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THE INTERTEMPORAL CHOICE BEHAVIOUR: CLASSICAL AND ALTERNATIVE DELAY DISCOUNTING MODELS AND CONTROL TECHNIQUES

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ABSTRACT. Hyperbolic discounting refers to the tendency for people to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner rather than later in time. When offered a larger reward in exchange for waiting a set amount of time, people act less impulsively (i.e., choose to wait) as the rewards happen further in the future. Put another way, people avoid waiting more as the wait nears the present time.

1. Introduction

The Discounting Utility Model introduced by Samuelson (1937) has dominated the economic analysis of Intertemporal Choice for a long time, assuming an exponential delay discounting function, with a constant discount rate that implies dynamic consistency and stationary intertemporal preferences. Nevertheless the model fails in being both normative and descriptive, as shown by several studies especially carried out in the fields of psychology and neuroeconomics, that revealed the existence of relevant anomalies, that violate the traditional model axioms, such as the delay effect, magnitude effect, sign effect and sequence effect, we will deal with in the following.

As empirically shown, people discount delayed rewards applying a hyperbolic delay discounting (declining as the length of the delay increases), so, they have the tendency to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time. This entails intertemporal inconsistency and preferences reversal, as formulated by Mazur (1987), Laibson (1996;1997) and Sozou (1998). Even so, an impatient behavior not necessarily is incoherent. Hyperbolic discounting, as well as exponential discounting, can formalize consistent preferences under restrictive assumptions.

Moreover, hyperbolic discounting allows us to explain several inconsistent behaviors usually manifested as temporary preference for options that are extremely costly or harmful in the long run, as typically seen in alcoholism, drug abuse, gambling, but also in overeating, money mismanagement and other bad habits too popular to be diagnosed as pathological. These behaviors are characterized by decreasing impatience and myopia. Impulsivity is justified in the Theory of Rational Addiction (Becker, Grossman and Kevin

M. Murphy, 1988) as an economically rational behavior, even if clinically problematic, for people who have large discount rates but with an exponential discount function.

Takahashi (2007) attempts to dissociate impulsivity and inconsistency in their econophysical studies proposing the Q-exponential Delay Discounting Function, and he also proposes the Exponential Discounting with Logarithmic time perception, where he asserts that the dynamic inconsistency is due to a distortion in time perception, often found in substance abusers and psychopathological subjects. Other behavioral economists propose Multiple Selves Models attempting to measure the strength of the internal conflict within the decision maker, best known as Quasi-hyperbolic discounting model first introduced by Laibson (1997). In many cases a dynamic inconsistent behavior is attributed to the existence of contingent temptations that increase impulsivity and induce a deviation from the desirable behavior. To fight impulsivity, Strotz (1956) proposes two strategies that might be employed by a person who foresees how her preferences will change over time: the "strategy of precommitment" and the "strategy of consistent planning".

Finally, Thaler and Shefrin (1981) model incoherent purpose by treating an individual as if he contained two distinct psyches denoted as planner and doer, and they suggest that the conflict arisen within the decision maker can be solved through the Theory of self-control. They maintain that this model can be compared with the principal-agent problem present in any organization, so the individual may adopt many of the same strategies to solve self-control problems in intertemporal choice.

2. The intertemporal choice: Rationality and time consistency

The standard economic model of discounted utility (DU model), introduced in (Samuelson, 1937), supposes that the value of a future reward is discounted because of the risk that waiting for its reception implies. Given a contingent relationship between the choice of a reward and its eventual reception, it is supposed that a constant hazard rate exists in this relation, then the temporal discounting function will be exponential. A discount function assumes the form

$$D(k) = \prod_{n=0}^{k-1} \left(\frac{1}{1+p_n} \right)$$

where p_n represents the per-period discount rate for period n - that is the discount rate applied between periods n and $n+1$. If the discount function is

$$D(k) = \left(\frac{1}{1+p} \right)^k$$

the DU model assumes a constant per-period discount rate ($p_n = p$, for every n). It means that delaying or accelerating two dated outcomes by a common amount, preferences between the outcomes should not change - if in period t a person prefers X at τ to Y at $\tau + d$ for some z, then in period t she must prefer X at τ to Y at $\tau + d$, for all z. The assumption of constant discounting permits a person's time preference to be summarized as a single discount rate and it implies that intertemporal preferences are time-consistent, i.e., later preferences "confirm" earlier preferences.

Diminishing marginal utility and positive time preference. Consider now a concave instantaneous utility function $u(c_t)$ and a positive discount rate p. These two assumptions create opposite forces in intertemporal choice: diminishing marginal utility motivates a person to spread consumption over time, while positive time preference motivates a person to concentrate consumption in the present. Since people do, in fact, spread consumption over time, the assumption of diminishing marginal seems strongly justified. The assumption of positive time preference, on the other hand, is more questionable. But some Authors (Koopmans, 1967; Koopmans, Diamond and Williamson, 1964) have argued that a zero or negative time preference, combined with a positive real rate of return on saving, would command the infinite deferral of all consumption, which means, unrealistically, that individuals have infinite life-spans and linear (or weakly concave) utility functions.

> We care less about our further future... because we know that less of what we are now – less, say, of our present hopes or plans, loves or ideals – will survive into the further future... [if] what matters holds to a lesser degree, it cannot be irrational to care less (Derek, 1971).

Integration of new alternatives with existing plans. A person evaluates new alternatives by integrating them with her existing plans. Consider a person with an existing consumption plan $(ct, \ldots, cT)$ who is offered an intertemporal choice prospect X. The person must choose what her new consumption path $(c't, \ldots, c'T)$ would be if she were to accept prospect X, and should accept the prospect if

$$U^t(c'_t, \ldots, c'_T) > U^t(c_t, \ldots, c_T)$$

If the person's initial endowment is E_0, then accepting prospect X would change her endowment to $E_0 \cup X$. Letting $B(E)$ denote the person's budget set given endowment E, the DU model says that the person should accept prospect X if:

$$\max_{(c_t,\ldots,c_T)\in B(E_0\cup X)} \sum_{\tau=t}^{T} \left(\frac{1}{1+p}\right)^{\tau-t} u(c_\tau) > \max_{(c_t,\ldots,c_T)\in B(E_0)} \sum_{\tau=t}^{T} \left(\frac{1}{1+p}\right)^{\tau-t} u(c_\tau)$$

A person may not have well-formed plans about future consumption streams, or be unable (or unwilling) to recompute the new optimal plan every time she makes an intertemporal choice (Frederick, Loewenstein and O'Donoghue, 2002).

