higher dimensions from the string lines into the membranes called D-Branes (Johnson 2006). Formally, the generalized mapping onto the 2D brane with the $(h_1, h_2) \in < 0, l_{\rm s} > \times < 0, l_{\rm s} >$ coordinates which vary along two extra dimensions could be proposed in the following form:

$$\begin{aligned} P_{{\rm 2D},q}(\tau, h_1, h_2) \;=\; & f_q \left(\left(\frac{p_{\rm ask}(\tau + h_1) - p_{\rm ask}(\tau)}{p_{\rm ask}(\tau + h_1)} \right) \left(\frac{p_{\rm ask}(\tau + l_{\rm s}) - p_{\rm ask}(\tau + h_1)}{p_{\rm ask}(\tau + l_{\rm s})} \right) \right. \\ \times \; & \left. \left(\frac{p_{\rm bid}(\tau) - p_{\rm bid}(\tau + h_2)}{p_{\rm bid}(\tau)} \right) \left(\frac{p_{\rm bid}(\tau + h_2) - p_{\rm bid}(\tau + l_{\rm s})}{p_{\rm bid}(\tau + h_2)} \right) \right) . \end{aligned} \tag{23}$$

The map constituted by the combination of "bid" and "ask" quotes is constructed to satisfy the Dirichlet boundary conditions

$$P_{{\rm 2D},q}(\tau, h_1, 0) = P_{{\rm 2D},q}(\tau, h_1, l_{\rm s}) = P_{{\rm 2D},q}(\tau, 0, h_2) = P_{{\rm 2D},q}(\tau, l_{\rm s}, h_2)\,. \tag{24}$$

In addition, the above construction, Eq.(23), has been chosen as an explicit example, where the action of $\hat{\mathscr{T}}_{\rm id}^{\rm ab}$ becomes equivalent to the permutation of coordinates

$$\hat{\mathscr{T}}_{\rm id}^{\rm ab} P_{{\rm 2D},q}(\tau, h_1, h_2) = P_{{\rm 2D},q}(\tau, h_2, h_1)\,. \tag{25}$$

Thus, the symmetry with respect to interchange of extra dimensions h_1, h_2 can be achieved through $P_{{\rm 2D},q} + \hat{\mathscr{T}}_{\rm id}^{\rm ab} P_{{\rm 2D},q}$. In a straightforward analogous manner one can get an antisymmetric combination $P_{{\rm 2D},q} - \hat{\mathscr{T}}_{\rm id}^{\rm ab} P_{{\rm 2D},q}$. For a certain instant of time we proposed illustration which is depicted in Fig.(1)(b).

At the end of this subsection, we consider the next even simple example, where mixed boundary conditions take place. Now let the 2-end-point string be allowed to pass to the 1-end-point string by means of the homotopy $P^{(1,2)}_{q_1,q_2}(\tau, h, \eta) = (1-\eta) P^{(1)}_{q_1}(\tau, h) + \eta P^{(2)}_{q_2}(\tau, h)$

driven by the parameter η which varies from 0 to 1. In fact, this model can be seen as a variant of the 2D brane with extra dimensions h and η.

5.1. Partial compactification. In the frame of the string theory, the compactification attempts to ensure compatibility of the universe based on the four observable dimensions with twenty-six dimensions found in the theoretical model systems. From the standpoint of the problems considered here, the compactification may be viewed as an act of the information reduction of the original signal data, which makes the transformed signal periodic. Of course, it is not very favorable to close strings by the complete periodization of real input signals. Partial closure would be more interesting. This uses pre-mapping

$$\tilde{p}(\tau) = \frac{1}{N_{\mathrm{m}}} \sum_{m=0}^{N_{\mathrm{m}}-1} p(\tau + l_{\mathrm{s}} m) \,, \tag{26}$$

where the input of any open string (see e.g. Eq.(3), Eq.(7)) is made up partially compact.

Thus, data from the interval $< \tau, \tau + l_{\mathrm{s}}(N_{\mathrm{m}} - 1) >$ are being pressed to occupy "little space" $h \in < 0, l_{\mathrm{s}} >$. We see that as N_{m} increases, the deviations of $\tilde{p}$ from the periodic signal become less pronounced. The idea is illustrated in Fig.(1)(c),(d). We see that the states are losing their original form (a),(b) are starting to create ripples.

