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DOMINANT TYPE OF DETERMINISTIC POLARIZATION TRANSFORMATION FOR INHOMOGENEOUS ELLIPTIC BIREFRINGENT MEDIUM

S. SAVENKOV,[a*] YE. OBEREMOK,[a] I. KOLOMIETS,[a]
V. YAKUBCHAK,[a] AND R. MUTTIAH [b]

ABSTRACT. The features of spectral properties of the Jones matrix model of the dominant deterministic polarization transformation for inhomogeneous elliptic birefringent medium have been studied.

1. Introduction

The Jones matrices of the dominant type of deterministic polarization transformation for inhomogeneous linear and elliptic birefringent media have been obtained in [1, 2]. In particular, we showed that for a given direction of observation, an initially lossless medium exhibits strong effective dichroism: an initial linear birefringent crystal displays linear dichroism, and circular birefringence displays circular dichroism. It is important to note that if for inhomogeneous linear birefringent medium the values of azimuths of effective linear dichroism θ and initial linear birefringence α are the same and in the case of weakly depolarization (i.e., when entropy $H < 0.5$) do not depend on the value of inhomogeneity, then it is not true for inhomogeneous elliptic birefringent medium, see Figs. 1 and 2.

The aim of this paper is to study the features of spectral properties of the Jones matrix model of the dominant deterministic polarization transformation for inhomogeneous elliptic birefringent medium.

2. Model of coaxial orientations of linear anisotropic mechanisms

Note that from Figs. 1 and 2 it can be directly verified that for both inhomogeneous linear and elliptical birefringent media the dominant Jones matrices possess orthogonal eigenpolarizations.

For further analysis we use the matrix model of arbitrary homogeneous anisotropic medium based on generalized equivalence theorem [3], which in terms of Jones matrices

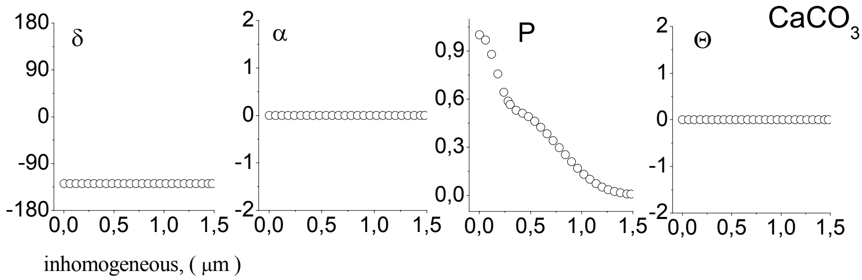

Figure 1. Dominant type of deterministic polarization transformation for inhomogeneous linear birefringent medium [1].

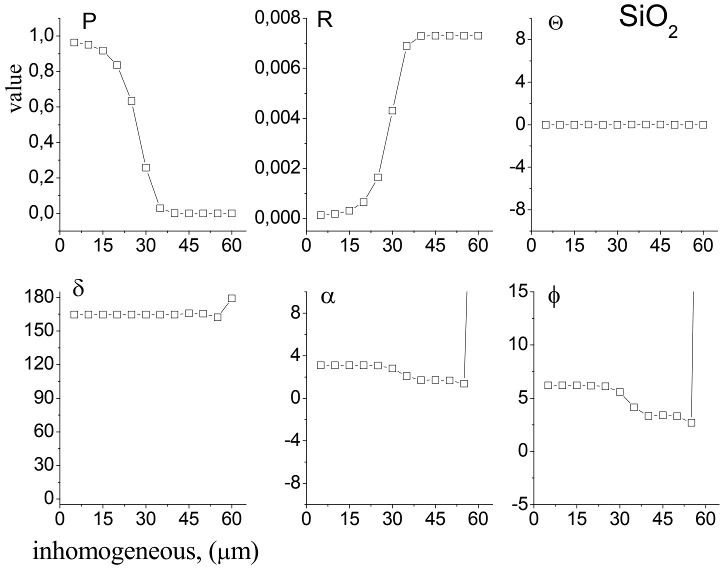

Figure 2. Dominant type of deterministic polarization transformation for inhomogeneous elliptic birefringent medium [2].

is:

$$J = J^{CP}\left(\varphi\right)\ J^{\mathrm{LP}}\left(\delta{,}\alpha\right)\ J^{\mathrm{LA}}\left(\mathrm{P}{,}\theta\right)\ J^{CA}\left(\mathrm{R}\right) \tag{1}$$

Firstly we determine the conditions for eigenpolarizations of arbitrary Jones matrix [3] to be orthogonal in the case when azimuths of linear dichroism θ and linear birefringence α are coincide. For the sake of definiteness we can evidently set $\alpha = \theta = 0$. This can always be done by corresponding choice of laboratory coordinate system and is illustrated in Fig.1. Then the Jones matrix Eq.(1) for this case takes the form:

$$J|_{\substack{\alpha = 0^0 \\ \theta = 0^0}} = \begin{pmatrix} \cos\left(\varphi\right)\left(\varphi\right) + ie^{-i\delta}PR\sin(\varphi) & e^{-i\delta}P\sin(\varphi) - iR\cos(\varphi) \\ -\sin(\varphi) + ie^{-i\delta}PR\cos(\varphi) & e^{-i\delta}P\cos(\varphi) + iR\sin(\varphi) \end{pmatrix}. \tag{2}$$