Utility independence. The overall value, or "global utility" of a sequence of outcomes is equal to the (discounted) sum of the utilities in each period. Hence, the distribution of utility across time makes no difference beyond that dictated by discounting, which (assuming positive time preference) penalizes utility experienced later.

Consumption independence. It can be assumed that the utility of an outcome utility is unaffected by outcomes experienced in prior or expected in future periods. But, as Samuelson and Koopmans both recognized, there is no compelling rationale for such an assumption; Samuelson (1952, p. 674) noted that, *"the amount of wine drank yesterday and will drink tomorrow can be expected to have effects upon my today's indifference slope between wine and milk."*

3. Behavioral finance: empirical anomalies violating the traditional discounting model

Several empirical studies, mainly arisen from the field of psychology, have described the individual behavior when discounting real or hypothetical rewards, showing the existence of "anomalies" or violations of the traditional discounting models (DU and EU) axioms (Cruz Rambaud and Muñoz Torrecillas, 2004). First, empirically observed discount rates are not constant over time, but appear to decline – a pattern often referred to as hyperbolic discounting. Furthermore, even for a given delay, discount rates vary across different types of intertemporal choices: gains are discounted more than losses, small amounts more than large amounts, and explicit sequences of multiple outcomes are discounted differently than outcomes considered singly. Delay effect, magnitude effect, sign effect and sequence effect are among the relevant anomalies in intertemporal choice, we will deal with.

The delay effect. As waiting time increases, the discount rates tend to be higher in short intervals than in longer ones. This effect has been shown for both monetary and non-monetary decisions (Chapman, 2000; Thaler, 1981). Delay effect can derive in preference reversals (Kirby and Herrnstein, 1995), whose modelling can be obtained with a hyperbolic discount function better than the exponential one specified by the normative theory (Kirby and Marakovic, 1995). In fact, unlike exponential discount functions, hyperbolic functions can intersect for higher and lower rewards, indicating preference reversals.

Prelec and Loewenstein (1991) propose the property of decreasing absolute sensibility: for instance, the difference between years 0 and 2 seems greater than the difference between years 6 and 8. They call this anomaly common difference effect and immediacy effect. We can set out this effect as follows:

$$(x, s) \sim (y, t) \text{ but } (x, s + h) < (y, t + h), \text{for } y > x, s < t \text{ and } h > 0$$

If two capitals (x, s) and (y, t), are indifferent, $(x, s) \sim (y, t)$, their projections onto a common instant p (usually, p is taken as 0) have to coincide:

$$xA(s, p) = yA(t, p) \text{ if and only if } \frac{x}{y} = \frac{A(t, p)}{A(s, p)} = v(s, t, p).$$

being $A(t, p)$ the discount function which represents the amount available at p instead of one euro available at t, and $v(s, t, p)$ the corresponding financial factor. In the same way, if $(x, s + h)$ $(y, t + h)$, this implies that

$$xA(s + h, p) < yA(t + h, p) \text{ if and only if } \frac{x}{y} < \frac{A(t + h, p)}{A(s + h, p)} = v(s + h, t + h, p).$$

Then:

$$v(s, t, p) < v(s + h, t + h, p)$$

DEFINITION 1. The financial factor associated to a discount function A(t, p) is said to be increasing (resp. decreasing) if

$$v(s, t, p) \leq v(s + h, t + h, p), \ h > 0$$

$$(v(s, t, p) \geq v(s + h, t + h, p), \ h > 0)$$

A necessary and sufficient condition for a financial factor to be increasing is that the discount rate will be decreasing. The immediacy effect means that decision makers give special importance to the immediate results, that is to say: (x, s) (y, t) implies $(x, s + h)(y, t + h)$, for $t = 0$ and $y > x$, $h > 0$. This can be seen in the extremely high discount rates estimated for short delays in several empirical studies about discounting (Thaler, 1981; Benzion, Rapaport and Yagil, 1989). If we consider the equivalent capitals (x, p) and (y, t), being $y > x$ and $p < t$, we can state the following relationship between the amounts with the discount function:

$$xA(p, p) = yA(t, p) \text{ if and only if } \frac{x}{y} = A(t, p)$$

since $A(p, p) = 1$.

Increasing the appraisal instant in a constant $h > 0$, there will be a more preferred capital, $(x, p + h)$ $(y, t + h)$, and the relationship between the amounts with the discount functions will be:

$$xA(p + h, p) < yA(t + h, p) \text{ if and only if } \frac{x}{y} < \frac{A(t + h, p)}{A(p + h, p)} = v(p + h, t + h, p)$$

and so, we can conclude that:

$$A(t, p) < v(t + h, p + h, p)$$

This means that the average discount rate spot in the interval $[p, t]$ is lower than the average discount rate forward at p for the interval $[p + h, t + h]$. We can also consider that the appraisal instant, p, is variable; then the discounting function will be contractive, implying a decreasing instantaneous discount rate in the direction of the vector $(1, 1)$.

DEFINITION 2. A discounting function $A(t, p)$ is said to be contractive (resp. expansive) if

$$A(t, p) \leq A(t + h, p + h) \ h > 0$$

$$(\text{resp. } A(t, p) \geq A(t + h, p + h) \ h > 0)$$

A sufficient condition for a discounting function to be a contractive (resp. expansive) discounting function is that the instantaneous discount rate is decreasing (resp. increasing) in the direction of the vector $(1, 1)$.

The magnitude effect. The subjective discount rates vary not only with the period until the reward is got, but also with the magnitude of the result or reward. Smaller rewards tend to result in higher discount rates. Let us suppose that the instantaneous discount rate is inversely proportional to the discounted amount:

$$\delta(z) = \frac{k}{c'} \text{ with } k = 100.$$

It follows that

$$A(c, z) = c \cdot e^{-\int_o^z \frac{k}{c} dx} = c \cdot e^{-\frac{k}{c} z}$$

Prelec and Loewenstein (1991) formulate the magnitude effect as follows:

$$(x, s) \sim (y, t) \text{ implies } (\alpha x, s) < (\alpha y, t), \text{ for } y > x > 0 \text{ and } s < t$$

and

$$(-x, s) \sim (-y, t) \text{ implies } (-\alpha x, s) > (-\alpha y, t)$$

In order to explain it they propose the property of proportional increasing sensibility: if we increase the absolute magnitude of all attribute values by a common multiplicative constant, the attribute weight will increase. Specifically, if:

$$(a_1, b_1) \sim (a_2, b_2) \text{ and } \alpha a_1 > 0, |\alpha| > 1$$

then

$$(\alpha a_1, b_1) < (\alpha a_2, b_2) \text{ if and only if } (\alpha a_1, b_2) < (\alpha a_2, b_1)$$

THEOREM 1. If the magnitude effect is verified, for all x, y, s and t, such that $x < y$ and $s < t$, if $(x, s) \sim_p (y, t)$, then $(\alpha x, s) >_p (\alpha y, t)$, for all α between 0 and 1.