For example, one might consider the construction of the $(\tilde{D}+1)$-brane

$$f_q \left(\frac{p(\tau+h_0) - p(\tau)}{p(\tau+h_0)} \right) \prod_{j=1}^{\tilde{D}} f_q \left(\frac{\tilde{p}_j^{(\pm)}(\tau+h_j) - \tilde{p}_j^{(\pm)}(\tau)}{\tilde{p}_j^{(\pm)}(\tau+h_j)} \right) \tag{27}$$

maintained by combining $(\tilde{D}+1)$ 1-end-point strings, where partial compactification in $\tilde{D}$ extra dimensions is supposed. Of course, the construction introduces auxiliary variables $\tilde{p}_j^{(\pm)}(\tau) = \sum_{m=0}^{N_{\mathrm{m}},j-1} p(\tau \pm m\, l_{\mathrm{s},j})$.

6. Inter-currency study: map onto rotating strings

The incorporating of the mutual relations between the pairs into the mapping procedure represents a very challenging task. Let us study trading activity in the (I,J) plane, where I,J stands for indices of the currency pair described by two 2-end-point strings. The real time data are used instead of tick by tick (see Sec. **2**) in order to maintain the consistency of prices quoted.

As some examples of the generalized strings distance concept we can introduce the inter-currency momentum distance function

$$d_{q,I,J}(t) \quad = \quad \frac{1}{l_{\mathrm{s}}+1} \sum_{h=0}^{l_{\mathrm{s}}} \left| P_{q,I}^{(2)}(t,h) - P_{q,J}^{(2)}(t,h) \right| . \tag{28}$$

At higher dimension, it is tempting to deal with angular momentum (see Zwiebach (2009))

$$M_{q,I,J}(t) = \sum_{h=0}^{l_{\mathrm{s}}} \left[P_{q,I}^{(2)}(t,h) X_{q,J}^{(2)}(t,h) - P_{q,J}^{(2)}(t,h) X_{q,I}^{(2)}(t,h) \right] . \tag{29}$$

The momentum calculation can be interpreted as a measure of the rotational information flows between the currency pairs. In Fig.(3) we present the results achieved for two alternatives $(I,J) = (\mathrm{EUR/USD}, \mathrm{GBP/USD})$, and $(I,J) = (\mathrm{GBP/USD}, \mathrm{USD/JPY})$. We see that calculation of $D_{q,I,J}$ and $M_{q,I,J}$ yields oscillatory (i.e. seasonal) behavior. Simultaneously, the concept of distance and moment has been extended to analyze the impact of spread. Analogously, as in the previous cases, the distance between the ask and bid strings may be defined

$$\begin{aligned} d^{\mathrm{ab}}_{q,I}(t) &= \frac{1}{l_{\mathrm{s}}+1}\sum_{h=0}^{l}\left|P^{(2)}_{q,\mathrm{ask}}(t,h)-P^{(2)}_{q,\mathrm{bid}}(t,h)\right|, && (30)\\ M^{\mathrm{ab}}_{q,I}(t) &= \sum_{h=0}^{l_{\mathrm{s}}}\left[P^{(2)}_{q,\mathrm{ask}}(t,h)X^{(2)}_{q,\mathrm{bid}}(t,h)-P^{(2)}_{q,\mathrm{bid}}(t,h)X^{(2)}_{q,\mathrm{ask}}(t,h)\right]. && (31)\end{aligned}$$