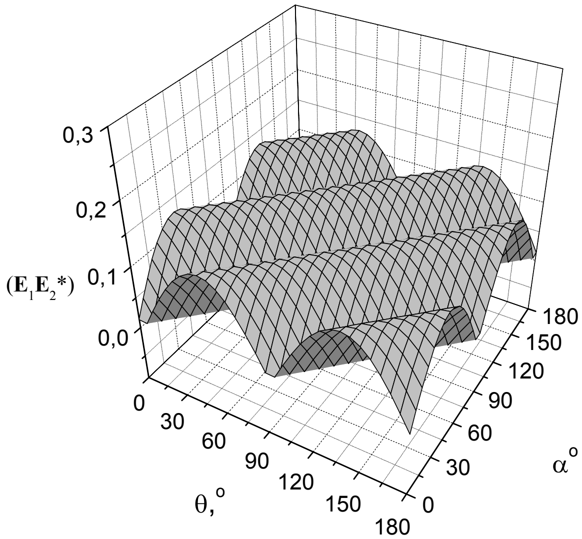

Figure 3. The dependence of the product of eigenpolarizations $E_1E_2^*$ on values of azimuths of linear dichroism θ and linear birefringence α.

In general case the conditions for eigenpolarizations of arbitrary deterministic medium to be orthogonal have been derived in [4]:

$$\begin{cases} (1-P)\left(\left(1+R^2\right)\cos 2\left(\alpha-\theta-\phi\right)-\left(1-R^2\right)\cos 2\left(\alpha-\theta\right)\right)=0 \\ R\tan\left(\delta/2\right)=\frac{1}{2}\frac{1-P}{1+P}\left(\left(1-R^2\right)\sin 2\left(\alpha-\theta\right)-\left(1+R^2\right)\sin 2\left(\alpha-\theta-\phi\right)\right) \end{cases} \tag{3}$$

For Jones matrix Eq.(2) condition Eq.(3) takes the form

$$\begin{cases} (1-P)\left(R^2-1+\left(1+R^2\right)\cos 2\varphi\right)=0 \\ -\frac{(P-1)\left(1+R^2\right)\cos\varphi\sin\varphi}{1+P}-R\tan\frac{\delta}{2}=0 \end{cases} \tag{4}$$

From Eq.(4), the Jones matrix Eq.(2) is characterized by orthogonal eigenpolarizations in the following cases:

(**i**) four components in Eq.(1) conditions

$$\left[\begin{array}{l} \delta=\pm\arccos\left(\frac{|P-1|}{1+P}\right) \\ \begin{cases} \varphi=\pm\arccos\left(-\frac{1}{\sqrt{1+R^2}}\right) \\ \varphi=\pm\arccos\left(\frac{1}{\sqrt{1+R^2}}\right) \end{cases} \end{array}\right., \tag{5}$$

(**ii**) two components in Eq.(1) conditions

$$\begin{cases} P=1,\quad \delta=\pm 2\pi n,\quad R=\forall \\ R=0,\quad \phi=0,\quad P=\forall \end{cases}, \tag{6}$$

Note that there no exist three components conditions at all. Conditions Eq.(6) relate to the cases of initially circularly and linear birefringent media respectively. It can be seen that results presented in Fig.2 for elliptical case do not satisfy the derived four components conditions Eq.(5). Thus, given condition $\alpha = \theta = 0$, in the case of presence of all types of anisotropy, which is observed for inhomogeneous elliptical birefringent medium (SiO_2), eigenpolarizations can not be orthogonal.

3. Conclusions

In this paper, for the case when azimuths of linear dichroism θ and linear birefringence α are coincide, the conditions for eigenpolarizations of the arbitrary Jones matrix defined by Eq.(1) to be orthogonal have been derived. We show that the dominant type of deterministic polarization transformation for inhomogeneous elliptic birefringent media (SiO_2) do not satisfy the condition Eq.(5), i.e., in this case azimuths of linear dichroism θ and linear birefringence α can not coincide.

Indeed, using arbitrary Jones matrix Eq.(1) consider the dependence of the product of eigenpolarizations $E_1 E_2^*$ on values of azimuths of linear dichroism θ and linear birefringence α. Results for the case of inhomogeneity $25\,\mu m$ (see Fig.2) are presented in Fig.3. The solution for quartz is a central line $E_1 E_2^* = 0$ corresponding to rotation of laboratory coordinate system.

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[a] Kiev Taras Shevchenko University
Radiopysics Department
Vladimirskaya str. 64
01 033 Kiev, Ukraine

[b] University of Texas
Dept. Civil Engineering
Arlington, Texas, U.S.A.

* To whom correspondence should be addressed | Email: sns@univ.kiev.ua

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