THEOREM 2. A necessary condition for the magnitude effect is that the underlying discount function is subadditive with respect to the amount.

THEOREM 3. A necessary and sufficient condition for the magnitude effect is that for all x, y, s and t, such that $x < y$ and $s < t$, if $(x, s) \sim p(y, t)$, then the following relationship between the directional derivatives is verified

$$D_{(x,0,0)}A(c, s, p) < D_{(y,0,0)}A(c, t, p)$$

The sign effect. The discount rates for losses are lower than the discount rates for gains: this is what is meant as the sign effect also called gain-loss asymmetry. Many subjects showed negative discounting, since they preferred an immediate to a delayed loss of the same magnitude. Like the magnitude effect, the sign effect can be explained in terms of the value function for money.

Prelec and Loewenstein (1991) proposed the "amplification loss property" implying that, changing the sign of an amount from gains to losses, the weight of this amount increases; that is, the ratio of subjective values for losses is higher than the ratio of equivalent gains:

$$(x, s) \sim (y, t) \text{ implies } (-x, s) > (-y, t), \text{ for } y > x > 0 \text{ and } s < t.$$

The improving sequence effect. While for individual results it is shown a positive time preference, for sequences of results it is shown a negative time preference. In the short term, decision makers prefer increasing sequences of both money and health because they expect to improve their position over time and, hence, they show a negative time preference. However, for long (lifetime) sequences, they continue preferring increasing sequences of money (negative time preference), but they prefer decreasing sequences of health (positive time preference), since most people expect to experience health that decreases as they age and not the contrary. The subjects show a negative time preference (which implies a negative discount) that is justified for most of them by the pleasure of experiencing an increasing payment and consumption stream (Loewenstein and Prelec, 1991).

Preferences for sequences of outcomes depend on both the domain (health or money) and the length of the sequence (Chapman, 2000). These quantitative differences in the discount of different categories of goods constitute what has been called framing effect (Lázaro et al., 2001).

The improving sequence effect can be characterized as: for all s and t, and $s < t$, there is a c_0 such that, for all $y > x > c_0$, the following relation holds

$$\{(x, s), (y, t)\} >_p \{(y, s), (x, t)\}$$

For instance, let x and y be amounts, with $x < y$. If $A(t, p)$ is a discount function at p and $s < t$, then, being A strictly decreasing with respect to t, it must be verified that:

$$A(s, p) > A(t, p)$$

that implies:

$$A(s, p) - A(t, p) > 0.$$

As $x < y$, then

$$xA(s, p) + yA(t, p) < yA(s, p) + xA(t, p),$$

or, equivalently,

$$\{(x, s), (y, t)\} <_p \{(y, s), (x, t)\}$$

This means that, independently of the preference relation between the financial capitals (x, s) and (y, t) – even verifying that (x, s) is preferable at p to (y, t) – the previous presence of a higher amount capital, determines the preference direction. However, in case of preference for increasing sequences, we can check:

$$A(x, s, p) + A(y, t, p) > A(y, s, p) + A(x, t, p)$$

and, as $t - s > 0$

$$\frac{A(x, s, p) - A(y, t, p)}{t - s} > \frac{A(y, t, p)}{t - s}.$$

If $t - s$ tends to 0, then:

$$-\frac{\partial A(x, t, p)}{\partial t} > -\frac{\partial A(y, t, p)}{\partial t}$$

i. e., a necessary and sufficient condition for the sequence effect is that the instantaneous discount rate is decreasing with respect to the amount.

4. The hyperbolic discounting and the time preferences reversal

The hyperbolic discount model. Hyperbolic discounting is a DU anomaly that consists in the tendency of the individuals to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time. The individual avoids waiting more as the wait nears the present time and so, the rate at which he discounts future rewards declines as the length of the delay increases. There are several reasons why people might rationally choose a smaller reward now over a larger reward later: they may like the sure thing, their preferences could change, or they may have an urgent need such as hunger or paying the rent. Even so, people still seem to show inconsistencies in their choices over time: even when facing the same exact choice, people act impulsively in the short-term, but exhibit greater patience in the long term.

Strotz (1955-56) recognized that for any non exponential discount function a person would have time-inconsistent preferences. The literature has used a particularly simple functional form to describe hyperbolic discounting, introduced by Phelps and Pollak

(1968), to study intergenerational altruism, and applied to individual decision making, namely:

$$D(k) = \beta\delta^k, \text{ if } k > 0$$

$$D(k) = 1, \text{ if } k = 1$$

(Some authors proposed different hyperbolic discounting functions: Ainslie (1975) suggests the function: $D(t) = l/t$; Herrnstein (1981) and Mazur (1987) suggest $D(t) = 1/(1 + at)$. This functional assumes that the per-period discount rate between now and the next period is

$$\frac{1-\beta\delta}{\beta\delta}$$

while the per-period discount rate between any two future periods satisfies inequality:

$$\frac{1-\delta}{\delta} < \frac{1-\beta\delta}{\beta\delta}$$

The (β, α) formulation is highly tractable and captures many of the qualitative implications of hyperbolic discounting. Laibson (1996) used the (β, α) formulation to explore the implications of hyperbolic discounting for consumption-saving behavior.

Hyperbolic discounting has been applied to a wide range of phenomena lapses in willpower, hyperbolic phenomena, health outcomes, consumption choices over time, personal finance decisions. Of particular importance, the sumption for personal well-being, hyperbolic discounting has been linked to the problems of addiction and self-being, control.

Laibson (1996) has also used hyperbolic discounting to explain why people simultaneously have large credit-card debts at a high interest rate and pre-retirement wealth growing at a lower interest rate. As predicted by hyperbolic discounting, the rewards provided by buying something today often outweigh the discounted displeasure of future payments. This leads to sizeable credit card debt payments. However, when thinking about their retirement savings in the far future, people use a much smaller discount rate for delayed rewards.