Here

$$P^{(2)}_{q,\mathrm{ask}} \equiv P^{(2)}_{q}|_{p\to p_{\mathrm{ask}}}\,, \quad P^{(2)}_{q,\mathrm{bid}} \equiv P^{(2)}_{q}|_{p\to p_{\mathrm{bid}}} \qquad (32)$$

are obtained by substituting expressions above in Eq.(7). With the help of Eq.(9) and $P^{(2)}_{q,\mathrm{ask}}$, $P^{(2)}_{q,\mathrm{bid}}$ we construct iteratively $X^{(2)}_{q,\mathrm{ask}}$ and $X^{(2)}_{q,\mathrm{bid}}$. As one can see from Fig.(3), the differences measured in terms of $M_{q,I,J}(t)$ are very subtle. There is evidence of intercoupling of the spread and currency dynamics. The fundamental role in the string theory is played by the *Regge slope parameter* α' (or inverse string tension (Zwiebach 2009)). This has a proper analogy with our approach where we introduced a slope in terms of the angular momentum

$$\alpha'_{q,I,J} = \frac{<|M_{q,I,J}|>}{2\pi\, l_{\mathrm{s}}^2}\,. \qquad (33)$$

For the $l_{\mathrm{s}} = 1$hour string pair we obtained $\alpha'_{6,\mathrm{EUR/USD,\ GBP/USD}} = 5.07.10^{-54}\ (2\pi)^{-1}\ \mathrm{hour}^{-2}$, $\alpha'_{6,\mathrm{GBP/USD,\ USD/JPY}} = 1.55.10^{-53}\ (2\pi)^{-1}\ \mathrm{hour}^{-2}$, $\alpha'_{6,\mathrm{USD/JPY,\ EUR/USD}} = 1.34.10^{-53}\ (2\pi)^{-1}\ \mathrm{hour}^{-2}$ much larger than the spread $\alpha'^{\mathrm{ab}}_{6,\mathrm{GBP/USD}} = 1.16\cdot10^{-55}\,(2\pi)^{-1}\mathrm{hour}^{-2}$. However, it is worth noting that relation, Eq.(33), should be understood as an estimate since there is no statistical mean of the type $<|M_{\ldots}|>$ in the original specification. The problem of estimation of the *slope parameter* arises from the fact that in the original model nonaveraged angular momentum is divided by the square of the mass instead of l_{s}^2. Herein, we have no idea how to measure the mass of the string, or how to verify the fact that the string mass is proportional to l_{s}.

7. Differentials of string map

Gâteaux derivative (Gamelin and Greene 1999) is a generalization of the concept of a directional derivative in the differential calculus. In our study the concept can be viewed as a systematic way in the generation of more structured maps expressing more information about the structure of data we deal with. Given the string map $P(.)$, the m-th *Gâteaux derivative* of $P(.)$ in the "direction" of $\psi(.)$ (unspecified yet series) is defined as follows:

$$\mathrm{d}^m P(\{p\};\{\psi\})(\tau,h) = \frac{\mathrm{d}^m}{\mathrm{d}\varepsilon^m}P(\{p(\tau,h)+\varepsilon\psi(\tau,h)\})\Big|_{\varepsilon\to0}\,. \qquad (34)$$

For $q=1$ the calculation gives

$$\mathrm{d}P_1^{(1)}(\{p\};\{\psi\})(\tau,h) = \frac{1}{p(\tau+h)}\left[\frac{p(\tau)\psi(\tau+h)}{p(\tau+h)}-\psi(t)\right] \qquad (35)$$