This makes it more attractive to invest in alternatives providing a higher expected return in the long run. Consistent with hyperbolic discounting, people's investment behavior exhibits patience in the long run and impatience in the short run. The classical economic view of exponential discounting can cannot easily account for these personal saving decisions using a single constant discount rate.

Preference reversal. According to Sozou (1998), a plausible reason for the inconsistent behavior is the risk that a future reward will not be realized. The present value of a given reward when the reward is due after a delay of τ can be expressed as a time-preference function $v(\tau)$. If the agent is risk-neutral, this will satisfy

$$v(\tau) = v_0 s(\tau) \tag{1}$$

where v_0 is the value of an immediate risk free reward and $s(\tau)$ is a survival function specifying the probability that the reward can be realized after a delay of τ. Let's define $h(\tau)$ as the risk per unit time of the hazard occurring, given that it has not occurred before τ. The risk that a reward which is still available after a delay of τ is lost between a delay of τ and $\tau + d\tau$ is $h(\tau)dt$.

The absolute risk per unit time of the hazard occurring at a delay of τ is given by $-ds/d\tau$. The value $h(\tau)$ is obtained by dividing this absolute risk by the probability that the hazard has not already occurred before τ, i.e., dividing by the survival function $s(\tau)$:

$$h(\tau) = -\frac{1}{s}\frac{ds}{d\tau} \tag{2}$$

If the hazard rate h has a constant value λ for all τ, equation(2) yields

$$s(\tau) = \exp(-\lambda\tau)$$

And hence, from equation(1), $v(\tau) = v_0 \exp(-\lambda\tau)$.

In experiments, animals exhibit time preferences which are not exponential, but instead fall off with delay at a decreasing proportional rate.

Mazur (1987) employed an adjusting-delay titration procedure on pigeons, and found that a good empirical fit to the data is given by a hyperbolic function

$$v(\tau) = \frac{v_0}{1 + k\tau}$$

where k is a constant, with a larger value of k denoting more rapid discounting. The hyperbolic time- preference function becomes

$$s(\tau) = \frac{1}{1 + k\tau}$$

A survival function of this form, which falls off more slowly than the exponential, implies a hazard rate which falls with increasing delay (Green and Myerson, 1996). Non-exponential time-preference curves can cross (Strotz, 1955-1956) and consequently the preference for one future reward over another may change with time. This is illustrated in Fig. 1.

Some authors have interpreted this time-preference reversal effect as indicating non-rational time preferences (Ainslie, 1975; Strotz, 1955-1956): I may appear to be temporally inconsistent if, for example, I prefer the promise of a bottle of wine in 3 months over the promise of a cake in 2 months, but I prefer a cake immediately over a promise of a bottle of wine in one month. There is, however, no inconsistency if I perceive a promised future reward not as a sure thing, having a probability attached to it.

Time consistency with hyperbolic discounting. N. Dimitri (2005) proved that hyperbolic discounting, as well as exponential discounting, can formalize consistent preferences and it can be conveniently used to formalize dynamically inconsistent choices only for specific payoffs streams.

Consider a discrete three periods time horizon, $t = 0, 1, 2$ and an individual having to choose between payoff $x(1) > 0$, available at $t = 1$, and $x(2) > 0$ available at $t = 2$. According to Frederick, Loewenstein and O'Donoghue (2002), we can verify the decision maker's time consistency with respect to the two following payoff profiles,

$$a = (x'(0) = 0, x(1) > 0, x'(2) = 0); b = (x'(0) = 0, x'(1) = 0, x(2) > 0).$$

At $t = 0$, he would prefer $x(1)$ $(x(2))$, i. e. profile $a(b)$, if

$$x(1) \geq (<)[d(2)/d(1)]x(2) \tag{3}$$

and would still prefer $x(1)$ $(x(2))$ at $t = 1$ if

$$x(1) \geq (<)d(1)x(2) \tag{4}$$

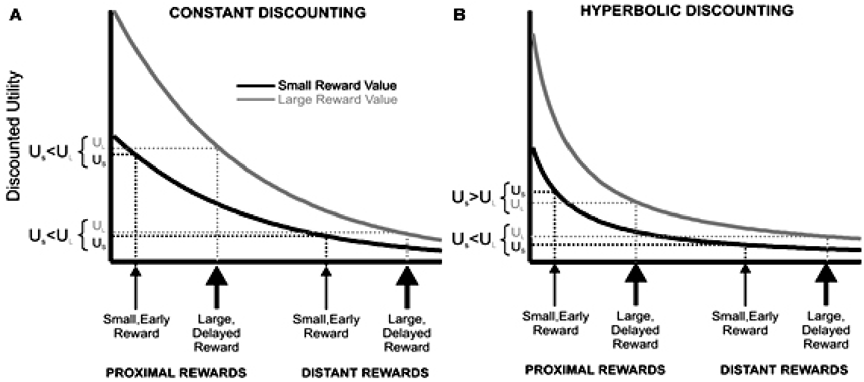

Figure 1. *Constant vs. hyperbolic discounting of future events.* The figure describes a choice between a small, short-term outcome or a large, long-term outcome (proximal), and another situation in which both outcomes are deferred into the future term by the same time interval (distant). (A) Constant (here: exponential) utility function of a large, delayed (gray line) and small, short-term commodity (black line). With exponential discounting, preference stationarity holds when the rewards are deferred term by the same time interval into the future. (B) People seem to place a premium on short-term availability of rewards, deflecting term the discount into an upward direction for temporally close rewards. The resulting hyperbolic discount f function can explain preference reversals over time. Due to the steeper utility decay for short delays, the utility of the small, short short-term commodity is higher than the large, delayed reward for temporally proximal outcomes, but the utility order reverses when both outcomes are deferred into the future (Kalenscher and van Wingerden, 2011).

Conventionally, if in both in cases (3) and (4) equalities hold, $x(1)$ should be preferred. When both (3) and (4) are satisfied the individual preferences are time consistent, since his decision would be the same independently of the time at which it's taken. To discuss time consistency it's convenient to reformulate the two inequalities as follows, where $x = [x(1)/x(2)]$

$$d(2) \leq (>) x d(1)$$

$$d(1) \leq (>) x$$

If $x \geq 1$, then time consistency is the case for all pairs $[0 \leq d(1) \leq 1, d(2) \leq \min(1, d(1)x)]$; consistency, however, can only concern $x(1)$ but not $x(2)$. Inconsistent choices realize when $d(1)$ and $d(2)$ are sufficiently different, with $d(2) > d(1)$. As x gets large, i.e., $x(1)$ is much higher than $x(2)$, choices tend to become dynamically consistent for almost all discounting functions. On the contrary, if the condition $d(1) \geq d(2)$ is imposed, then any discounting function can represent time consistent preferences. In general, if rewards are decreasing with time, then no inconsistency can arise (Fig. 2). Hence,

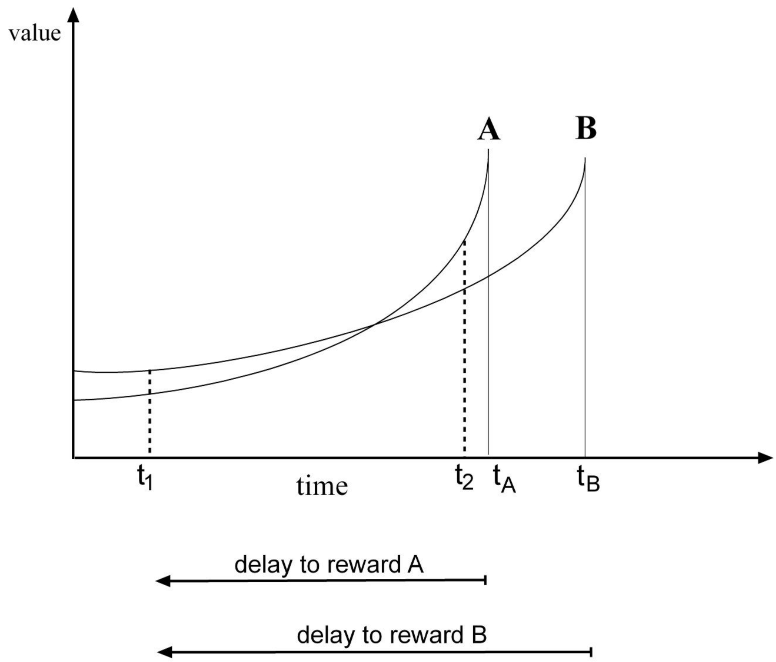

Figure 2. *The time-preference reversal effect.* The curves show how the value to a recipient of a reward A due at a time t_A and a larger reward B due at a later time t_B change over time. At a very early time t_1, when there is a long delay to both rewards, B is preferred over A. At a later time t_2, when reward A is imminent, it is preferred over B.

the study of time consistency is meaningful when $x(1) < x(2)$, $x < 1$, that is when the higher reward is available later in time. If $x < 1$ then time consistency realizes for all $[d(1) \leq x, d(2) \leq d(1)x]$ or $[d(1) > x, d(2) > d(1)x]$, namely as long as $d(1)$ and $d(2)$ are not too different. Moreover, quite intuitively, with $x(1) < x(2)$ "high values" of the discount function imply consistent preferences for $x(2)$ while "low values" for $x(1)$.

If $x(1) < x(2)$, then hyperbolic discounting can represent time consistent choices when the two rewards are either close to each other or else quite different. This suggests that the most common interpretation of hyperbolic discounting, as a model formalizing inconsistent choices, is correct but only for some rewards structures. Indeed, any non-exponential discounting model could represent time consistency preferences.

The study appears to suggest time consistency to depend upon:

- similarity-diversity of rewards available in time
- length of time horizon.

5. Neuroeconomics: Hyperbolic discounting explains impulsivity and inconsistency in investment and addictive behaviour

Rational choice theory maintains that all choice's problems must maximize the expected utility, which is the selective factor for all choices. Psychology looks at the process by which utility is realized, referring to it as reinforcement or reward.

There is a number of behavior patterns that violate the rational choice theory (Kahneman et al., 1982; Thaler, 1991).

The greatest contradiction to this theory is inconsistent preference, usually manifested as temporary preference for options that are extremely costly or harmful in the long run. This behavior can be typically seen in alcoholism, drug abuse, gambling and other psychiatric disorders, but also in more ordinary phenomena such as overeating, credit card debt, overconsumption of passive entertainment and other bad habits too popular to be diagnosed as pathological. Addiction is an intertemporal consumption phenomena, when the current consumption of a commodity cause an increase in the future consumption of the same commodity.

Elevated delay discounting characterizes impulsivity associated with harmful behaviors: loss of self- control, failure in formerly-planned abstinence from addictive substances and relapse, a dead-line rush due to procrastination, failure in saving enough before retirement and risky sexual behavior. Addiction and financial mismanagement frequently co-occur, and elevated delay discounting may be a common mechanism contributing to both of these problematic behaviors.

Neuroeconomics is a multidisciplinary approach to the study of how economic behavior is related to neuronal processes and structures. The hyperbolic discount model can explain the widely- observed tendency of human and animal intertemporal choice, i.e., decreasing impatience. In particular, it has been observed that addicted populations are more myopic (have higher k values, that is a constant describing the individual subject's degree of impatience) than non-addicted populations (Ainslie, 1975; Bickel, et al. 1999).

A heterogeneous group of substance dependent subjects discounted more steeply than controls (Ainslie and Haendel, 1983); heavy social drinkers and problem drinkers both discounted delayed rewards more steeply than did light drinkers (Vuchinich and Simpson, 1998); smokers discounted the future more steeply than non-smokers (Bickel et al., 1999); and opioid dependent patients discounted money more steeply than controls.

It is to be noted that the preference for more immediate rewards per se is not irrational or inconsistent, because there are opportunity costs and risk associated with non-gaining in delaying the rewards. Therefore, impulsivity in intertemporal choice is rationalizable for several kinds of people.

In (Becker et al., 1988) addicts are supposed to have large discount rates, leading to ignoring future delayed health loss and preferring immediate euphoria obtained from drug intake, in a completely consistent manner. This behavior is clinically problematic, but economically rational when their choices are time-consistent (if they have large discount rates with an exponential discount function). However, it is known that addicts also discount delayed outcomes hyperbolically, suggesting the intertemporal choices of addicts are time-inconsistent, resulting in a loss of self-control (Bickel et al., 1999) even if an agent had made patient and forward-looking plans regarding the distant future, as the time of executing the plan approaches the present she will cancel the patient plan and act more impulsively at the moment of the choice, against his/her own previously-intended plan. There is a discrepancy between the decision-maker's intentions and behavior, indicating that most people cannot act as they planned in advance (Ainslie, 2005; Frederick et al., 2002; Bickel et al., 1999; Takahashi et al., 2007). The next question is whether these substance abusers and addicts were originally impulsive in intertemporal choice or have become impulsive due to the neuropsychopharmacological effects of habitual drug intake. Recent studies have examined the stability of addicts' discount rates over time after abstinence. If large discount rates are due to habitual drug intake, it is expected that discount rates decreased after long-term abstinence. However, it has recently been reported that for alcoholics and smokers, abstinence did not dramatically reduce discount rates of former alcoholics and smokers (Takahashi, 2010).