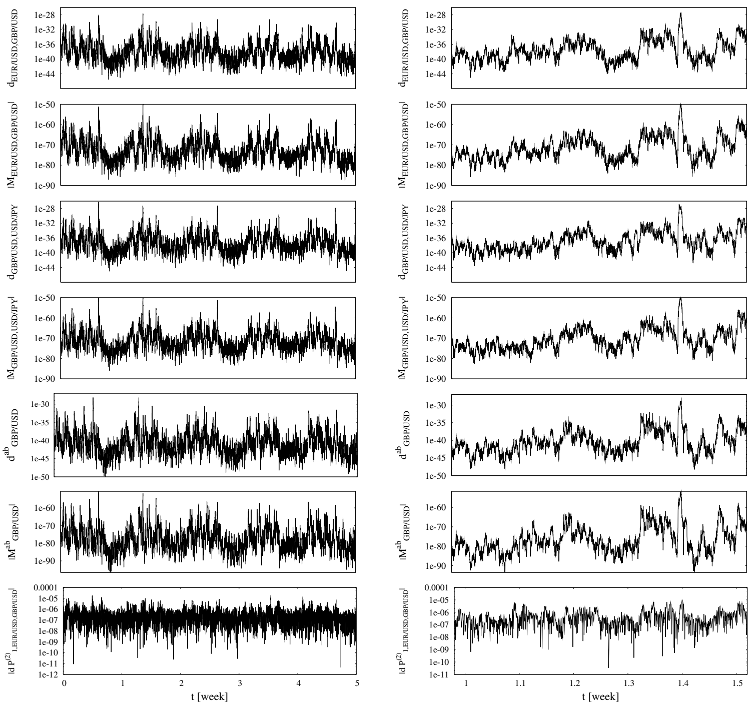

FIGURE 3. The inter-string distance and angular momentum (see Eqs.(28) and (29)) for specified currency pairs. The long term outlook compared with a detailed one-week view supplemented by the results obtained for spread according to Eq.(30) and Eq.(31). The results are compared with the G$\hat{a}$teaux derivative (see Eq.(36), where p corresponds to EUR/USD and ψ to GBP/USD pair), and the derivative is evaluated at $h = l_{\rm s}/2$. The calculation is carried for the string of the 1-hour time length and investigation of the Aug-Sep period.

and

$$
\begin{aligned}
\mathrm{d}P_1^{(2)}(\{p\};\{\psi\})(\tau,h) &= \psi(\tau)\left(\frac{1}{p(\tau+l_{\rm s})} - \frac{1}{p(\tau+h)}\right) \\
+ \quad \psi(\tau+h)&\left(\frac{p(\tau)}{p^2(\tau+h)} - \frac{1}{p(\tau+l_{\rm s})}\right) + \frac{\psi(\tau+l_{\rm s})}{p^2(\tau+l_{\rm s})}\left(p(\tau+h)-p(\tau)\right).
\end{aligned}
\tag{36}
$$

By going to the second order we obtained

$$
\mathrm{d}^2P_1^{(2)}(\{p\};\{\psi\})(\tau,h) = \frac{2\psi(\tau+h)}{p^2(\tau+h)}\left[\psi(\tau) - \frac{p(\tau)\psi(\tau+h)}{p(\tau+h)}\right] \tag{37}
$$

and

$$\mathrm{d}^2 P_1^{(2)}(\{p\};\{\psi\})(\tau,h) = \frac{2\psi(\tau+l_{\mathrm{s}})}{p^2(\tau+l_{\mathrm{s}})}\left[\psi(\tau+h)-\psi(\tau)\right] \tag{38}$$
$$+ \quad \frac{2\psi(\tau+h)}{p^2(\tau+h)}\left[\psi(\tau)-\frac{p(\tau)\psi(\tau+h)}{p(\tau+h)}\right]+\frac{2\psi^2(\tau+l_{\mathrm{s}})}{p^3(\tau+l_{\mathrm{s}})}\left[p(\tau)-p(\tau+h)\right].$$

Surprisingly, the generalized differentiation generates maps which satisfy the Dirichlet boundary conditions

$$\mathrm{d}^m P_1^{(1)}(\{p\};\{\psi\})(\tau,0) = 0\,, \qquad m=1,2; \tag{39}$$
$$\mathrm{d}^m P_1^{(2)}(\{p\};\{\psi\})(\tau,0) = \mathrm{d}^m P_1^{(2)}(\{p\};\{\psi\})(\tau,l_{\mathrm{s}}) = 0\,. \tag{40}$$

Many alternative ways exist to exploit the models with the auxiliary field $\psi(.)$. The field can be related to, e.g., (i) models which place emphasis on the currency margins determined by some adaption process; (ii) on the spread with $\psi(\tau)=p_{\mathrm{bid}}(\tau)-p_{\mathrm{ask}}(\tau-l_{\mathrm{s}})$. (iii) The benchmark setting represents $\psi=1$, (iv) the periodic function $\psi(\tau)$ can model the action of the compact. In this intuition supporting case, one can see that the generalized derivative modifies the original map as follows:

$$\mathrm{d}P_1^{(1)}(\{p\};\{\psi\})(\tau,h)|_{\psi=1} \quad = \quad -\frac{P_1^{(1)}(\tau,h)}{p(\tau+h)}\,, \tag{41}$$
$$\mathrm{d}P_1^{(2)}(\{p\};\{\psi\})(\tau,h)|_{\psi=1} \quad = \quad \left(\frac{1}{p(\tau+h)}+\frac{1}{p(\tau+l_{\mathrm{s}})}\right)P_1^{(2)}(\tau,h)\,. \tag{42}$$

In Fig.3, we present the idea, where p, ψ are represented by two currencies, their mutual influence is studied within the first-order differential model described by Eq.(36).