In order to describe human and animal subject's intertemporal choice behaviors in a manner which we can dissociate impulsivity and inconsistency, recent econophysical studies (Takahashi et al., 2007) have proposed and examined the following q-exponential discount function for subjective value V(D) of delayed reward:

$$V(D) = A/\exp_q(k_q D) = A/[1 + (1-q)k_q D]^{\frac{1}{1-q}}$$

where D denotes a delay until receipt of a reward, A the value of a reward at $D = 0$, and k_q a parameter of impulsivity at delay $D = 0$ (q-exponential discount rate) and the q-exponential function is defined as:

$$\exp_q(x) = (1 + (1-q))^{\frac{1}{(1-q)}}$$

The important point here is that the q-exponential discount model can distinctly parametrize impulsivity and dynamic consistency in intertemporal choice. Conventional models of temporal discounting (i.e., exponential and hyperbolic) cannot achieve this result.

Takahashi et al. (2007) have shown that human agents with smaller q values are more inconsistent in intertemporal choice. If $q < 0$, the intertemporal choice behavior is more inconsistent than hyperbolic discounting (the discount rate of the q-exponential function decreases more rapidly than that of the simple hyperbolic discount function). Behavioral economists have proposed that the inconsistency in intertemporal choice may be attributable to an internal conflict between "multiple selves" within a decision maker. The existence of the feeling of regret may be a strong evidence of human behavior's inconsistency and internal conflict between desires inconsistent with each other. This hypothesis states that (a) there are (at least) two exponential discounting selves (i.e., two exponential discount rates) in a single human individual and (b) when delayed rewards are at the distant future (> 1 year), the self with a smaller discount rate wins; while delayed rewards approach to the near future (within a year), the self with a larger discount rate wins, resulting in preference reversal over time (Laibson, 1997). This intertemporal choice behavior has been referred to as quasi-hyperbolic discounting (also as a $\beta - \delta$ model). In the discrete time, the quasi-hyperbolic discounting $F(\tau)$ for discrete time τ (the unit has been assumed to be one year) is defined as (Laibson, 1997):

$$F(\tau) = \beta\delta^{\tau} (\text{for } \tau = 1, 2, 3, \dots) \text{ and } F(0) = 1 (0 < \beta < \delta < 1)$$

The discount factor between the present and one-time period later β is smaller than that between two future time-periods (δ). So, people are patient in planning their intertemporal choice in the distant future, but impulsive in intertemporal choice action occurring at delay $D = 0$. A recent neuroeconomic study on temporal discounting for primary rewards in thirsty subjects utilizing functional magnetic resonance neuroimaging (McClure et al., 2007), has proposed a generalized quasi- hyperbolic discount model in which "dual selves" are linearly weighted at each delay. In the continuous time, the proposed model is equivalent to the linearly-weighted two-exponential functions (generalized quasi-hyperbolic discounting):

$$V(D) = A[w \exp(-k_1 D) + (1-w)\exp(-k_2 D)]$$

where w, $0 < w < 1$, is a weighting parameter and k_1 and k_2 are two exponential discount rates $(k_1 < k_2)$. Note that the larger exponential discount rate of the two k_2, corresponds to an impulsive self, while the smaller discount rate k_1 corresponds to a patient self. In the q-exponential discount model, the discount rate is defined as:

$$(q\text{-exponential discount rate}) = k_q/(1 + k_q(1-q)D)$$

When $q = 1$, the discount rate is independent of delay D, corresponding to exponential discount model (consistent intertemporal choice); while for $q < 1$, the discount rate is a decreasing function of delay D, resulting in preference reversal. This can be seen by a direct calculation of the time-derivative of the q-exponential discount rate:

$$(d/dD)(q\text{-exponential discount rate}) = -k_q^2(1-q)/(k_q(1-q)D + 1)^2$$

which is negative for $q < 1$, indicating "decreasing impatience" for q smaller than 1. Also, impulsivity at delay $D = 0$ is equal to k_q irrespective of q. Therefore, k_q and q can parametrize impulsivity and consistency, respectively, in a distinct manner. Notably, when q is negative, the speed of a decrease in the q-exponential discount rate is faster than the hyperbolic discount rate (i.e., "*hyper-hyperbolic*").

For the generalized quasi-hyperbolic discount model, the discount rate is:

$$\left[\frac{k_2(1-w)\exp(-k_2 D) + k_1 w \exp(-k_1 D)}{(1-w)\exp(-k_2 D) + w\exp(-k_1 D)}\right]$$

It is to be noted that at delay $D = 0$, the generalized quasi-hyperbolic discount rate $wk_1 + (1-w)k_2$. This indicates that impulsivity in an intertemporal choice action (at delay $D = 0$) corresponds to linearly- weighted discount rates at delay D. The time-derivative of the discount rate, i.e., (d/dD) (a generalized quasi-hyperbolic discount rate) is:

$$\frac{-w(1-w)(k_2-k_1)^2\exp(k_2 D + k_1 D)}{w\exp(k_2 D) + (1-w)\exp(k_1 D)}$$

which is negative because $1 - w > 0$. This also indicates that the discount rate is a decreasing function of delay, again indicating decreasing impatience. Neuropsychologically, the weighted difference between discount rates $k_2 > k_1$:

$$(1-w)k_2 - wk_1$$

may indicate the strength of "*internal conflict*" between impulsive and patient selves in intertemporal choice, and this can be regarded as an internal conflict parameter. Recent behavioral, neuroeconomic, and neuropharmacological studies collectively stress the importance of time-perception in intertemporal choice. Takahashi (2007) has proposed the Exponential Discounting with Logarithmic Time-Perception:

$$\tau(D) = \alpha \log(1 + \beta D)$$

it may explain dynamic inconsistency in intertemporal choice. If a subject discounts a delayed reward exponentially, but with the logarithmic time-perception, his temporal discount function has a hyperbolic form:

$$F(\tau) = \exp(-k\tau) = 1/(1 + \beta D)^{k\alpha}$$

Subjects try to discount a delayed reward exponentially (i.e., rationally and consistently), but actual intertemporal choice behavior may be hyperbolic and dynamically inconsistent, due to a distortion in time-perception. It is to be noted that the exponential discount model with logarithmic time-perception is mathematically equivalent to the q-exponential discount model based on Tsallis' statistics. This hipotesis is supported by neuroimaging studies, which observed that when subjects make intertemporal choices, brain regions for time-perception such as the caudate nucleus (a type of dopamine systems) are activated in association with the delay length; subjects with large discount rates, for example, substance abusers have overestimated time-perception; furthermore, behavioral economists have reported that if the time of receiving a delayed reward is presented in the form of a calendar date (instead of time durations of delay length until receipt) the functional form of their temporal discounting becomes exponential, rather than hyperbolic. This "*delay/date effect*" can be explained by considering that the presentation of a calendar date may reduce the non-linearity of the perception of delay length.

6. Impulsivity, self control and precommitment, the Strotz model

In many cases a dynamic inconsistent behavior is attributed to the existence of contingent "temptations" that increase impulsivity and induce a deviation from the desirable behavior. Our modern societies provide ever-increasing opportunities for impulse spending: developments in technology as cash machines, shop at home television programs, internet shopping, drive to act immediately and buy around the clock highly difficult to resist.

Sociologists and psychologists have persistently studied impulsivity relative to its resultant behaviors such as drug addiction, suicide, aggression and violence. Murray (1938) defined impulsivity as "an inclination to react swiftly without reflection". Oas (1985) defined impulsive behavior as "socially inappropriate or maladaptive", and as being "emitted quickly and without forethought". This definition suggests that individuals who frequently engage in impulsive behavior may fail to evaluate the consequences of their behavior appropriately. The discounting hypothesis of impulsivity suggests that the degree of discounting due to delay is a measure of an individual's impulsivity.

A number of mechanism of self-control are predicted by hyperbolic discounting. Strotz (1956) proposed two strategies that might be employed by a person who foresees how her preferences will change over time: the *strategy of precommitment* (when she commits to some plan of action) and the "strategy of consistent planning" (when she chooses her behavior ignoring plans that she knows her future selves will not carry out). He starts his article quoting the Odissey and describing Ulysses' problem as one of changing tastes, or time varying preferences, i.e., if at time t the individual reconsiders a plan formulated at time $t_1 < t$, he will change the plan. The solution Ulysses adopts is that his crew must tie him to the mast. Strotz refers to this type of solution as precommitment. He hypothesizes a non-exponential discount function which produces dynamic inconsistency. Consider a consumer with an initial endowment K_0, of consumer goods which has to be allocated over the finite interval $(0, T)$. At time period t he wishes to maximize his utility function:

$$J_0 = \int_0^T \lambda(t-0) U[\bar{c}(t), t] dt$$

where $[\bar{c}(t), t]$ is the instantaneous rate of consumption at time period t, and e $\lambda(t-0)$ is a discount factor, the value of which depends upon the elapse of time between a past or future date and present. The maximization of the function over the interval $(0, T)$, denoted by $[0, c(t), T]$, is subject to the budget constraint:

$$\int_0^T c(t) dt = K_0$$

And this implies that the discounted marginal utility of consumption should be the same for all periods. The resulting path of consumption is optimal from the point of view of $\tau = 0$, but, at a later date, the consumer may reconsider his consumption plan. The problem then is to maximize

$$\int_0^T \lambda(t-\tau) \cdot U[(c(t), t] dt$$

subject to $[0, c(t), \tau]$, already given as:

$$\int_0^T c(t) dt = K_1 = K_0 \int_0^\tau c(t) dt$$

The optimal pattern of consumption will change with changes in τ.

An important issue concerns the conditions under which the optimal plan at time $\tau > 0$ will be continuation of the plan formulated at $\tau = 0$. It is required that if $[0, c^*(t), \tau']$ and $[\tau', c(t), T]$ maximize:

$$\int_0^{\tau'} \lambda(t-0) U[c(t), t] dt + \int_{\tau'}^T \lambda(t-0) U[c(t), t] dt$$

subject to

$$\int_0^{\tau'} c(t) dt + \int_{\tau'}^T c(t) dt = K_0$$

then $[\tau', c(t), T]$ should maximize

$$\int_\tau^T \lambda(t-\tau') U[c(t), t]$$

subject to

$$\int_\tau^T c(t) dt = K_0 - \int_0^T c^*(t) dt$$

A necessary and sufficient condition for the properties above to be equal is that all future dates are discounted at a constant rate, i.e., $c(t) = kt$.

A conclusion by Strotz is that the individual will not alter the original plan if $\lambda(t, \tau)$ is exponential in $|t - \tau|$; otherwise he will. If the original plan is altered, then the individual is said to display dynamic inconsistency. Since precommitment is not always a feasible solution to the problem of intertemporal conflict, the man may adopt a different strategy and reject any plan which he will not follow through. His problem is then to find the best plan among those that he will actually follow i.e., the strategy of the consistent planning.

But while the change of preferences hypothesized by Strotz is necessary for precommitment to be rational, it is not sufficient, we must add another condition which is implicit in Strotz reasoning: i.e., that the earlier preferences are judged in some sense to be "right".

7. An economic theory of self-control

Conflict arises because the individual recognizes his own weaknesses. Plans made in advance are consistently broken because temptation becomes too great. What the person knows to be his best long run interests conflict with his short run desires. Shefrin and Thaler (1978) model incoherent purpose by treating an individual as if he contained two distinct psyches which we will denote the planner and the doer: the current doer's preferences are always myopic relative to the planner's because the doer is concerned only with short-term satisfactions, while the planner pursue longer-run results. This conflict creates a control problem of the same variety as those present in any organization with a principal-agent problem, so, individuals may adopt many of the same strategies for solving these problems. Consider an individual with a fixed income stream $y = [y_1, y_2, ...y_T]$. He is assumed to choose a nonnegative level of consumption c_t in period t; call $c = [c_1, c_2, ...c_T]$ a consumption plan. The individual, just like an organization, consists of $T + l$ components: T distinct doers (one for each period) and a single planner. The period t doer is assumed to exercise direct control over the period t consumption level c_t and his utility function is $z_t(\cdot)$. The value $z_t(c_t)$ denotes the degree of immediate or short-term satisfaction. Strotz recognized some of these features, in fact he says "The individual over time is an infinity of individuals". But, unlike Thaler and Shefrin, Strotz considered an individual to be a system of doers with no planner, and each Strotz doer has some concern for the other doers while their doers are completely selfish. The present value budget constraint is

$$\sum_t c(t) \leq \sum_t y_t = Y \tag{5}$$

This implies the existence of a perfect capital market. The multiplicity of drive mechanisms $[Z_1, Z_2, \ldots, Z_T]$ are in mutual conflict as a result of (5). The planner's preferences are represented by a utility function $V(Z_1, Z_2, \ldots, Z_T)$. If the individual were fully integrated, then the planner would choose a consumption plan to maximize $V(Z_1, Z_2, \ldots, Z_T)$, subject to

$$\sum_{t=1}^{t} c_t \leq Y$$

where t is oriented towards achieving maximum short-term satisfaction Z_t, not longer-run gain V. In fact, an unrestrained doer 1 would borrow $Y - y_1$ on the capital market and therefore choose $c_1 = Y$; the resulting consequence is naturally $c_2 = c_3 = \cdots = c_T = 0$. Such action would suggest a complete absence of psychic integration. What can the planner do to exert some control over the doers? In general he has two instruments he can use. First, he can impose rules on the doers behavior, for example, he could purchase an annuity which allocates each doer a specific consumption level, or he could simply forbid borrowing. The second instrument available to the planner is to alter the doers utility function directly introducing a modification parameter $\theta = \theta_1, \theta_2, \ldots, \theta_T$. Z is assumed to be a function of two arguments, c_t and If $\theta_T = 0$, then the doer is completely unrestrained. As θ_t increases, both Z and $(\delta Z_t)/(\delta c_t)$ are reduced. θ might be thought of as a guilt parameter. The higher is θ_t, the more guilt the doer feels for any level of c_t.

Define $c_t^*(\theta_t)$ to be the consumption level where t chooses to maximize $Z_t(c_t, \theta_t)$ when the planner picks θ_t. If sufficient modification has taken place so that has an internal maximum, then $c_t^* < Y_t = Y - \sum_{s<t} c_s^*$. We can now write down the planner's problem in the discretionary mode. Let $Z(c^*(\theta), \theta) = [Z_1(c_1^*(\theta_1), \theta_1) \ldots Z_T^*(c_T^*(\theta_T), \theta_T)]$. Then the planner wants to solve

$$\max_{\theta} V(Z(c^*(\theta), \theta)$$

subject to

$$\sum_{t=1}^{T} c_t^*(\theta) \leq Y$$

In this case the planner will increase θ_t until the marginal loss to the planner from the resulting decrease in doer t's utility is equal to the marginal gain to the planner from the increases in utility to all future doers. Both the gain and the loss have two components. Doer t is worse off from a rise in θ_t because he consumes less and because he enjoys each unit of consumption less. Similarly future doers gain both because there is more income remaining and because less future modification will be employed.

To explain the planner-doer theory, Shefrin and Thaler show the case of a bank that is run by an owner-manager. One of the functions which the owner serves is that of loan officer. He determines which applicants should be granted loans. Two kinds of procedures are used. The applicant fills out a report which predicts the probability of default. This rule can also be supplemented by the judgment of the owner based on a personal interview. It could permit to gather further information but it is more costly to conduct, so, the owner will utilize it only if sufficient extra profits will cover the extra costs. Now let's assume that the owner hires an employee to process loan applications. The employee is as skillful as the owner in judging loan applications, but he is not motivated to make interviews because he doesn't keep the profits, so he may become careless, lazy or even dishonest. This will create an incentive to adopt rules to reduce the opportunities for the employee to misbehave.

The two basic instruments the planner can use are rules and discretion. Discretion must be accompanied by some method of altering the incentives or rewards to the doer without any self-imposed constraints. Rules operate by altering the constraints imposed on any given doer. Pure rules, like precommitment, can be a very effective self-control strategy because they eliminate all choice. The advantage of these strategies is that once in place they require little or no self enforcement. However, pure rules require external help which may be unavailable or too expensive.

Between these extremes there are other intermediate strategies, called internal rules, that generally are self-enforced rules-of-thumb, rather than externally enforced precommitments. They have some desirable characteristics with respect to the pure rules: simplicity, flexibility, dynamic stability, reasonableness, lower monetary costs. We can identify some likely rules-of-thumb between pure discretion and a pure rule, for example, introducing a ban on borrowing

$$\left(S_t = \sum_{t=1}^{t-1} (y_t - c_t) \right)$$

where y_t is current income, Y_t is the present value of remaining future income and S_t as the accumulated saving up to period t. On a no-borrowing regime the budget constraint is simply $c_t \leq y_t + S_t$. If borrowing is banned then the budget constraint becomes $c_t \leq y_t$. If borrowing is permitted up to some level B, then the constraint becomes $c_t \leq y_t + B$. Each of the above rules may be combined with a savings plan. For example, saving at least s percent of income in each period, with the discretion to save more but not less, the budget constraint becomes $c_t \leq (1 - s)y_t$ until retirement. Since the plan can be stopped at any time and the savings are very liquid it is obviously an internal rule.

Seductive goods will require opportunities manipulations. This implies that ct should be considered a vector, so we denote the level of good i consumed in period t by c_{it}. If θ is also good specific then we

$$\frac{-\delta c^*_{it}}{\delta \theta_{it}}$$

as a measure of how seductive good i is. If a good is highly seductive then in utility terms discretion will be very costly. The best alternative may be to avoid the good altogether. Without outright prohibition, variations in

$$\frac{\delta c^*_{it}}{\delta \theta_{it}}$$

imply that the individual will appear more impatient with goods that are more seductive and the implicit discount functions for specific goods will be dynamically unstable, just like for the addictive or habit-forming goods. A possible measure of the degree to which a good is seductive/addictive would then be

$$\frac{\delta}{\delta c_{it}} \left[\frac{\delta c^*_{it+1}}{\delta \theta_{it+1}} \right]$$

Addictive goods present a special problem in self-control, The current doer receives all the benefits of consuming an addictive good while the costs, in terms of future attempts to control behavior and harmful side-effects are all imposed on future doers. Pure rules may be an successful strategy and for those who find the good initially seductive.

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