8. Conclusions

We shown that the string theory may motivate the adoption of the nonlinear techniques of the data analysis with a minimum impact of justification parameters. The numerical study recovered interesting fundamental statistical properties of the maps from the data onto string-like objects. The remarkable deviations from the features known under the notion of the efficiently organized market have been observed, namely, for high values of the deformation parameter q.

The main point here is that the string map gives a geometric interpretation of the information value of the data. The model of the string allows one to manipulate with the information stored along several extra dimensions. We started from the theory of the 1-end-point and 2-end-point string, where we distinguished between the symmetric and antisymmetric variants of the maps. In this context, it should be emphasized that duality is a peculiar property of the suggested maps, not data alone. On the contrary with call-put duality in option pricing where could be some relation with string duality.

We presented here physics and geometry motivated methods of analysis of the coupling between the currency pairs. The results led us to believe that our ideas and methodology can contribute to the solution of the problem of the robust portfolio selection. As we have seen, the complex multi-string structures produced by the generalized derivatives of strings cannot be easily grasped by the intuitive principles. We believe, the presented method affords potential to be used in the practical applications.

References

Eryiğit, M. and Eryiğit, R. (2009). "Network structure of cross-correlations among the world market indices". *Physica A: Statistical Mechanics and its Applications* **388**(17), 3551–3562. DOI: 10.1016/j.physa.2009.04.028.

Gamelin, T. W. and Greene, R. E. (1999). *Introduction to topology*. 2nd ed. Mineola, N.Y: Dover Publications.

Grassberger, P. and Procaccia, I. (1983). "Measuring the strangeness of strange attractors". *Physica D: Nonlinear Phenomena* **9**(1–2), 189–208. DOI: 10.1016/0167-2789(83)90298-1.

Green, M. B. (1988). *Superstring Theory: Volume 1, Introduction*. Cambridge University Press.

He, J. and Deem, M. W. (2010). "Structure and Response in the World Trade Network". *Phys. Rev. Lett.* **105**(19), 198701. DOI: 10.1103/PhysRevLett.105.198701.

Johnson, C. V. (2006). *D-Branes*. Cambridge University Press.

José Santiago Fajardo, M. E. (2006). "Symmetry and duality in Levy markets". *Quantitative Finance* **6**(3), 219–227.

Jung, W.-S., Kwon, O., Wang, F., Kaizoji, T., Moon, H.-T., and Stanley, H. E. (2008). "Group dynamics of the Japanese market". *Physica A: Statistical Mechanics and its Applications* **387**(2–3), 537–542. DOI: 10.1016/j.physa.2007.09.022.

Krishnaiah, P. R. and Kanal, L. N. (1982). *Classification, pattern recognition, and reduction of dimensionality*. Handbook of statistics v. 2. Amsterdam ; New York: North-Holland Pub. Co. ; Sole distributors for the U.S.A. and Canada, Elsevier Science Pub. Co.

McMahon, D. (2008). *String Theory Demystified*. McGraw Hill Professional.

Polchinski, J. (1998). *String Theory*. 1st ed. Cambridge University Press.

Roll, R. (1984). "A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market". *The Journal of Finance* **39**(4), 1127–1139. DOI: 10.1111/j.1540-6261.1984.tb03897.x.

Wagner, W. H. and Edwards, M. (1993). "Best Execution". *Financial Analysts Journal* **49**(1), 65–71. DOI: 10.2469/faj.v49.n1.65.

Yongwei, D., Xiaohu, Y., Kavs J., A., and Juefeng, L. (2010). "Financial data representation and similarity model". *International Journal of Trade, Economics and Finance* **1**(4), 320–324.

Zwiebach, B. (2009). *A First Course in String Theory*. 2nd. Cambridge University Press.

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